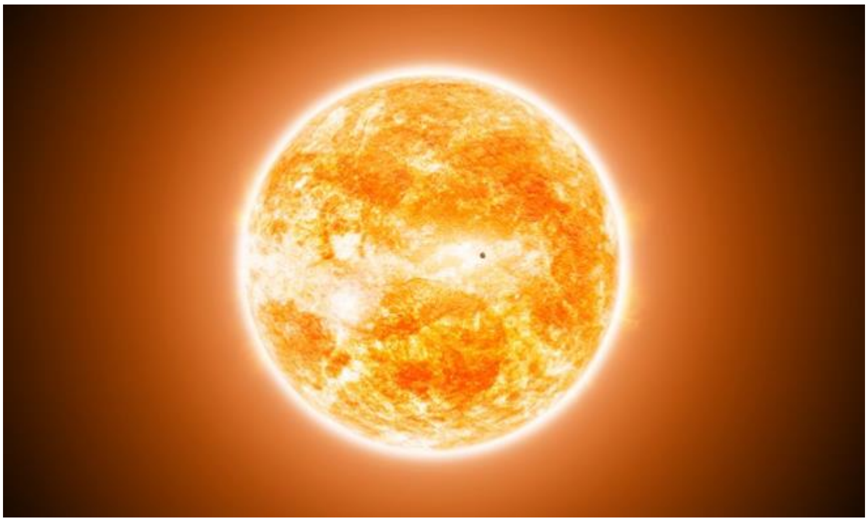
H2 Topic 20 – Nuclear Physics



The Sun is powered by nuclear fusion in its core. The core converts approximately 10^{38} protons/second into helium at a temperature of 14 million K. This process releases energy in the form of photons, neutrinos, and other particles.
(*EIT - Extreme ultraviolet Imaging Telescope Consortium, The Solar and Heliospheric Observatory, NASA*)

Content

- The nucleus
- Isotopes
- Nuclear processes
- Mass defect and nuclear binding energy
- Radioactive decay
- Biological effects of radiation

Learning Outcomes
Candidates should be able to:

(a) infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the atomic nucleus
(b) distinguish between nucleon number (mass number) and proton number (atomic number)
(c) show an understanding that an element can exist in various isotopic forms each with a different number of neutrons in the nucleus
(d) use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$
(e) state and apply to problem solving the concept that nucleon number, charge and mass-energy are all conserved in nuclear processes
(f) show an understanding of the concept of mass defect
(g) recall and apply the equivalence relationship between energy and mass as represented by $E = mc^2$ to solve problems
(h) show an understanding of the concept of nuclear binding energy and its relation to mass defect
(i) sketch the variation of binding energy per nucleon with nucleon number
(j) explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission
(k) show an understanding of the spontaneous and random nature of nuclear decay
(l) infer the random nature of radioactive decay from the fluctuations in count rate
(m) show an understanding of the origin and significance of background radiation
(n) show an understanding of the nature of α, β and γ radiations (knowledge of positron emission is not required)
(o) show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino (knowledge of antineutrino and antiparticles is not required)
(p) define the terms activity and decay constant and recall and solve problems using the equation $A = \lambda N$
(q) infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_0 \exp(-\lambda t)$ where x could represent activity, number of undecayed particles or received count rate
(r) define and use half-life as the time taken for quantity x to reduce to half its initial value
(s) solve problems using the relation $\lambda = \dfrac{\ln 2}{t_{1/2}}$
(t) discuss qualitatively the effects, both direct and indirect, of ionising radiation on living tissues and cells.

20.0 Introduction

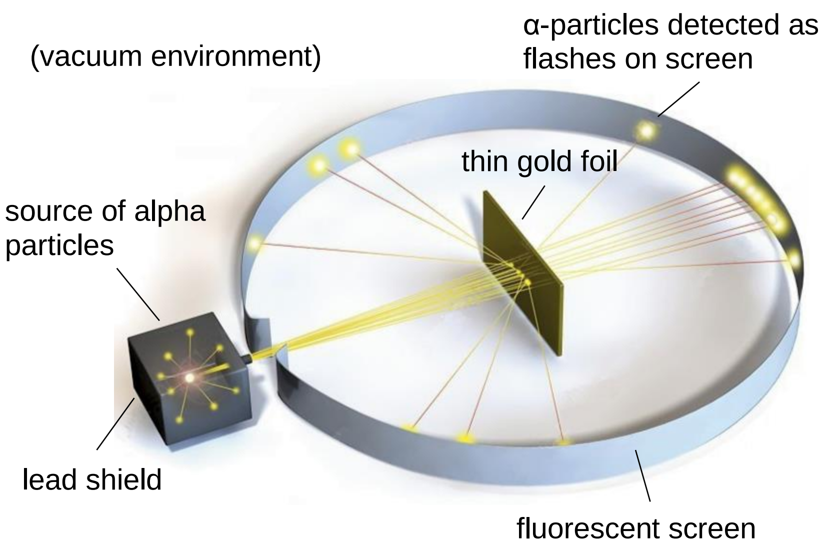
In our studies of Physics at the microscopic dimensions so far, we have remained at the atomic level, with the atomic electrons being the main participants in interactions. The nucleus, on the other hand, has been assumed to be dormant.

This chapter on Nuclear Physics offers us a first look into the inner workings of a nucleus, and the interaction between nuclei. We will first look at the reactions between nuclei, and the changes in mass or energy involved. The spontaneous disintegration of a nucleus, which leads to the phenomena of radioactivity, will be discussed next.

20.1 The Rutherford α-Scattering Experiment

Atoms, the basic building blocks of matter, were once thought to be the smallest indivisible particle. However, with his discovery of the electrons in 1897, J.J. Thomson concluded that electrons are part of an atom. He further postulated that these very light and negatively charged electrons were distributed throughout a uniform sea of positive charges in an atom, much like the way that plums are evenly distributed in a pudding. He thus named this model as the 'plum-pudding' model.

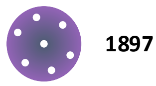
In 1909, under the direction of Ernest Rutherford, Hans Geiger and Ernest Marsden investigated the structure of an atom. As shown in the figure below, a beam of α-particles ($^{4}_{2}\text{He}$) having an energy of 7.7 MeV is emitted by the decay of radium, and is directed towards a thin gold foil. [Gold was chosen because the element is stable (will not undergo radioactive decay with collisions), inert (will not *chemically* react with the alpha particles), and as a metal is malleable as well as ductile (so that it can be made to fewest possible layers of atoms).] The deflected α-particles were detected as flashes on the fluorescent screen. To minimise the scattering of the α-particles by air molecules, their experimental set up was enclosed in vacuum.



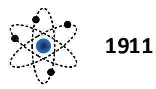
Based on the 'plum-pudding' model of the atom, Rutherford and his collaborators had expected the α-particles to be deflected by only a very small angle (~1° at most). The experimental results, as summarized in the table below, however, were much to their surprise.

Models of Atom

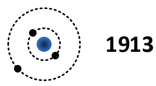
Indivisible Atom
An atom was thought to be the smallest component of matter and could not be further broken down into smaller constituents.



Plum Pudding Atom
Atoms composed of the negatively charged electrons evenly distributed within a cloud of positive charges



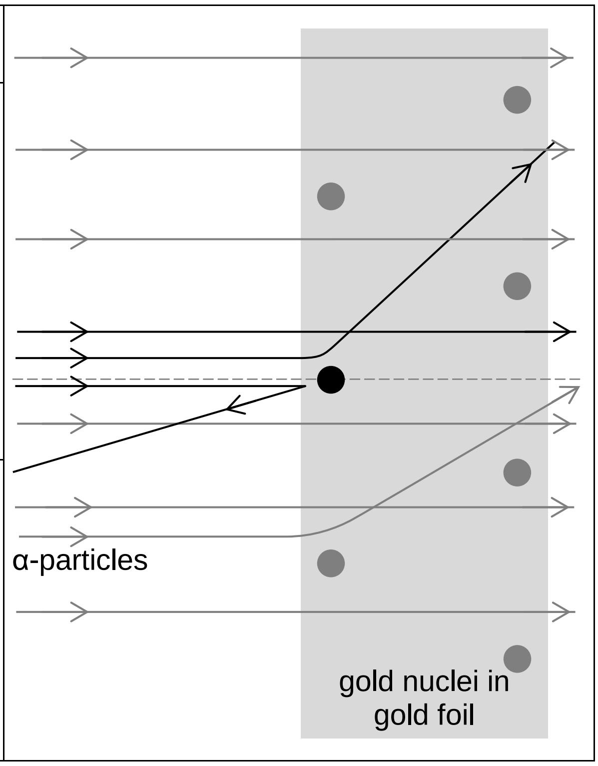
Planetary Model
Atoms consist of negatively charged electrons orbiting around a very small, dense, positively charged nucleus.



Bohr Model
Similar to Rutherford's model, but electrons are in stationary states, hence do not radiate EM energy.

Quantum Model
Electrons are distributed in space described by a probability density function around a positively charged nucleus. The greatest probability of locating the electron is at the densest region.

Experimental Observations	Deductions
Majority of α-particles went straight through or were deviated by small angles of less than 10°	- most of atom is empty space, mass of atom is concentrated in a very small nucleus. (size of atom ~ 10^{-10} m, size of nucleus ~ 10^{-15} m) - nucleus is positively charged, majority of α-particles pass through atom far enough from nucleus to experience negligible electrostatic repulsion
a small proportion (about 1 in 8000) of the α-particles deflected through large angles of more than 90° or came straight back	- a small number of α-particles that come close enough to the positively charged nucleus experienced significant electrostatic repulsion and are deflected through large angles



Note that in Physics, to *scatter* means to change in the direction of motion of a particle due to a collision with another particle. The "collision" need not involve direct contact between the particles, e.g. in this case the collision is due to electrostatic repulsion at a distance. The closer the bombarding alpha particle is to the gold nucleus, the larger the angle of deflection.

The Rutherford alpha scattering experiment established the presence of a massive, positively charged nucleus that occupies a tiny fraction of the volume of an atom.

20.2 The Nucleus

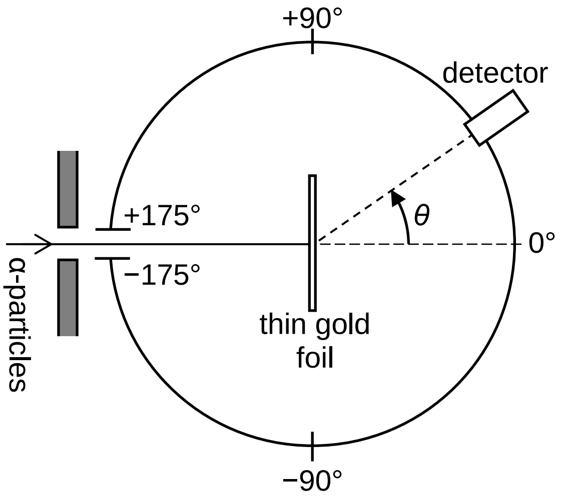
After the discovery of the nucleus of an atom, Rutherford turned his attention to the scattering of α-particles on lighter nuclei. By 1917, Rutherford had succeeded in producing the hydrogen nuclei by bombarding hydrogen gas with α-particles. When he repeated the experiment with pure nitrogen gas, hydrogen nuclei were also detected. He concluded that the hydrogen nucleus (hydrogen being the simplest and lightest element), is the building block of the nucleus. Rutherford named this particle the **proton**.

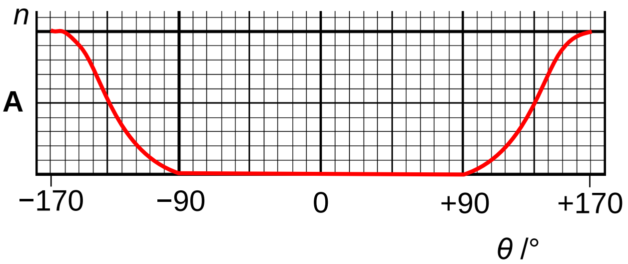
However, the disparity between the atomic number of an atom and its atomic mass led Rutherford to hypothesize the existence of a neutral particle in the nucleus. In 1931, physicists detected an extremely penetrating radiation produced when very energetic α-particles bombarded the nuclei of beryllium, boron and lithium. This radiation was initially thought to be gamma radiation. In 1932, James Chadwick, a former student of Rutherford, performed a series of experiments that conclusively dismissed the gamma radiation hypothesis. Instead, he found that the new radiation consisted of neutral particles which had approximately the same mass as the protons. Chadwick named this particle the **neutron**.

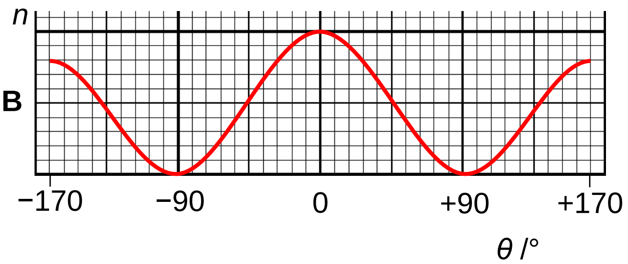
It is now an established fact that the nucleus of an atom consists of protons and neutrons, collectively known as **nucleons**. The following table summarizes the properties of the common subatomic particles.

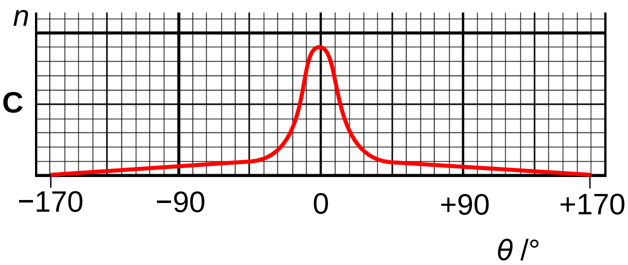
Properties	**proton**	**neutron**	**electron**
charge/ C	1.60×10^{-19}	0	-1.60×10^{-19}
rest mass/ kg	1.67×10^{-27}	1.67×10^{-27}	9.11×10^{-31}

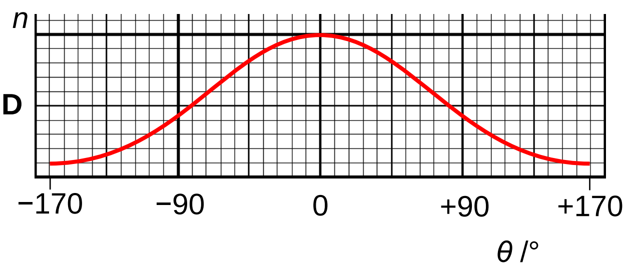
Example 1
In a repeat of the Rutherford's α-scattering experiment, the set-up shown below is used to measure *n*, the number of α-particles incident per unit time on a detector held at various angular positions *θ*. Which graph best represents the variations of *n* with *θ*?



A


B


C


D


Solution:

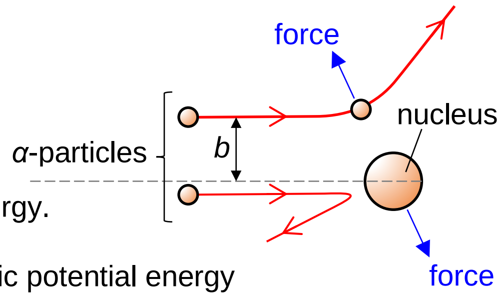
Majority of alpha particles go straight through,
so eliminate **A**

Only a small proportion of alpha particles are deflected through large angles of more than 90° so eliminate **B**

Majority of alpha particles are deviated by angles of less than 10°
so eliminate **D**

Answer is **C**

Example 2
In the Rutherford's α-scattering experiment, the angle of deflection of the α-particle is dependent on the impact parameter *b*, which is illustrated for one of the α-particles in the diagram below. The electrostatic repulsion between a nucleus and an incoming α-particle is the primary interaction that causes the deflection.
(a) State the value of *b* for which an α-particle will be at the distance of closest approach to a nucleus.
(b) Find the distance of closest approach for an α-particle of energy 7.7 MeV.
(c) Suggest why the gold foil must be made very thin.



Solution:
(a) $b = 0$ (head-on collision) [Can you explain why?]

(b) At distance of closest approach, the kinetic energy of the α-particle is entirely converted into electric potential energy.
By Principle of Conservation of Energy,

loss in kinetic energy of alpha particle = gain in electric potential energy

$$E_{\text{K, initial}} - E_{\text{K, final}} = E_{\text{P, final}} - E_{\text{P, initial}} \Rightarrow E_{\text{K, initial}} - 0 = E_{\text{P, final}} - 0$$

$$\Rightarrow E_{\text{K, initial}} = \frac{q_\alpha q_{\text{Au}}}{4\pi\varepsilon_0 d} = \frac{(2e)(79e)}{4\pi\varepsilon_0 d}$$

$$(7.7\times10^6)\left(1.6\times10^{-19}\right) = \frac{\left(2\left(1.6\times10^{-19}\right)\right)\left(79\left(1.6\times10^{-19}\right)\right)}{4\pi\left(\frac{1}{36\pi}\times10^{-9}\right)d}$$

$$d = 2.95\times10^{-14}\ \text{m}$$

(c) To reduce the possibility of multiple deflections of alpha particles with gold nuclei, so that only single interactions between them can be studied.

Note: The distance of closest approach forms the upper-bound for the estimated radius of a gold nucleus.

Example 3
Studies reveal an atomic nucleus is roughly spherical, of constant density, and its radius given by

$$R = \left(1.25\times10^{-15}\right)\sqrt[3]{A}$$

where *A* is the nucleon number, the total number of nucleons in a nucleus (also called mass number). The expression above is valid for "heavier" nuclides with *A* > 20. Estimate nuclear density.

Solution:

$$\rho = \frac{m_{\text{total}}}{V_{\text{sphere}}} = \frac{Au}{\frac{4}{3}\pi R^3}$$

$$= \frac{3Au}{4\pi\left[\left(1.25\times10^{-15}\right)^3 A\right]} = \frac{3\left(1.66\times10^{-27}\right)}{4\pi\left(1.25\times10^{-15}\right)^3}$$

$$= 2.03\times10^{17}\ \text{kg m}^{-3}$$

Note: In contrast, the densest of metals that are naturally occurring on Earth have densities of approximately $22\ \text{kg m}^{-3}$, 16 orders of magnitude less dense than the nucleus.

20.3 Notations

The nucleus of any atom can be represented using the notation ${}^{A}_{Z}\mathrm{X}$, where X is the chemical symbol, *A* is the nucleon number or mass number, and *Z* is the atomic number or proton number. The number of neutrons in the nucleus ${}^{A}_{Z}\mathrm{X}$ is given by (*A* – *Z*). For example, a uranium-235 (${}^{235}_{92}\mathrm{U}$) nucleus contains 235 nucleons (*A* = 235), among them 92 protons (*Z* = 92), and 143 neutrons (*A* – *Z* = 235 – 92 = 143). Uranium-238 (${}^{238}_{92}\mathrm{U}$), an isotope of uranium, contains 238 nucleons (*A* = 238), among them 92 protons (*Z* = 92) and 146 neutrons (A – Z = 146). Notice that ${}^{235}_{92}\mathrm{U}$ and ${}^{238}_{92}\mathrm{U}$ have the same number of protons (hence the same chemical symbol), but different number of neutrons. These nuclides are called isotopes. Carbon-12 (${}^{12}_{6}\mathrm{C}$) and carbon-13 (${}^{13}_{6}\mathrm{C}$) is another example of isotopes.

name	symbol	meaning and example
nuclei	(nil)	plural form of *nucleus*
nuclide	${}^{A}_{Z}\mathrm{X}$	a specific combination of protons and neutrons in a nucleus
nucleon	(nil)	a proton or a neutron in a nucleus
nucleon number (mass number)	A	total number of protons and neutrons in a nucleus
proton number (atomic number)	Z	number of protons in a nucleus
neutron number	$N = A - Z$	number of neutrons in a nucleus
isotopes	${}^{A_1}_{Z}\mathrm{X}$, ${}^{A_2}_{Z}\mathrm{X}$	nuclei of the same element containing the same number of protons but different number of neutrons (hence different mass number A_1 and A_2)
unified atomic mass constant	u	1u is one-twelfth of the mass of a carbon-12 atom

The symbols for particles commonly involved in nuclear processes are listed in the table below.

name	proton	neutron	electron	antineutrino	α-particle
symbol	${}^{1}_{1}\mathrm{H}$ or ${}^{1}_{1}\mathrm{p}$	${}^{1}_{0}\mathrm{n}$	${}^{0}_{-1}\mathrm{e}$	${}^{0}_{0}\bar{\nu}$	${}^{4}_{2}\mathrm{He}$

Example 4
Complete the table.

isotopes	${}^{12}_{6}\mathrm{C}$	${}^{13}_{6}\mathrm{C}$	${}^{196}_{80}\mathrm{Hg}$	${}^{198}_{80}\mathrm{Hg}$	${}^{199}_{80}\mathrm{Hg}$	${}^{200}_{80}\mathrm{Hg}$	${}^{201}_{80}\mathrm{Hg}$	${}^{202}_{80}\mathrm{Hg}$	${}^{204}_{80}\mathrm{Hg}$
atomic number	6	6	80	80	80	80	80	80	80
neutron number	6	7	116	118	119	120	121	122	124
mass number	12	13	196	198	199	200	201	202	204

Example 5
The mass of a proton m_p, a neutron m_n and an electron m_e, as well as the unified atomic mass constant u is provided to a high degree of precision. Find the total mass of 6 protons, 6 neutrons and 6 electrons in terms of u.

	mass / kg
m_p	1.672623×10^{-27}
m_n	1.674929×10^{-27}
m_e	9.109383×10^{-31}
u	1.660539×10^{-27}

Solution:

$$\begin{aligned}\text{total mass} &= \frac{6\left(m_p + m_n + m_e\right)}{u} \\ &= \frac{6\left(1.672623\times10^{-27} + 1.674929\times10^{-27} + 9.109383\times10^{-31}\right)}{1.660539\times10^{-27}} \\ &= 12.09895u\end{aligned}$$

Note 1: If provided with data of 7 s.f., give final answer to 7 s.f. as well.
Note 2: The above total mass (12.09895u) is that of free nucleons and electrons that are far apart from each other. When these subatomic particles form a single particle, a carbon-12 atom (of mass exactly 12u, by virtue of the definition of u, which is 1/12 the mass of a carbon-12 atom), the total mass decreases. The difference in mass between the free constituent parts (nucleons and electrons) and the atom is called the **mass defect** of the atom, which is an important concept discussed in the next section. The mass defect of the carbon-12 atom is 0.09895u.

Example 6
The radius of an atomic nucleus is ranges from 1 to 10 fm, depending on its mass number. For 2 protons spaced 1 fm apart, estimate the magnitude of **(i)** electric force of repulsion and **(ii)** the gravitational force of attraction.

Solution:

(i)
$$\begin{aligned}\left|F_E\right| &= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \\ &= \frac{1}{4\pi\left(\frac{1}{36\pi}\times10^{-9}\right)}\frac{\left(1.6\times10^{-19}\right)^2}{\left(1\times10^{-15}\right)^2} \\ &= 230\text{ N}\end{aligned}$$

(ii)
$$\begin{aligned}\left|F_g\right| &= G\frac{m_1 m_2}{r^2} = G\frac{m_p^2}{r^2} \\ &= \left(6.67\times10^{-11}\right)\frac{\left(1.67\times10^{-27}\right)^2}{\left(1\times10^{-15}\right)^2} \\ &= 1.86\times10^{-34}\text{ N}\end{aligned}$$

20.4 Mass Defect & Nuclear Binding Energy

Example 6 reveals two facts: there exists strong Coulomb repulsion between the protons in a nucleus, and the gravitational attraction is too weak to hold the nucleons together. There must exist yet another type of attractive force responsible for binding the nucleons together in a nucleus. Indeed, this other attractive force, now called the strong force (or strong interaction), was discovered later. The strong force acts similarly between proton-proton, proton-neutron and neutron-neutron. The range of the strong force is extremely short (on the order of 1 fm (femtometre) = 10^{-15} m). Because of its short range, the strong force only binds a nucleon to its nearest neighbours within the nucleus.

The mass defect discussed below Example 5 can also be explained using the strong force. To do so, let's recall that the gravitational potential energy between two masses m_1 and m_2, separated by a distance r, is $-Gm_1 m_2 / r$. Compared to when m_1 and m_2 are free from each other (infinitely far apart, at which the potential energy is 0), there is a decrease in energy by the amount

$$\left|\Delta E\right| = \frac{Gm_1 m_2}{r}.$$

This amounts to a decrease in mass, or a mass defect, via the mass-energy equivalence relation,

$$E = mc^2$$

where

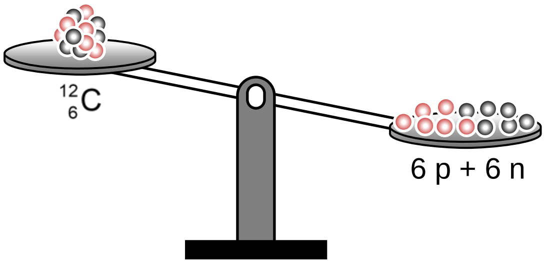
$$c = 3.0 \times 10^{8}\ \text{m s}^{-1}$$

is the speed of light in vacuum. This relation was established by Albert Einstein in 1905. Thus, the mass defect due to the gravitational attraction between m_1 and m_2 is

$$\Delta m = \frac{\Delta E}{c^2} = \frac{G m_1 m_2}{r c^2}.$$

The mass defect due to the gravitational attraction is extremely small. You may wish to estimate it using the above equation, for, e.g., the system consisting of you and the Earth.

The strong force, much like the gravitational force, is always attractive. Hence, it comes with a negative potential energy. The corresponding mass defect, due to its strength, is a much more appreciable fraction of the total mass. For example, in the case of carbon-12, the mass defect is 0.09895/12 = 0.8% of the mass of the atom.



Mass defect of a nucleus, Δm, is the difference between the total mass of individual, free nucleons and the mass of the nucleus.

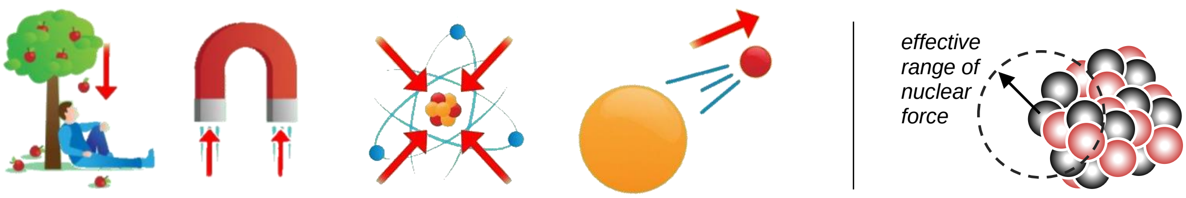
$$\Delta m = \left[Z m_{\text{p}} + (A - Z) m_{\text{n}} \right] - m_{\text{nucleus}}$$

Imagine the process in which free nucleons come together to form a nucleus. Since the final mass (of the nucleus) is less than the initial mass (of the free nucleons), the difference in mass must be released as some form of energy ($E = \Delta m c^2$) [What form can this energy assume?], if mass-energy is to be conserved. This is similar to what we learned in Chemistry, where we refer to energy being released when chemical bonds are formed.

The reverse is also true, i.e. we need to provide energy (to do work) if we wish to break up a nucleus into its individual, free nucleons. This amount of energy is called the binding energy of the nucleus, and is the same amount as the energy that is released during the formation of the said nucleus. This energy input is converted into part of the mass of the free nucleons (hence free nucleons add up to a larger mass than the nucleus), via $E = \Delta m c^2$.

Nuclear binding energy is the minimum energy required to separate the nucleons in a nucleus to infinity.

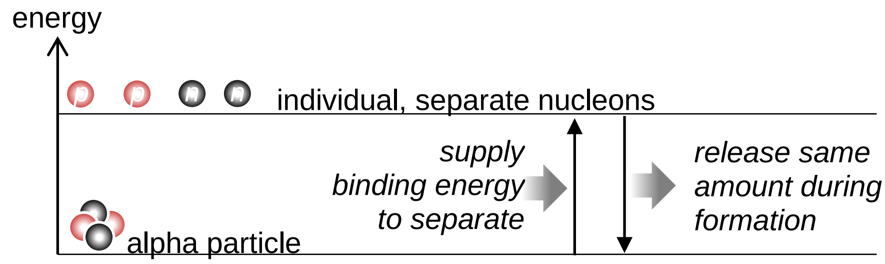
$$\begin{pmatrix}\text{nuclear}\\ \text{binding}\\ \text{energy}\end{pmatrix} = (\Delta m) c^2$$



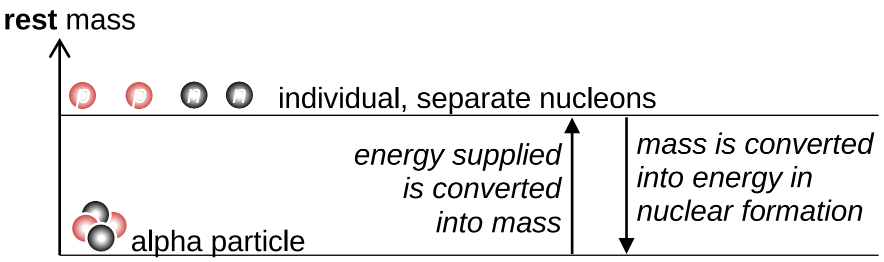
There are 4 fundamental forces in nature: gravitational force, electromagnetic force, strong force (describes nuclear binding), and weak force (describes β-decay). In terms of strength, gravitational force is the weakest, followed by weak force, then electromagnetic force. Strong force is the strongest. In terms of range, both gravitational and electromagnetic forces have infinite range, whereas both weak and strong forces are effective only over very short ranges (~ 1–3 fm).

Please take note that, when describing energy released in a nuclear reaction, it is ***incorrect*** to say "~~*nuclear binding energy is released*~~". Please describe it simply as "energy release" or "nuclear energy released".

Mass-energy equivalence, i.e. (E = mc^2), was proposed by Albert Einstein when he developed his theory of Special Relativity in 1905.



At risk of over-simplification, **rest mass** can be thought of as the mass when the particle is *stationary*.



Example 7

(a) Find the nuclear binding energy, in MeV, of
 (i) iron-56
 (ii) uranium-238

(b) Hence, in MeV to 3 s.f., find binding energy **per nucleon** of iron-56 and uranium-238.

	nuclear mass / *u*
iron-56 $\left({}^{56}_{26}\mathrm{Fe}\right)$	55.934936
uranium-238 $\left({}^{238}_{92}\mathrm{U}\right)$	238.050788
hydrogen $\left({}^{1}_{1}\mathrm{H}\right)$	1.007825
neutron $\left({}^{1}_{0}\mathrm{n}\right)$	1.008665

Solution:

(i) mass defect of iron

$$\begin{aligned}\Delta m_{\mathrm{Fe}} &= 26m_{\mathrm{p}} + (56-26)m_{\mathrm{n}} - m_{\mathrm{Fe}} \\ &= 26m_{\mathrm{H}} + 30m_{\mathrm{n}} - m_{\mathrm{Fe}} \\ &= 26(1.007825u) + 30(1.008665u) - 55.934936u \\ &= 0.528464u\end{aligned}$$

$$\begin{aligned}\text{binding energy} &= \Delta m_{\mathrm{Fe}}c^2 \\ &= (0.528464)(1.66\times10^{-27})(3.0\times10^{8})^2 \\ &= 7.895252\times10^{-11}\ \mathrm{J} = 4.934533\times10^{8}\ \mathrm{eV} = 493.4533\ \mathrm{MeV}\end{aligned}$$

$$\text{binding energy per nucleon} = 493.4533 \div 56 = 8.81\ \mathrm{MeV}$$

(ii) binding energy (in MeV) of $^{238}_{92}U = \dfrac{\Delta m_U c^2}{e \times 10^6} = \dfrac{(92m_H + 146m_n - m_U)c^2}{(1.6\times10^{-19})\times10^6}$

$$= \frac{(92(1.007825u)+146(1.008665u)-238.050788u)c^2}{(1.6\times10^{-19})\times10^6}$$

$$= \frac{(1.934202)uc^2}{1.6\times10^{-13}} = \frac{(1.934202)(1.66\times10^{-27})(3.00\times10^8)^2}{1.6\times10^{-13}}$$

$$= 1806.061 \text{ MeV}$$

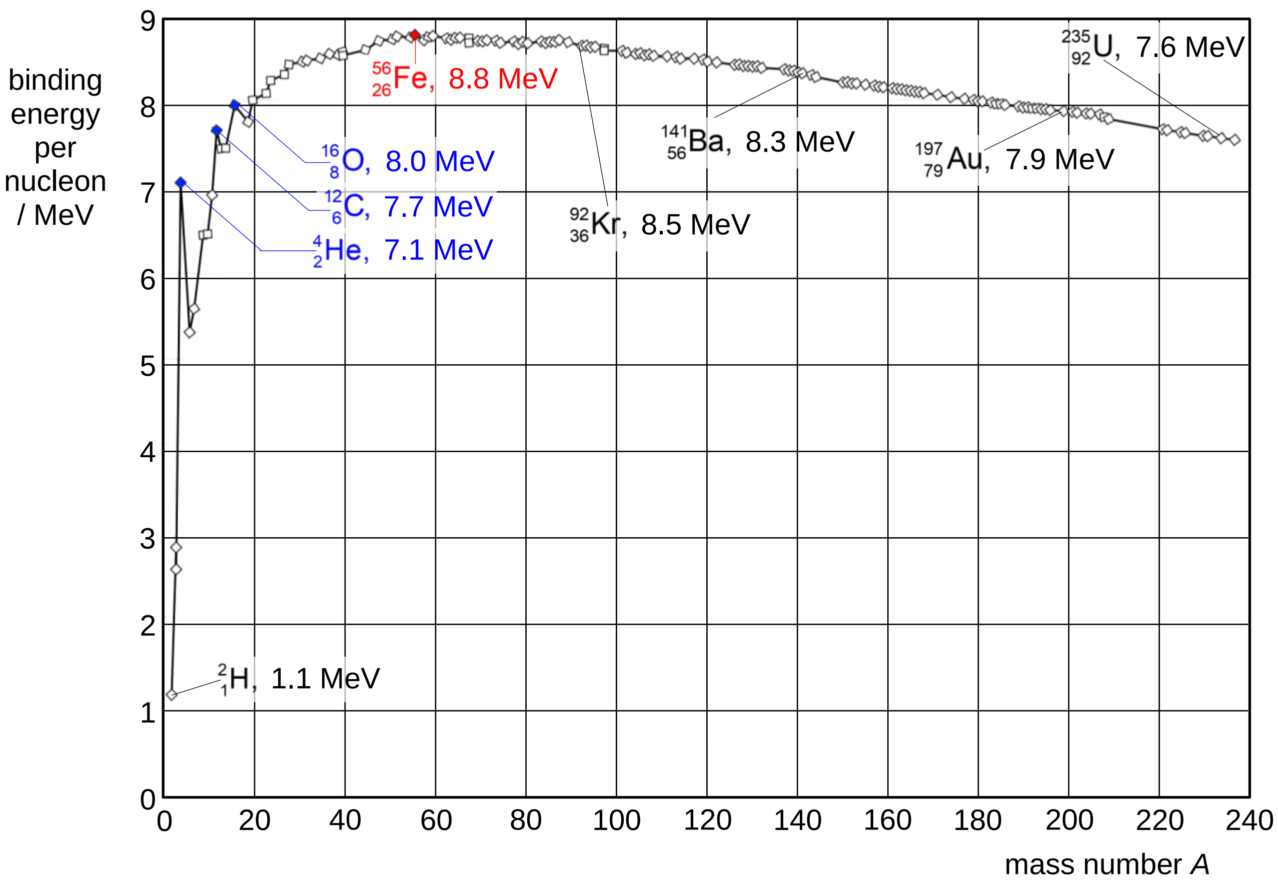
binding energy per nucleon $= 1806.061 \div 238$

$= 7.59$ MeV

Note: Recall from electric field that $1\text{ eV} = 1.60\times10^{-19}$ J.

20.5 Nuclear Stability: Binding Energy per Nucleon

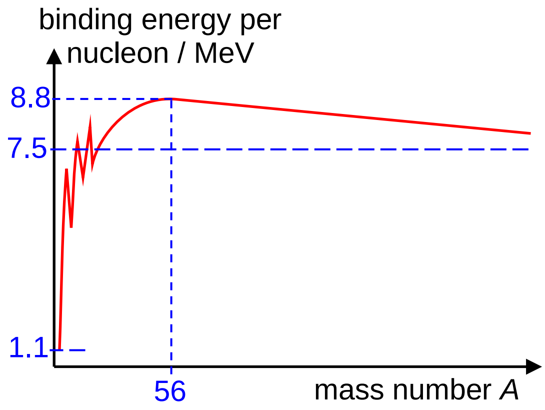
The stability of nuclides is reflected by the **binding energy per nucleon**. The higher the binding energy per nucleon in a nuclide (which means, in a loose sense, more energy is needed to remove one nucleon from a nucleus), the more stable the nuclide is. Thus, the results of Example 7 say that uranium-238 is less stable than iron-56. The graph below shows the variation of binding energy per nucleon with nucleon number.



We can observe the following:

- Hydrogen ^{1}H does not appear on the graph. The atom comprises an electron orbiting a proton. The binding energy due to the electric attraction between the electron and the nucleus (a single proton) is negligible. The binding energy of a lone proton free from any strong interaction is, by definition, zero.

- ^{4}He, ^{12}C, and ^{16}O lie above the main curve; they are more stable than nuclides adjacent in mass number.
- Graph has a maximum of about 8.8 MeV at the ^{56}Fe nuclide. Iron-56 is the most stable nucleus.
- When sketching, note the
  - start point at A = 2, ~ 1.1 MeV
  - maximum point at A = 56, ~ 8.8 MeV
  - binding energy per nucleon for 'heavier' nuclides is generally larger than 7.5 MeV



The binding energy per nucleon graph helps us determine which nuclear process (nuclear fission or fusion, which we discuss in the next section) is likely to occur naturally. Natural nuclear processes tend to result in the increase in binding energy per nucleon of nuclides involved (hence more stable nuclides (or nuclides of smaller total mass)); energy is released in such processes. If a particular process results in *decreases* in binding energy per nucleon, energy input is required for the process to occur. We can supply the additional energy required in the form of kinetic energy of the reactant nuclei. The final (product) nuclei of such processes will be less stable (or have a larger total mass).

20.6 Important Nuclear Processes

A **nuclear reaction** is a process during which two nuclei, or one nucleus and an external subatomic particle (e.g electron, proton, neutron), collide with each other to produce different nuclei. Some nuclear reactions result in products that are more stable than the reactants, hence yield a net energy release. Nuclear fission and fusion, are examples of such processes. Some other nuclear reactions result in products that are less stable than the reactants. Such nuclear reactions require a net energy input, typically in the form of initial kinetic energy of the reactants. See Example 9.

20.6.1 Nuclear Fission

> **Nuclear fission** is the splitting of a heavy nucleus into two lighter nuclei *of approximately the same mass*.

The fission of a heavy nucleus is typically initiated by a slow (or thermal) neutron and the fission products include two or three slow neutrons and gamma ray photons. An enormous amount of energy is released during nuclear fission of a very heavy nucleus.

An example of a nuclear fission is when $^{235}_{92}\text{U}$ (the **parent nucleus**, or the reactant) is bombarded by a neutron ($^{1}_{0}\text{n}$). The **daughter nuclei** (or the products) are $^{141}_{56}\text{Ba}$ and $^{92}_{36}\text{Kr}$ (along with three free neutrons):

$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}.$$

According to the Binding Energy per Nucleon graph above, the daughter nuclei in this fission event have higher binding energy per nucleon than the parent nucleus. The energy E released in a fission reaction can be shown to be equal to the difference in the total binding energies of the reactants and that of the products. That is,

$$\text{total energy released} = \text{total binding energy of products} - \text{total binding energy of reactants.}$$

Remember that the binding energy of the free neutrons is zero.

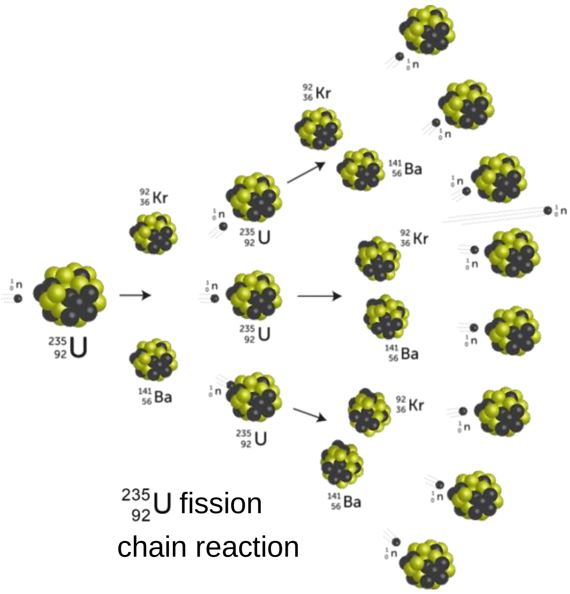
Another way to calculate the energy released is to look at the mass defects. The parent nucleus has a smaller mass defect (less stable) than the daughter nuclei (more stable). The energy release, using again the mass-energy equivalent relation,

total energy released
$= (\text{total mass defect of daughter nuclei} - \text{total mass defect of parent nuclei}) \times c^2$

Again, the mass defect of free neutrons is zero.

20.6.2 Chain Reaction

As shown in the fission reaction of $^{235}_{92}U$ (when colliding with a neutron) in the previous section, more neutrons are produced in the process. The neutrons produced in this initial fission reaction can each trigger further fission reaction, which produces more neutrons that can trigger even more fission events. This is called a chain reaction, as illustrated in the diagram on the right. The rate of energy release increases exponentially with time (which is what happens in an exploding atomic bomb, where a huge amount of energy is released quickly), unless steps are taken to control the rate of the reactions (using some material to absorb some of the neutrons produced, to slow down the rate of fission reaction; this is done in a nuclear reactor).



$^{235}_{92}U$ fission chain reaction

If the amount of the original sample of *fissile material* is too small, too many neutrons will escape without striking other nuclei and the chain reaction cannot be sustained. Critical mass is the minimum amount of fissionable material needed to sustain a chain reaction.

20.6.3 Nuclear Fusion

Nuclear fusion occurs when two light nuclei combine to form a nucleus of greater mass.

Fusion reactions typically requires extremely high temperature before they can take place. An example of fusion reaction is where two hydrogen nuclei collide and form a deuterium nucleus (an isotope of hydrogen, comprises of one proton and one neutron). A positron and a neutrino (discussed in the next topic) are also produced in the process:

$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{1}e + ^{0}_{0}\nu$$

The daughter nuclei in a fusion reaction are of higher binding energy per nucleon, or of less total mass (larger mass defect) than the parent nuclei. The energy released in nuclear fusion is calculated in the same way for nuclear fission, namely

total energy released = total binding energy of products − total binding energy of reactants

OR

total energy released

= (total mass defect of daughter nuclei − total mass defect of parent nuclei) × c^2

20.6.4 Conservation Laws in Nuclear Reactions

The conservation laws of **momentum** and **mass-energy** hold for nuclear reactions. Apart from these, a few other quantities are also conserved, the most important ones being the **total number of nucleons** and the **total charge** (protons plus neutrons).

Nuclear reactions are represented in equations similar to chemical reactions:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$

$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{1}e + ^{0}_{0}\nu$$

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

Conservation of nucleon number means that **the left superscript**, or the nucleon number, must be **balanced** between the two sides of the equation. Conservation of charge means that the **left subscript** must also be **balanced** between the two sides. One may check that the nuclear reactions above indeed satisfy both criteria. When you write down nuclear reaction equations, please remember to check whether these conservation laws are satisfied.

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nuclear processes

involve change in nuclear binding energies per nucleon; conserves (i) total number of nucleons, (ii) total electric charge, (iii) total mass-energy, and (iv) total momentum within isolated system of interacting nuclei

nuclear reactions

involves collisions between (i) two nuclei or (ii) nuclei and subatomic particle (e.g. proton, neutron, electron), resulting in nuclear transformation (product nuclides differ from reactant nuclides); can be represented by nuclear equations using notations similar to $^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}$

energy **released** per nuclear reaction:

$$\begin{pmatrix}\text{total}\\ \text{binding energy}\\ \text{of products}\end{pmatrix} - \begin{pmatrix}\text{total}\\ \text{binding energy}\\ \text{of reactants}\end{pmatrix} \quad \text{OR} \quad \left[\begin{pmatrix}\text{total}\\ \text{rest mass}\\ \text{of reactants}\end{pmatrix} - \begin{pmatrix}\text{total}\\ \text{rest mass}\\ \text{of products}\end{pmatrix}\right]c^2$$

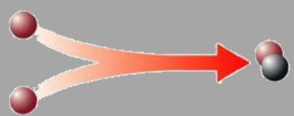
radioactive decay

Nuclear fusion is the

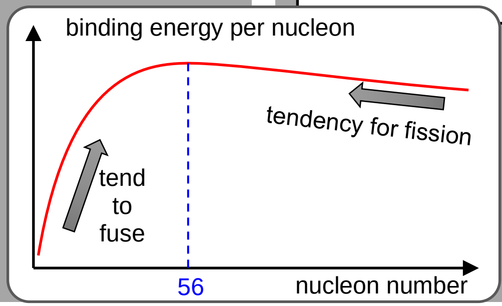
[action] combining of two or more light nuclei
[condition] under very high temperatures
[result] to form a single, more massive nucleus.

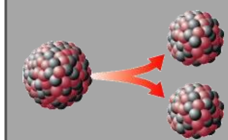
Nuclear fission is the

[action] splitting of a single heavy nucleus
[condition] when bombarded by neutrons
[result] to form two or more lighter nuclei of approximately same mass with neutrons emitted.



Energy is released when light nuclei undergo fusion or when heavier nuclei undergo fission, as these processes result in increases in binding energy per nucleon where the product nuclei are more stable.



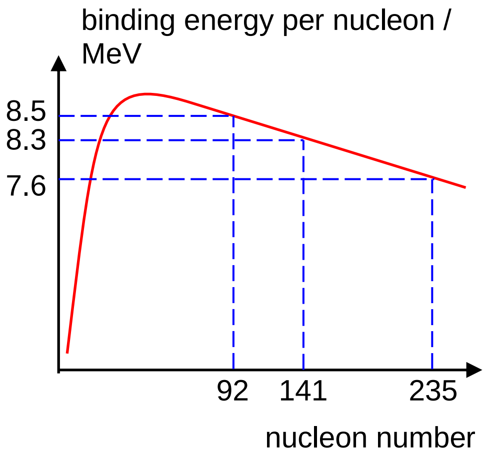


If a nuclear reaction is not energetically feasible (energy released is “negative”), it can still proceed (be induced) if reactant nuclei have enough KE.

Example 8

Fission of uranium-235 $\left({}^{235}_{92}\text{U}\right)$ can be initiated by a neutron to give, among its products, a krypton-92 $\left({}^{92}_{36}\text{Kr}\right)$ nucleus and a barium-141 $\left({}^{141}_{56}\text{Ba}\right)$ nucleus.

(i) Work out the nuclear equation. Hence identify the other fission products.
(ii) Find the energy released in a single fission process.
(iii) Explain why the other fission products in (i) need not be taken into account in the calculation of (ii).
(iv) Explain the release of energy despite there being an *increase* in the binding energy.



Solution:

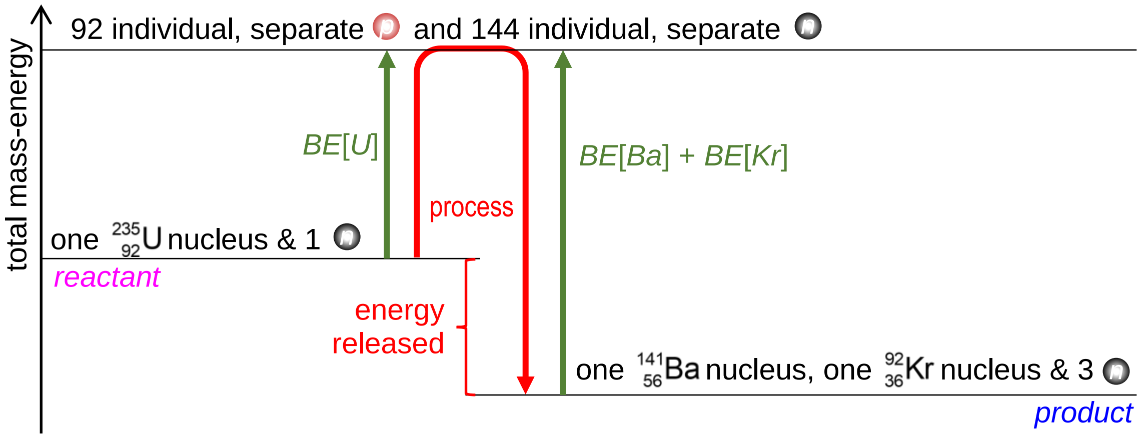
(i) $${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}$$

(ii) $$\begin{pmatrix}\text{energy}\\ \text{released}\end{pmatrix} = \begin{pmatrix}\text{total}\\ \text{binding energy}\\ \text{of products}\end{pmatrix} - \begin{pmatrix}\text{total}\\ \text{binding energy}\\ \text{of reactants}\end{pmatrix}$$

$$= (141)(8.3) + (92)(8.5) - (235)(7.6) = 166.3 \text{ MeV}$$

(iii) neutrons are emitted as products as well
emitted neutrons are single discrete particles
that are not part of any bound nuclei
so binding energy $= 0$

(iv) **[definition]** binding energy is the minimum energy required to separate nucleons in a nucleus to infinity
[stability] for the same number of nucleons, the binding energy per nucleon of the more stable product nuclei are higher than the reactant
[see reaction as 2-stage process, and "binding energy" describes separation process]
total binding energy required to separate the uranium nucleus is less than the total ~~binding~~ energy released in forming krypton and barium nuclei from the same constituent nucleons



Note: The meaning of the above diagram is the following. One may consider the nuclear fission as a two-step reaction: first from a uranium-235 nucleus to free nucleons, then the free nucleons form the daughter nuclides. From the diagram, it is clear that the energy released is equal to the difference between the total binding energies. Similar diagrams below should be interpreted in a similar way.

Example 9

The proton was discovered in the first laboratory-induced nuclear reaction by Rutherford:

$$^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

Find the net energy output of this reaction.

	atomic mass
nitrogen-14 $\left({}^{14}_{7}\text{N}\right)$	14.003074
alpha particle $\left({}^{4}_{2}\text{He}\right)$	4.002603
oxygen-17 $\left({}^{17}_{8}\text{O}\right)$	16.999131
hydrogen $\left({}^{1}_{1}\text{H}\right)$	1.007825

Solution:

$$^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

$$\begin{pmatrix}\text{energy}\\ \text{released}\end{pmatrix} = \left[\begin{pmatrix}\text{total}\\ \text{rest mass}\\ \text{of reactants}\end{pmatrix} - \begin{pmatrix}\text{total}\\ \text{rest mass}\\ \text{of products}\end{pmatrix}\right] c^2$$

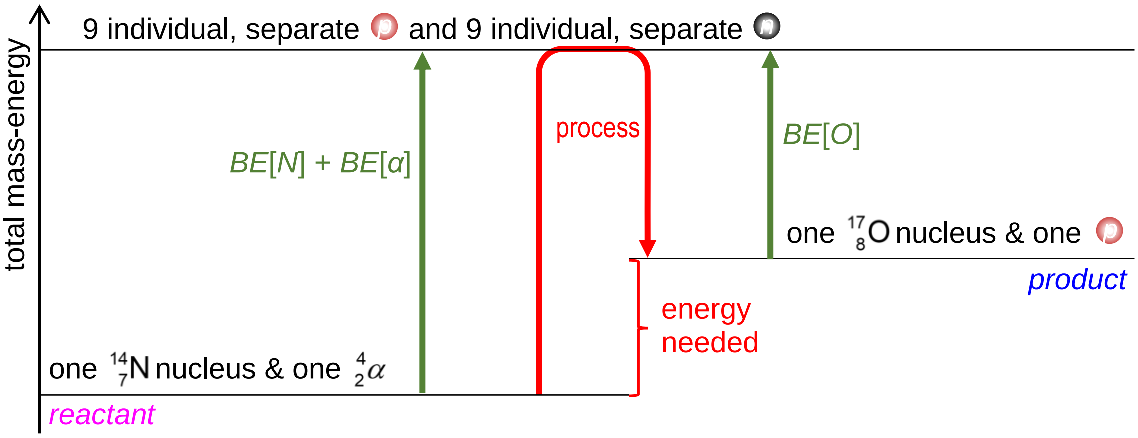
$$= \left[(14.003074 + 4.002603) - (16.999131 + 1.007825)\right] uc^2$$

$$= -0.001279\ uc^2$$

$$= -0.001279\ \left(1.66 \times 10^{-27}\right)\left(3.00 \times 10^{8}\right)^2$$

$$= -1.910826 \times 10^{-13}\ \text{J}$$

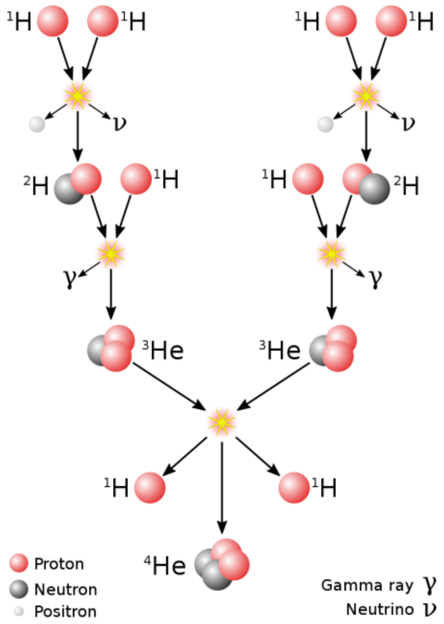
$$= -1.19\ \text{MeV}$$



Note: The net energy output of this reaction is negative, which means it requires a net energy input: the total mass-energy of the product is higher than that of the reactant (you can also check that the binding energy per nucleon of the product is lower). The reaction has to be *induced* as mentioned in the question. In this case, the kinetic energy of the alpha particles makes up for the energy deficit.

Example 10

Nuclear fusion powers our Sun. The generation of energy in the Sun is a multi-stage process starting with protons $\left({}^{1}_{1}\text{H}\right)$ and ending with helium-4 $\left({}^{4}_{2}\text{He}\right)$ nuclei (shown in the diagram on the right). The net effect is:



$$4\left({}^{1}_{1}\text{H}\right)+2\left({}^{0}_{-1}\text{e}\right)\rightarrow {}^{4}_{2}\text{He}$$

(i) Find the energy released per reaction.

The worldwide electricity consumption in 2020 was approximately 23 000 TWh.

(ii) Find the mass of hydrogen needed if the same amount of energy was to be generated via nuclear fusion of hydrogen.

Solution:

	atomic mass
${}^{4}_{2}\text{He}$	4.002603
${}^{1}_{1}\text{H}$	1.007825

(i) $4\left({}^{1}_{1}\text{H}\right)+2\left({}^{0}_{-1}\text{e}\right)\rightarrow {}^{4}_{2}\text{He}$

$$\begin{pmatrix}\text{energy}\\ \text{released}\end{pmatrix}=\left[\begin{pmatrix}\text{total}\\ \text{rest mass}\\ \text{of reactants}\end{pmatrix}-\begin{pmatrix}\text{total}\\ \text{rest mass}\\ \text{of products}\end{pmatrix}\right]c^2$$

$$=\left[4(1.007825)-(4.002603)\right]uc^2$$

$$=0.028697\ uc^2$$

$$=0.028697\ \left(1.66\times10^{-27}\right)\left(3.00\times10^{8}\right)^2$$

$$=4.2873318\times10^{-12}\ \text{J}$$

$$=26.8\ \text{MeV}$$

(mass of electrons negligible)

(ii) number of reactions, n, needed:

$$n=\frac{\left(23\ 000\times10^{12}\right)\left(60^2\right)}{4.2873318\times10^{-12}}$$

$$=1.931271\times10^{31}$$

number of hydrogen atoms required $=4n$

mass of hydrogen required $=(4n)(1.007825u)$

$$=129\ 000\ \text{kg}$$

Note: Producing 23 000 TWh of electricity will require 1.79×10^{12} kg of crude oil. It is therefore not difficult to see the lure of generating electricity via fusion, which in addition does not generate undesirable by-products. The technical hurdles are 1) to generate enough kinetic energy in the hydrogen atoms to overcome the electrostatic repulsion, so as to bring the reactant nuclei close enough and for long enough a duration, for the strong nuclear force to act; 2) to achieve the required kinetic energy (corresponding to hundreds of millions of degree in temperature) of the reactant nuclei without melting whatever is supporting/holding/containing the reactant.

20.7 Radioactive Decay & Stability of Nuclei

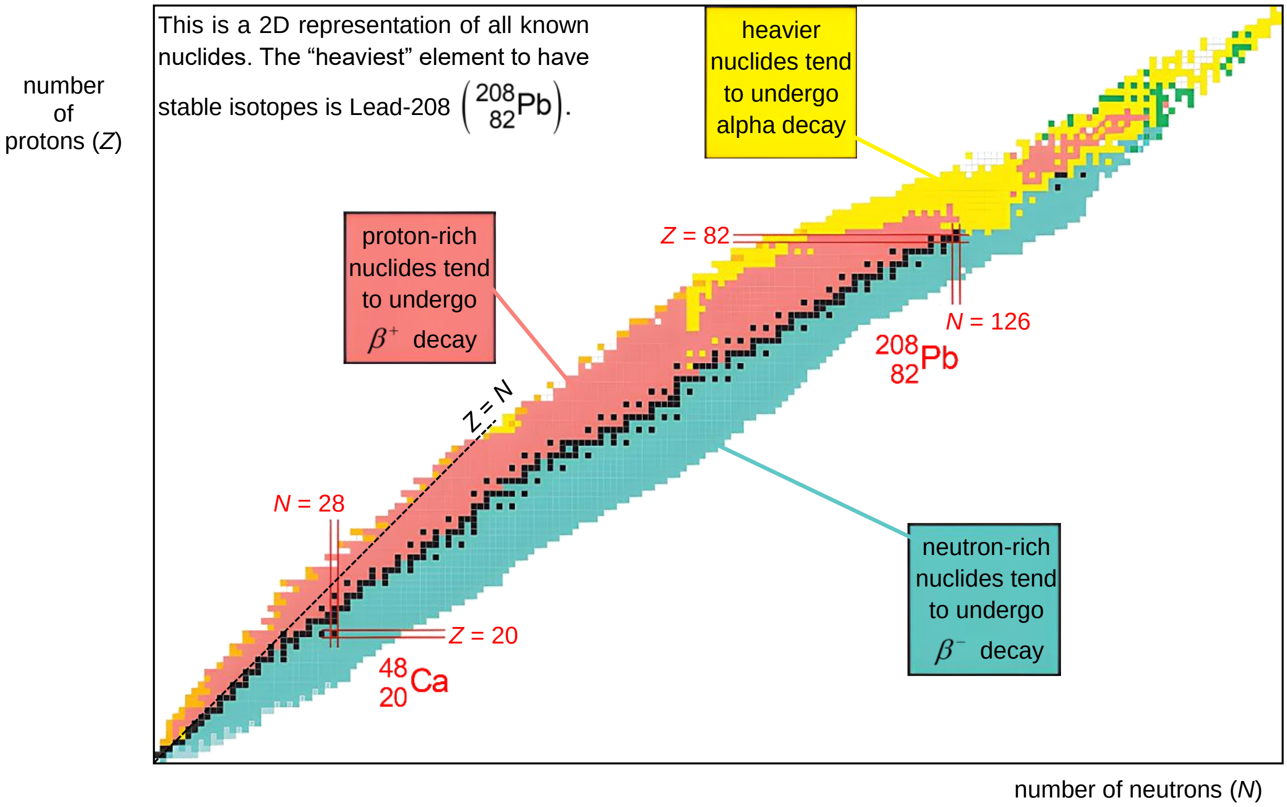
Some nuclear reactions, such as nuclear fission and fusion, release energy and result in product nuclei that are more stable (larger mass defect, or higher binding energy per nucleon). Such nuclear reactions typically require external conditions such as bombardment of neutrons, or high temperature and pressure, in order to take place.

Radioactivity, or **radioactive decay,** is the **spontaneous** and **random emission of ionising radiation** in the form of alpha particles, beta particles or gamma ray photons from **unstable nucleus** to become a **more stable nucleus**.

There is another class of nuclear process, **radioactive decay**, that releases energy and hence yield daughter nuclei that are more stable than the parent nuclei.

Radioactive decay, unlike nuclear fission and fusion, does not require any external condition to be met. In fact, no external factor (such as temperature and pressure) can exert any influence (neither induction nor deterrence) on radioactive decay. Such processes are said to be spontaneous.

There are at least 3300 (both natural and artificially created) nuclides, but only 254 are stable (black dots on the Karlsruhe Nuclide Chart below). Most nuclides undergo some form of decay over time.



20.8 Types of Radiation

There are three types of ionising radiation emitted by radioactive nuclei: alpha (α), beta (β) and gamma (γ). These radiations are said to be "ionising" because particles of these radiations have enough energy to detach electrons from atoms and molecules (thereby ionising them) along their paths. Alpha, beta and gamma are sequential letters of the Greek alphabet representing the increasing *penetrating power* (or decreasing *ionising power*) of each type of radiation.

Example 11

The β-particle can be either a positron (${}^{0}_{1}\text{e}$, also called β^{+} particle) or an electron (${}^{0}_{-1}\text{e}$, β^{-} particle). A positron is the antiparticle of an electron. It is the same as an electron in all its properties, except it has the opposite charge. When a positron and an electron meet, they annihilate each other and produce two (or sometimes more) gamma photons. In this process, the conservation laws (of momentum, mass-energy, charge, etc) hold.

An electron undergoes a head-on collision with a positron. The 2 particles move towards each other with the same initial speeds and are annihilated in the collision, resulting in the emission of 2 gamma photons:

$$ {}^{0}_{-1}\text{e} + {}^{0}_{1}\text{e} \rightarrow \gamma + \gamma $$

State and explain the properties of resulting gamma photons.

Solution:

By principle of **conservation of linear momentum**, the total linear momentum of the isolated system comprising the electron and position before and after the collision remains constant due to absence of net external force on the system.

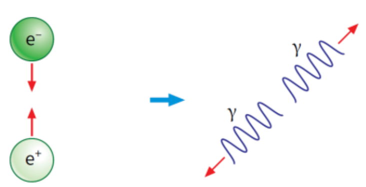
Since the particles are of the same mass, have the same speed and are travelling directly towards each other, the **total linear momentum of the system is zero**. Hence, the two photons must **move in opposite directions**, with the **same magnitude of momentum**. Because the momentum p and wavelength λ of a photon are related by $p = \dfrac{h}{\lambda}$, same magnitude of momentum implies the **same wavelength** of the photons. [Can you explain why it is not possible to produce a single gamma photon?]

By **conservation of mass-energy**, the total mass of positron and electron is converted into the energy of the 2 photons:

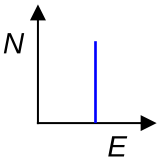
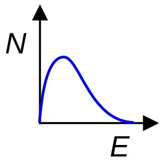
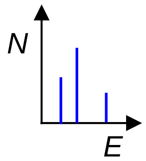
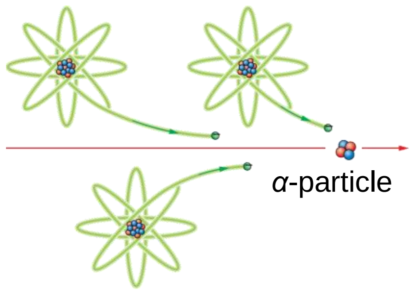
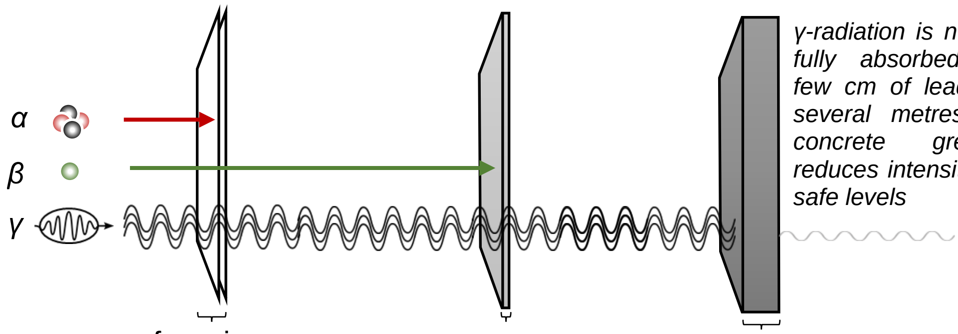
$$ (2m_e)c^2 = 2E_\gamma $$
$$ E_\gamma = m_e c^2 = (9.11 \times 10^{-31})(3.00 \times 10^{8})^2 $$
$$ = 8.20 \times 10^{-14}\ \text{J} = 0.512\ \text{MeV} $$

$$ E = hf = \frac{hc}{\lambda} $$
$$ f = \frac{E}{h} = 1.24 \times 10^{20}\ \text{Hz} $$
$$ \lambda = \frac{c}{f} = 2.43 \times 10^{-12}\ \text{m} $$

The process is illustrated in the diagram below.



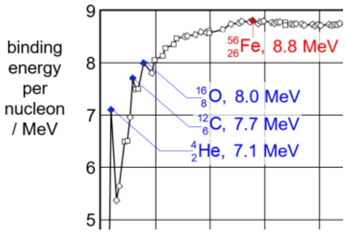
20.8.1 Properties of Radiations

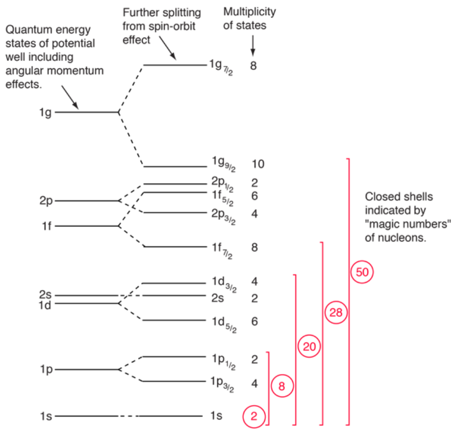
	alpha (α)	beta (β^-)	gamma (γ)
nature	helium nucleus $\left({}^{4}_{2}\text{He}\right)$	electron $\left({}^{0}_{-1}\text{e}\right)$	photon
electric charge	$+2e$	$-e$	0
approximate rest mass	$\sim 4\ u$	$\sim \dfrac{1}{1840}u$	0
typical speed	$\sim 0.05\ c$	range up to $0.9\ c$	c
distribution of energy from same nuclear process	 well-defined, 3 – 7 MeV	 range, up to 5 MeV	 discrete energy spectrum, (keV – MeV)
ionizing power	strong	moderate	weak
	Ionising power refers to the ability of the radiation to remove electrons from other atoms. Alpha particles have relatively larger mass, more charge and travel more slowly, so they interact with other atoms more strongly. *The radiation loses some kinetic energy in each ionisation process and will be completely absorbed by the surrounding material once the kinetic energy is expended. This is why strongly ionizing radiation has the least penetrating power / range.* *The particles' kinetic energy is converted into internal energy of the surrounding material in the process.*		
range in air	a few cm	a few metres	(long)
penetrating power	low	medium	high
stopped by	 a few pieces of paper	a few mm of aluminium	a few cm of lead *γ-radiation is never fully absorbed; a few cm of lead or several metres of concrete greatly reduces intensity to safe levels*

*The existence of **discrete energies of** alpha particles and **gamma photons** led to scientists hypothesizing that the **nucleus**, quite like (the electrons of) the atom, **exists in discrete energy levels too.***

The nuclear shell model was developed to integrate knowledge from quantum mechanics in a bid to account for the binding energies and in particular the abnormally-high binding energy per nucleon of oxygen, carbon and helium.

Newer nuclear models have since encompassed the nuclear shell model.

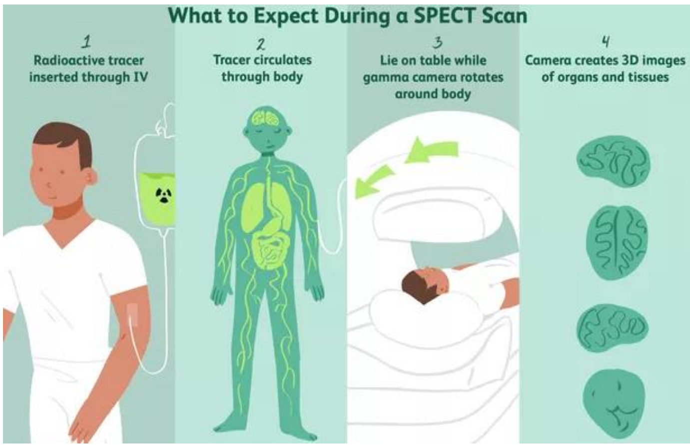




20.8.2 Safety? It Depends

Different ionising radiation has different ionising and penetration power. How 'safe' each type of ionising radiation is, depends on the 'dose' and how far away the source is.

If the source is a few metres away and there is a good probability of objects coming between source and the human, then a radioactive source emitting gamma radiation would be relatively the most dangerous, as the likelihood of any alpha or beta particles reaching the human subject will be low.

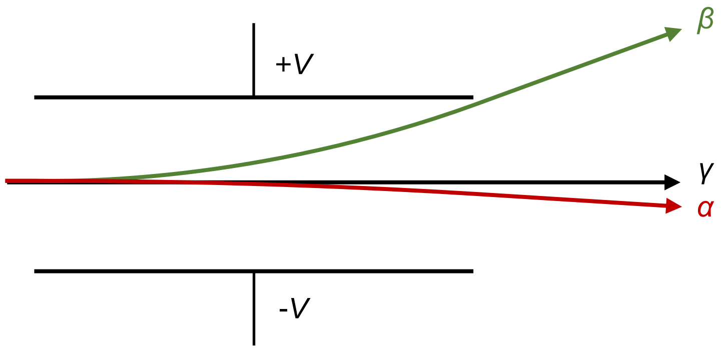


On the other hand, gamma radiation is the least ionising among the three kinds of radiation. If a source of gamma radiation is inside the body, most of the radiation will leave the body without causing harm. This is how a Single Photon Emission Computed Tomography (SPECT) scan works. A 'tracer' comprises a radioactive isotope (gamma emitters such as iodine-123, technetium-99) is injected into the blood stream. The blood flow through organs can then be visualized. However, gamma radiation is still dangerous since some photons will interact with tissues and cause damages.

An alpha source will be devasting to a body when inside the body due to its ionising power. However, due to its low penetration power, other humans near the affected person are safe from the alpha radiation.

However, in the affected person, the immediate tissue surrounding the alpha source will be strongly ionised and be damaged beyond repair. The damage will continue along the path of the alpha source as it circulates throughout the body in the blood stream.

20.8.3 Paths of Radiation

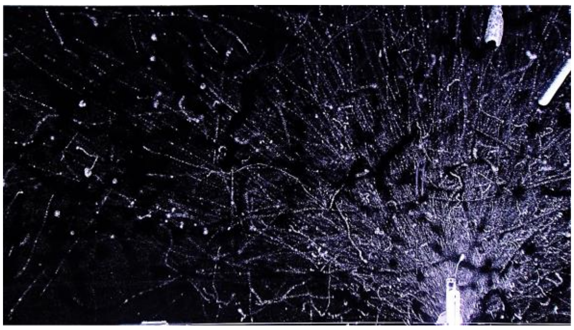


(deflection not to scale)

An electric field can distinguish between the three types of radiation. γ-radiation are photons without electric charge and are thus undeflected. A β particle carries one negative elementary charge, so deflect in a parabolic path towards the positive plate. An α particle carries two positive elementary charges, but it also is much more massive. Hence, it is deflected towards the negative plate, but to a much smaller degree than a β particle is.

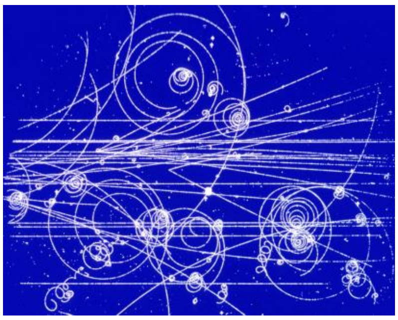
Cloud chambers or bubble chambers show the trail of radiation particles either as condensed liquid droplets or as bubbles of boiled-off liquid respectively.

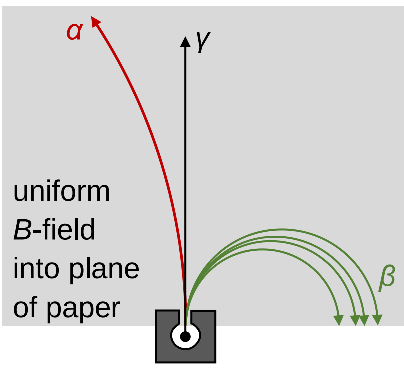
α particles leave thick tracks (they are strongly ionising) that are of similar lengths (well-defined energy)



β particles leave thin, wriggly (moderately ionising) tracks of varying lengths (range of energy)

When a magnetic field is applied perpendicular to the viewing plane of a cloud chamber, alpha and beta particles experience magnetic forces and undergo circular motion.





radioactive source
in lead-lined box

magnetic force provides centripetal force

$$Bq\cancel{v} = \frac{mv^{\cancel{2}}}{r} \Rightarrow r = \frac{mv}{Bq}$$

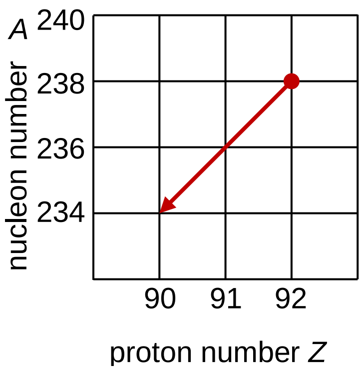
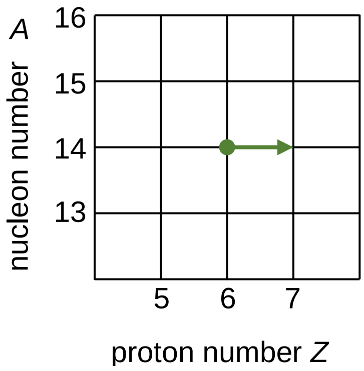
Radius for alpha particles is larger than those of beta particles due to the large mass and large momentum. Multiple radii show the *range* of kinetic energies (or momentum) for β.

γ radiation has low ionizing ability and so do **not** readily give rise to tracks.

Tracks, **if any**, are short, thin and scattered. They do not result directly from the γ photons but instead are from electrons of atoms (e.g. from the glass covering the chamber) ionised by γ photons.

20.9 Types of Decay

In the following nuclear equations, X denotes the parent nuclide and Y denotes the daughter nuclide.

	alpha (α)	beta (β^-)	gamma (γ)
nuclear equation	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \overline{\nu}$	${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$
example decay	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$	${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \overline{\nu}$	${}^{87}_{38}Sr \rightarrow {}^{87}_{38}Sr + \gamma$
			
distribution of energy from the same nuclear process	The kinetic energy of α-particles emitted by a particular nuclide is fixed. It can be uniquely determined using conservation of energy (since all the masses are fixed) and the conservation of momentum. See Example 16.	β particles carry a range of kinetic energies and momenta. To account for this, it was theorized and later confirmed that another particle (now known as anti-neutrino, ${}^{0}_{0}\overline{\nu}$) was also emitted in β decay.	γ photons can carry a few discrete energy (hence momenta, given by $p = E/c$), corresponding to the differences between the discrete nuclear energy levels (similar to atomic energy levels).
common in	massive nuclides with large number of nucleons	neutron-rich nuclides; neutron converts to proton and emits a β particle β naming distinguishes this particle originating from nucleus from an electron in atomic orbitals.	after α- or β-decay, nuclide left in excited state (the * on ${}^{A}_{Z}X^{*}$ indicates excited state), daughter nucleus de-excites by γ-emission

Example 12

Uranium-238 $\left({}^{238}_{92}U\right)$ undergoes a series of decays to reach a final stable nuclide. The particles emitted in succession are: α, β, β, α, α. Which nuclide is <u>not</u> produced during this series of decay?

A ${}^{228}_{88}Ra$ **B** ${}^{230}_{90}Th$ **C** ${}^{234}_{91}Pa$ **D** ${}^{234}_{92}U$

Solution:

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He \Rightarrow {}^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e + \overline{\nu} \Rightarrow {}^{234}_{91}Pa \rightarrow {}^{234}_{92}U + {}^{0}_{-1}e + \overline{\nu} \Rightarrow$$

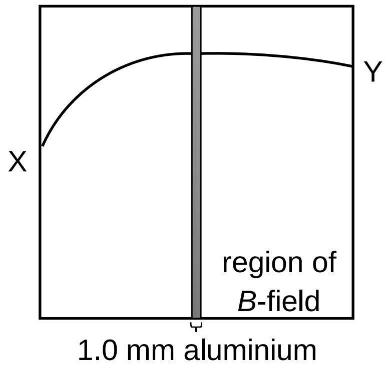
$${}^{234}_{92}U \rightarrow {}^{230}_{90}Th + {}^{4}_{2}He \Rightarrow {}^{230}_{90}Th \rightarrow {}^{226}_{88}Ra + {}^{4}_{2}He$$

Option **A** not produced.

Example 13

A region of uniform magnetic field directed normal to the plane of paper contains an aluminium sheet 1.0 mm thick. Ionizing radiation enters the region normal to the magnetic field. The path is shown.

(i) Explain which type of radiation is entering the region,
(ii) identify if X or Y is the point of entry, and
(iii) deduce the direction of magnetic field.



Solution:

(i) not gamma
because not electrically charged
and so not deflected by magnetic field
not alpha because
would have been completely stopped by aluminium
is beta because 1. would have been significantly slowed down by 1.0 mm of aluminium (to completely stop beta radiation, several millimetres of aluminium is required)
2. is electrically charged thus experiences magnetic force thus deflects within region of magnetic field

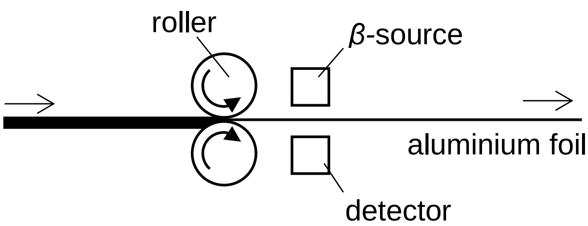
(ii) Y is entry. Radius of circular path before entering aluminium is larger due to higher velocity; radius of circular path after aluminium is smaller due to reduced velocity.

(iii) by Fleming's Left Hand Rule, magnetic field points up and away from plane of paper.

Example 14

The thickness of aluminium foil is monitored using β-radiation. The roller separation is controlled by the output from the detector so as to maintain a constant foil thickness.

(a) State and explain the change in roller separation if the detector reads an increased radiation.
(b) Suggest why the following are not suitable
(i) γ-source
(ii) α-source



Solution:

(a) increase in detector reading is due to less absorption of beta particles
implies less-than-intended thickness of foil
to compensate, roller separation will increase
to draw out thicker foil

(b) (i) not gamma because no appreciable decrease in intensity
as nearly all gamma radiation will pass through

(ii) not alpha because it would have been completely stopped by aluminium

Example 15

(a) Find the energy released when an initially stationary helium-6 nuclei undergoes beta decay.

(b) Explain why the β particles are emitted with a range of kinetic energies.

	mass / u
$m_{\text{He-6}}$	6.017788
$m_{\text{Li-6}}$	6.013475
m_e	0.000549

Solution:

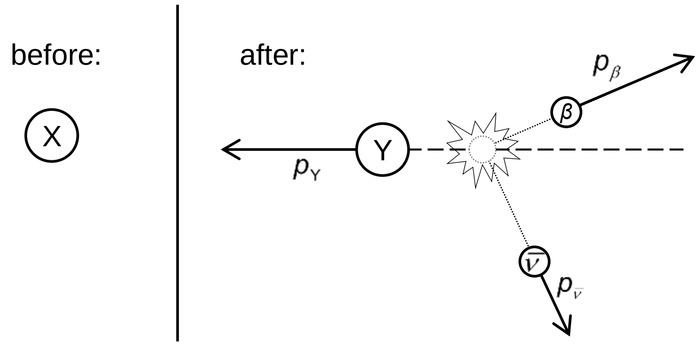
(a) $${}^{6}_{2}\text{He} \rightarrow {}^{6}_{3}\text{Li} + {}^{0}_{-1}\text{e}$$

$$\text{mass difference} = (\text{total mass of reactant}) - (\text{total mass of product})$$
$$= m_{\text{He-6}} - (m_{\text{Li}} + m_e) = (6.017788 - (6.013475 + 0.000549))u$$
$$= 0.003764u$$

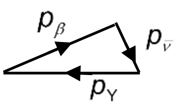
$$\text{energy released per decay} = (\text{mass difference})c^2 = 0.003764uc^2$$
$$= 0.003764(1.66\times10^{-27})(3.00\times10^{8})^2$$
$$= 5.62\times10^{-13}\text{ J} = 3.51\text{ MeV}$$

(b) An antineutrino $\bar{\nu}$ is also emitted in the beta decay. Energy and momentum is shared between the beta particle, the antineutrino and the daughter nuclei. So there are infinitely many possible combinations of momenta and kinetic energies.

beta decay: ${}^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z+1}\text{Y} + {}^{0}_{-1}\text{e} + \bar{\nu}$



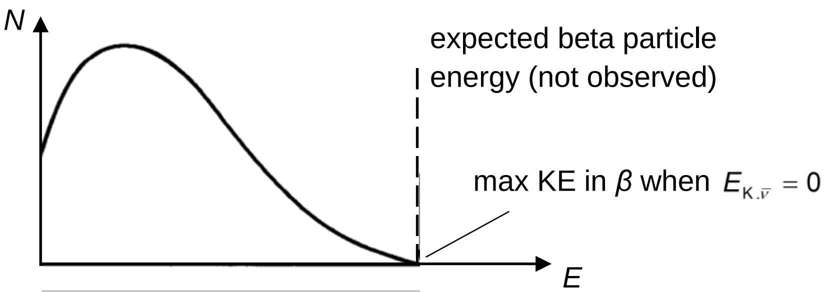
conserve linear momentum:
$0 = \vec{p_\beta} + \vec{p_{\bar{\nu}}} + \vec{p_Y}$



conserve energy:
$\Delta E = E_\beta + E_{\bar{\nu}} + E_Y$

Notes:

- Antineutrino $\bar{\nu}$ is the antiparticle of neutrino ν. Correspondingly a neutrino is emitted with β^+ (positron) decay.
- Neutrinos have very little mass and no electric charge. It was difficult to detect initially.
- Neutrinos were hypothesized by Wolfgang Pauli because otherwise:
  (i) total kinetic [energy] of lithium-6 and beta particle (in e.g. above) should be 3.51 MeV. Instead, beta particles are emitted with a *range* of speeds, up to a max speed corresponding to kinetic energy of 3.51 MeV (where the lithium-6 nuclei has almost no speed).



  (ii) Lithium-6 and beta particle should move directly away from each other along straight line. Instead, beta particles are emitted with varying angles.

Example 16

A stationary uranium-238 nucleus decays by emitting an alpha particle. Find the kinetic energy of the alpha particle.

	atomic mass / u
uranium-238 $\left({}^{238}_{92}\text{U}\right)$	238.000281
thorium-234 $\left({}^{234}_{90}\text{Th}\right)$	233.994192
alpha particle $\left({}^{4}_{2}\text{He}\right)$	4.001505

Solution:

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

$$\text{mass difference} = (\text{total mass of reactant}) - (\text{total mass of product})$$
$$= m_{\text{U}} - (m_{\text{Th}} + m_\alpha) = (238.000281 - (233.994192 + 4.001505))u$$
$$= 0.004584u$$

$$\text{energy released per decay} = (\text{mass difference})c^2 = 0.004584uc^2$$
$$= 0.004584(1.66\times10^{-27})(3.00\times10^{8})^2$$
$$= 6.85\times10^{-13}\text{ J} = 4.28\text{ MeV}$$

Uranium nuclei was originally stationary, so by principle of conservation of linear momentum, total linear momentum of thorium and alpha particle is zero.:

$$0 = m_{\text{Th}}v_{\text{Th}} + m_\alpha v_\alpha$$

considering only magnitudes of momentum:

$$p_{\text{Th}} = p_\alpha$$
$$m_{\text{Th}}v_{\text{Th}} = m_\alpha v_\alpha$$
$$\frac{v_{\text{Th}}}{v_\alpha} = \frac{m_\alpha}{m_{\text{Th}}}$$

consider ratio of kinetic energies:

$$\frac{E_{\text{K, Th}}}{E_{\text{K, }\alpha}} = \frac{\frac{1}{2}m_{\text{Th}}v_{\text{Th}}^2}{\frac{1}{2}m_\alpha v_\alpha^2} = \left(\frac{m_{\text{Th}}}{m_\alpha}\right)\left(\frac{v_{\text{Th}}}{v_\alpha}\right)^2 = \left(\frac{m_{\text{Th}}}{m_\alpha}\right)\left(\frac{m_\alpha}{m_{\text{Th}}}\right)^2 = \frac{m_\alpha}{m_{\text{Th}}}$$

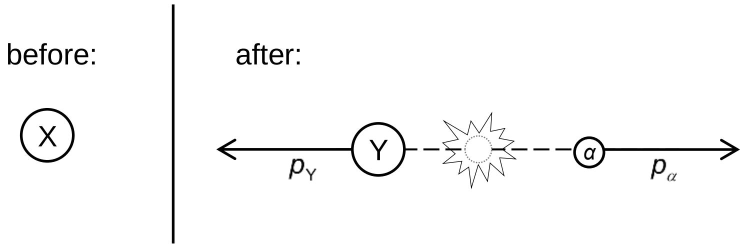
$$E_{\text{K}} = \frac{p^2}{2m} \rightarrow \frac{E_{\text{K, Th}}}{E_{\text{K, }\alpha}} = \frac{\frac{p^2}{2m_{\text{Th}}}}{\frac{p^2}{2m_\alpha}} = \frac{m_\alpha}{m_{\text{Th}}}$$

$$\frac{E_{\text{K, }\alpha}}{E_{\text{total}}} = \frac{E_{\text{K, }\alpha}}{E_{\text{K, }\alpha} + E_{\text{K, Th}}} = \left(\frac{E_{\text{K, }\alpha} + E_{\text{K, Th}}}{E_{\text{K, }\alpha}}\right)^{-1} = \left(1 + \frac{E_{\text{K, Th}}}{E_{\text{K, }\alpha}}\right)^{-1} = \left(1 + \frac{m_\alpha}{m_{\text{Th}}}\right)^{-1}$$

$$E_{\text{K, }\alpha} = \frac{E_{\text{total}}}{1 + \frac{m_\alpha}{m_{\text{Th}}}} = \frac{4.28}{1 + \frac{4.001505}{233.994192}} = 4.21\text{ MeV}$$

Note (i): the thorium nucleus carries away the rest of the kinetic energy (0.07 MeV)
Note (ii): this is why alpha emission has well-defined energy values ("mono-energetic")

alpha decay: ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$



conserve linear momentum:

$$|p_{\text{Y}}| = |p_\alpha|$$

consider energy:

$$\frac{E_{\text{K, }\alpha}}{E_{\text{K, Y}}} = \frac{\frac{p_\alpha^2}{2m_\alpha}}{\frac{p_{\text{Y}}^2}{2m_{\text{Y}}}} = \frac{m_{\text{Y}}}{m_\alpha}$$

20.10 Nature of Radioactive Decay

Previously we discussed nuclear reactions. Regardless of the reaction being energetically feasible or not, they always required human intervention to provoke the reaction – e.g. to create a reaction environment with higher enough temperature to provide the necessary kinetic energy, or to bombard the starting material with neutrons.

Radioactive *decay* on the other hand, occurs "unprovoked", and will lead to more stable nuclides (nuclides with higher binding energy per nucleon).

Radioactive decay is the

[nature]	**spontaneous** and **random**
[action]	emission of ionising radiation in the form of alpha particles, beta particles or gamma ray photons
[initial & final]	from unstable nucleus to become a more stable nucleus.

Radioactive decay is **spontaneous**, because the probability of decay is **unaffected by external factors** such as temperature, pressure or chemical composition.

Radioactive decay is a **random** process because it is impossible to predict if an unstable nucleus will decay at any point in time. All that is known is the **probability of decay** after a period of time.

If we look at a single unstable nucleus, we will have no methods of forcing it to undergo radioactivity decay (spontaneous). We can heat it up or compress it all we want but we will have no effect on its decay; it will decay on its own in time (random).

This random nature of radioactive decay results in the fluctuations in the count rate measured by a detector.

Spontaneity and randomness are distinctly different concepts. They are not interchangeable.

Because radioactive decay is spontaneous, radioactive waste can be compacted to minimize the volume required to store them. It is important to note that the process will concentrate the same amount of radioactivity material (and therefore radioactivity) into a more compact volume.

Another disposal method is cementation. Radioactive material is immobilised to prevent accidental dispersion into the wider environment.

20.11 Law of Radioactive Decay

Even though we are unable to predict which nucleus and when a particular nucleus will decay, due to the probabilistic nature of radioactivity decay and the large population (~10^{23}) of undecayed nuclei in a macroscopic sample, we can predict what proportion of nuclei would have decayed after a duration of time has elapsed.

Decay constant λ is the **probability per unit time** of the decay of a nucleus.

According to probability theory, for a sample containing N radioactive nuclei, the **number of radioactive decays per unit time** will be λN. Since each decay event reduced the total number N by 1, the number of radioactive decays per unit time is simply the rate of decrease of N, i.e.

$$\frac{dN}{dt} = -\lambda N$$

The negative sign in the above equation indicates that, since N is decreasing in time, dN/dt is negative. The solution to the above differential equation is

$$N = N_0 \exp(-\lambda t)$$

where N_0 is the number of the radioactive nuclei at $t = 0$. This is the **law of radioactive decay.**

20.11.1 Activity

The **activity *A*** of a radioactive source is the **number of radioactive decays per unit time** or **rate of radioactive decay** in the source.

$$A = -\frac{dN}{dt} = \lambda N$$

Using the law of radioactive decay, one has

$$A = -\frac{dN}{dt} = -\frac{d}{dt} N_0 \exp(-\lambda t) = \lambda N_0 \exp(-\lambda t) = A_0 \exp(-\lambda t),$$

where $A_0 = \lambda N_0$ is the activity at $t = 0$. Thus,

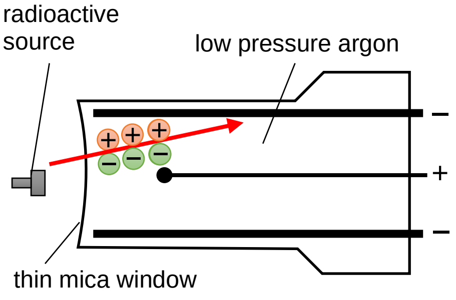
$$A = A_0 \exp(-\lambda t).$$

The SI unit for Activity is the becquerels (Bq), where

$$1\ \text{Bq} = 1\ \text{decay s}^{-1}$$

It can be shown that the number of moles, n, and the mass, m, of the radioactive nuclei both also decrease exponentially in time, similar to N and A, where n_0 and m_0 are respectively the number of moles and the mass of the radioactive nuclei at $t = 0$.

$$n = n_0 \exp(-\lambda t),$$
$$m = m_0 \exp(-\lambda t)$$



Schematic diagram of a GM tube

We can use a Geiger-Müller tube (GM tube) to measure the intensity of an ionizing radiation (alpha particles, beta particles and gamma radiation). GM tubes are robust and relatively inexpensive, but

(i) it cannot measure radiation energy so cannot distinguish radiation types;
(ii) it cannot last long when exposed to high radioactivity.

Inside the tube, low pressure argon gas is ionized by incoming radiation to form an avalanche of ions between the electrodes. For a brief moment, the gas conducts electricity as a pulse of current. A ratemeter counts the number of pulses per unit time.

The argon ions gain electrons when at the negative electrode, and may emit photons that cause undesirable secondary pulses. A quenching agent prevents this effect but renders the tube unable to respond for a short time. Therefore, the maximum measurable count rate is limited.

20.11.2 Half-life

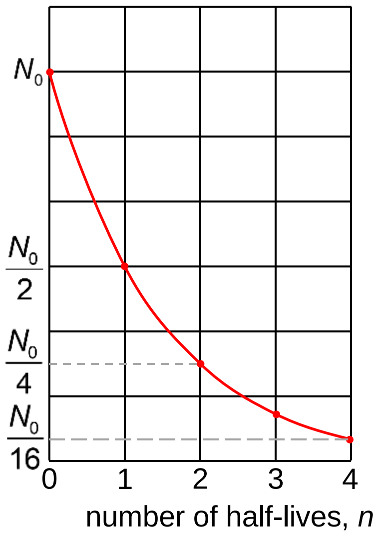
The **half-life**, $t_{1/2}$, of a radioactive nuclide is the time taken for the **number of undecayed nuclei** to be **reduced to half its original number**.

Using the law of radioactive decay,

$$\frac{N_0}{2} = N_0 \exp(-\lambda t_{1/2}) \Rightarrow \frac{1}{2} = \exp(-\lambda t_{1/2})$$

Upon taking natural log on both sides and rearranging,

$$t_{1/2} = \frac{\ln 2}{\lambda}$$



After one half-life, the number of radioactive nuclei and the activity are both reduced by a factor of 2. After n half-life, they are reduced by $(1/2)^n$. Hence, for an arbitrary duration t, one may count the number of half-lives $t_{1/2}$ in t, because each half-life reduces the number by 1/2:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

When using decay equations: $N = N_0 e^{-\lambda t}$ or $A = A_0 e^{-\lambda t}$, ensure that half-life (as in $\lambda = \frac{\ln 2}{t_{1/2}}$) and duration are same units (i.e. both in seconds or both in hours). However, when calculating the decay constant or the activity,

$$A = \lambda N = \frac{\ln 2}{t_{1/2}} N \ ,$$

since activity is in units of Bq (decay s^{-1}), it follows that the half-life needs to be in its SI unit (seconds).

20.12 Background Radiation

Background radiation is the ionising radiation in our environment that humans are constantly exposed to. The sources of background radiation can be natural and artificial. A list of sources is given in the table below.

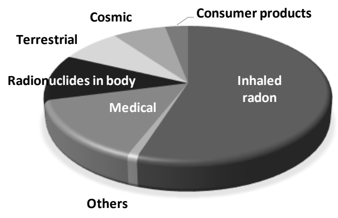
Natural sources	**Artificial sources**
Air, water & food	Medical imaging
Soil & rock (terrestrial)	Consumer products/materials
Cosmic rays (from outer space)	Nuclear accidents
Internal	

The Earth is a natural source of background radiation. Radionuclides such as thorium, uranium and radium exist naturally in soil and rock. These nuclides have extremely long half-life (billions of years). The activity has largely remained constant in the history of mankind. The decay of these radionuclides produces radon – a radioactive gas – which readily seeps out from the ground into underground spaces, buildings and the atmosphere. Radon is by far the biggest source of natural background radiation. Other radioactive sources include underground drinking water, banana, nuts and nearly every natural material.

Cosmic rays from outer space consist of very energetic positively charged ions ranging from protons to iron and even heavier nuclei originating from outside our solar system.

Radioactive nuclides such as potassium-40 and carbon-14 can also be found within our body so we are constantly being exposed to ionizing radiations from within. This is known as internal radiation.

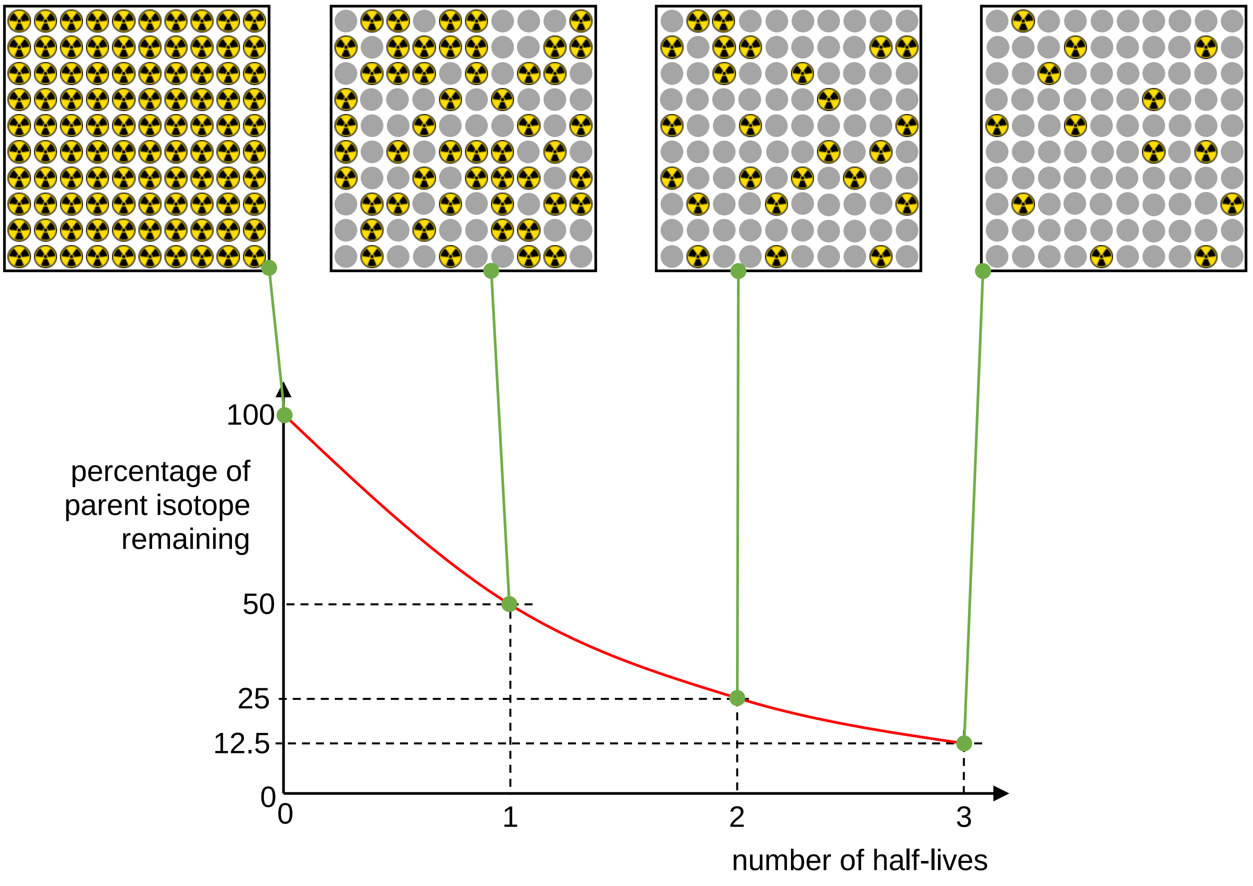
The most significant source of artificial background radiation comes from medical imaging – x-rays, computed tomography (CT), positron emission tomography (PET) and single-photon emission computed tomography (SPECT). Radioactive nuclides such as fluorine-18, iodine-123 and technetium-99m are often used as a tracer.



Contributions to background radiation from various sources.

Consumer products such as tobacco contain radionuclides such as Polonium-210. Heavy smokers are constantly dosed with significant level of radiations to localized spots in the lungs. Construction materials (concrete, stones and granite), petroleum and fuels are also radioactive.

The two most infamous nuclear accidents occurred in Fukushima, Japan (2011) and Chernobyl, Ukraine (1986) which resulted in significant loss of life and substantial contamination to the immediate vicinity and into the atmosphere. In the Fukushima disaster, millions of litres of contaminated radioactive water were discharged into the Pacific Ocean although the radioactivity level was still deemed to be safe.



Example 17

A particular Radioisotope Thermoelectric Generator (RTG) converts heat released by alpha decay of plutonium-238 $\left({}^{238}_{94}\text{Pu} \rightarrow {}^{234}_{92}\text{U} + {}^{4}_{2}\alpha\right)$ into electricity. The half-life of ${}^{238}_{94}\text{Pu}$ is 87.7 years. A spacecraft meant to operate continuously for 10 years requires a minimum of 110 W to run. Given a 5.0% efficiency in conversion of energy, find the mass of pure ${}^{238}_{94}\text{Pu}$ needed at launch.

	mass / *u*
plutonium-238	238.049553
uranium-234	234.040950
alpha particle	4.00151

Solution:

$$\begin{aligned}\text{energy released per reaction} &= (\text{mass difference})c^2 \\ &= (238.049553 - 234.040950 - 4.00151)uc^2 \\ &= (238.049553 - 234.040950 - 4.00151)(1.66\times10^{-27})(3.00\times10^{8})^2 \\ &= 1.059694\times10^{-12}\text{ J}\end{aligned}$$

$$\begin{aligned}\text{activity } A \text{ required after 10 y} &= \frac{110}{(5\%)(1.059694\times10^{-12})} \\ &= 2.076071\times10^{15}\text{ Bq}\end{aligned}$$

$$A = A_0 e^{-\lambda t} = A_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\begin{aligned}\text{initial activity } A_0 &= A\times 2^{\frac{t}{t_{1/2}}} \\ &= (2.076071\times10^{15})\times 2^{\frac{10}{87.7}} \\ &= 2.246814\times10^{15}\text{ Bq}\end{aligned}$$

$$A = \lambda N$$

$$\begin{aligned}&\text{initial number of radioactive nuclei } N_0 \\ &= \frac{A_0}{\lambda} = \frac{A_0}{\left(\frac{\ln 2}{t_{1/2}}\right)} \\ &= \frac{2.246814\times10^{15}}{\left(\frac{\ln 2}{87.7\times365\times24\times60\times60}\right)} = 8.964950\times10^{24}\end{aligned}$$

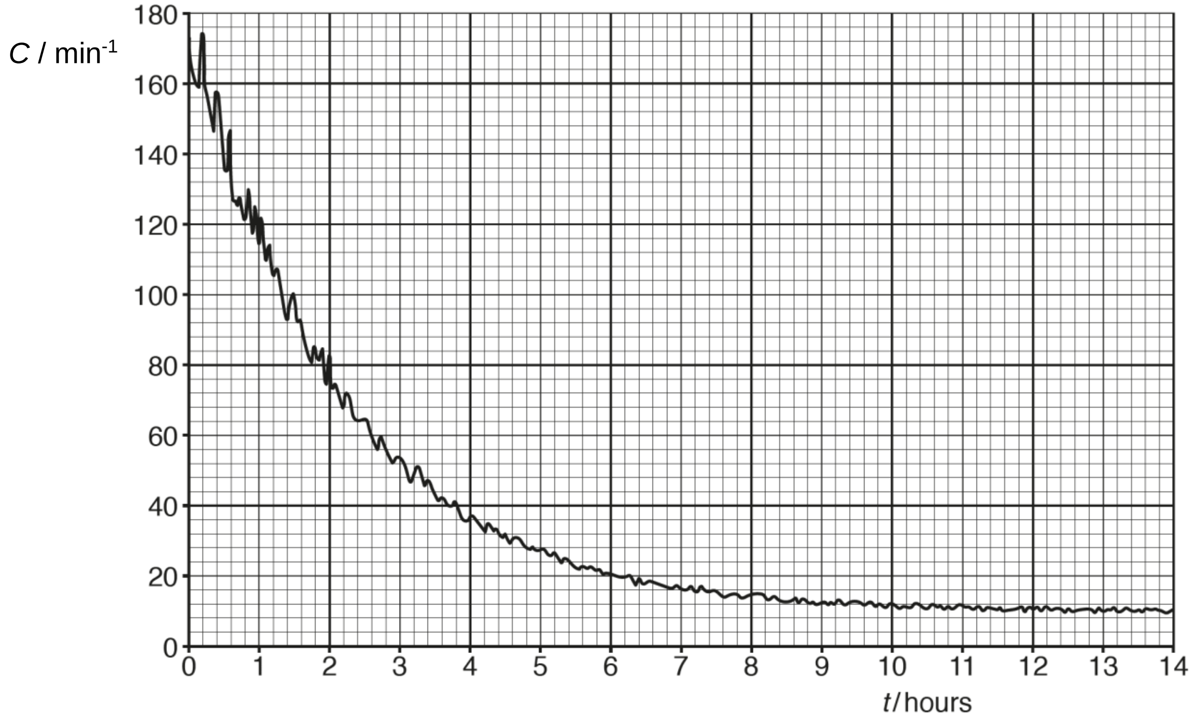
$$\begin{aligned}\text{initial mass} &= mN_0 \\ &= (8.964950\times10^{24})(238.049553)u \\ &= (8.964950\times10^{24})(238.049553)(1.66\times10^{-27}) \\ &= 3.54\text{ kg}\end{aligned}$$

Note: units of years left as-is earlier, but SI unit (seconds) is needed when finding decay constant

Example 18

(a) A GM tube is placed near a radioactive source.
Suggest reasons why the activity and measured count rate C are different.

(b) The variation with time t of C is shown:



(i) State the feature that indicates the random nature of radioactive decay.
(ii) Use the figure to find the half-life of the radioactive isotope in the sample.

(c) The readings were made at room temperature. A 2nd sample of this isotope is heated to 500 °C. The initial count rate, C_0 measured at t = 0 is the same as that in (b), and is tracked with time. Explain the difference, if any, in
(i) the half-life
(ii) the measured count rate at any specific time

Solution:

(a) radiation is emitted in all directions but detector only captures a proportion of radiation OR
radiation absorbed by air/window of detector/source through self-absorption OR
background radiation OR
daughter products are radioactive and so emit ionising radiations (see eg 21)

(b) (i) fluctuations along the curve
(ii) **[method]** draw a smooth curve that estimates the average value through the fluctuations
estimate the background radiation via flat portion after a long time has elapsed,
find **multiple half-lives and average**
estimating background count as 10 min^{-1},
total count-rate decreases from 170 min^{-1} to 30 min^{-1}, hence that due to the radioactive source alone changes from 160 min^{-1} to 20 min^{-1} (a factor of 1/8 = $(1/2)^3$), in about 4.5 h. This corresponds to 3 half-lives. Hence, Averaged half-life is 1.5 hours.

(c) (i) no change in half-life because radioactive decay is spontaneous
(ii) count rate different because radioactive decay is random

Note: Count rate is a (experimentally) measured quantity and therefore carries errors. By right it should decay with the same characteristic half-life but in a smaller proportion as total activity. However, it is subject to random errors (due to the nature of nuclear processes) and systematic errors (due to a constant background radiation from environment).

Below is a little summary on radioactivity.

$\frac{\mathrm{d}N}{dt} = -\lambda N$ $-\frac{\mathrm{d}N}{N} = \lambda dt$	Decay constant λ is the probability of decay per unit time. It has the SI unit of per second (s^{-1}).
$t_{1/2} = \frac{\ln 2}{\lambda}$	Half-life $t_{1/2}$ of a radioactive nuclide is the duration in which its population decreases to half its initial value. It has a SI unit of seconds. Half-life is characteristic to a particular nuclide, and is not affected by external factors such as pressure or temperature.
$A = -\frac{\mathrm{d}N}{\mathrm{d}t}$	Activity is the number of decay events per unit time. Its SI unit is Bq (decay s^{-1}).
$A = \lambda N$	Activity is proportional to the number of radioactive nuclei.
$N = N_0 e^{-\lambda t}$ $A = A_0 e^{-\lambda t}$	Both the number of radioactive nuclei and the activity decay in time exponentially.
$C = C_0 e^{-\lambda t} + C_{\mathrm{bg}}$	Real-world measurement (via e.g. a GM-tube connected to a rate meter) of decay events include the decay due to the radioactive sample ($C_0 e^{-\lambda t}$), whose activity decreases exponentially in time, and the background radiation that is constant in time (C_{bg}).
$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$ $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$	Another way of writing the law of radioactive decay. The population of the radioactive nuclei, or the activity, decreases by $\frac{1}{2}$ after each half-life.

Example 19

As part of a medical procedure, a radioactive tracer of initial activity 204 Bq is injected into a patient. The radioactive nuclide has a half-life of 15 hours. 30 hours after injection, a 1.0 cm^3 blood sample was drawn from the patient and the activity of the sample was found to be 8.5 × 10^{-3} Bq. Find the volume of blood present in the patient.

Solution:

Method 1:

$$\text{total activity after 30 h} = A_0\left(\frac{1}{2}\right)^{\frac{30}{15}} = 51\,\text{Bq}$$

$$\text{total volume} = \frac{A_{\text{total}}}{A_{\text{sample}}}\left(V_{\text{sample}}\right)$$

$$= \left(\frac{51}{8.5\times10^{-3}}\right)(1.0)$$

$$= 6000\text{ cm}^3$$

Method 2:

$$\text{initial activity of 1cm}^3 = A \div \left(\frac{1}{2}\right)^{\frac{30}{15}}$$

$$= \left(8.5\times10^{-3}\right) \div \left(\frac{1}{2}\right)^2$$

$$= 34\times10^{-3}\text{ Bq}$$

$$\text{total volume of blood} = \frac{A_{0,\text{ total}}}{A_{0,\text{ sample}}}\left(V_{\text{sample}}\right)$$

$$= \left(\frac{204}{34\times10^{-3}}\right)(1.0)$$

$$= 6000\text{ cm}^3$$

Note: We assume that the radioactive nuclei are evenly distributed (dissolved) in the blood. Hence, the activity is proportional to the volume of the blood.

Example 20

The ratio of carbon isotopes ^{14}C:^{12}C in living organisms is maintained naturally via process such as respiration and photosynthesis. The ratio decreases after death because ^{14}C is not replaced and undergoes beta decay with a half-life of 5600 years. The measured count rate of a 5.0 g sample of living wood is 400 per min. Find the approximate age an ancient wood if a 10 g sample of it exhibits a count rate of 70 per min. The ratemeter shows 20 counts per min when no sample is present.

Solution:

Since $A = \lambda N$, a more massive sample will exhibit more radioactivity. We consider 10 g of living wood to exhibit 2 × (400 – 20) = 760 counts per min. This is the activity of 10 g of ancient wood when it was alive and undergoing photosynthesis (C_0). The count rate of 10 g of ancient wood at the moment of measurement is 70 – 20 = 50 counts per min.

$$C = C_0 e^{-\lambda t} + C_{\text{bg}}$$

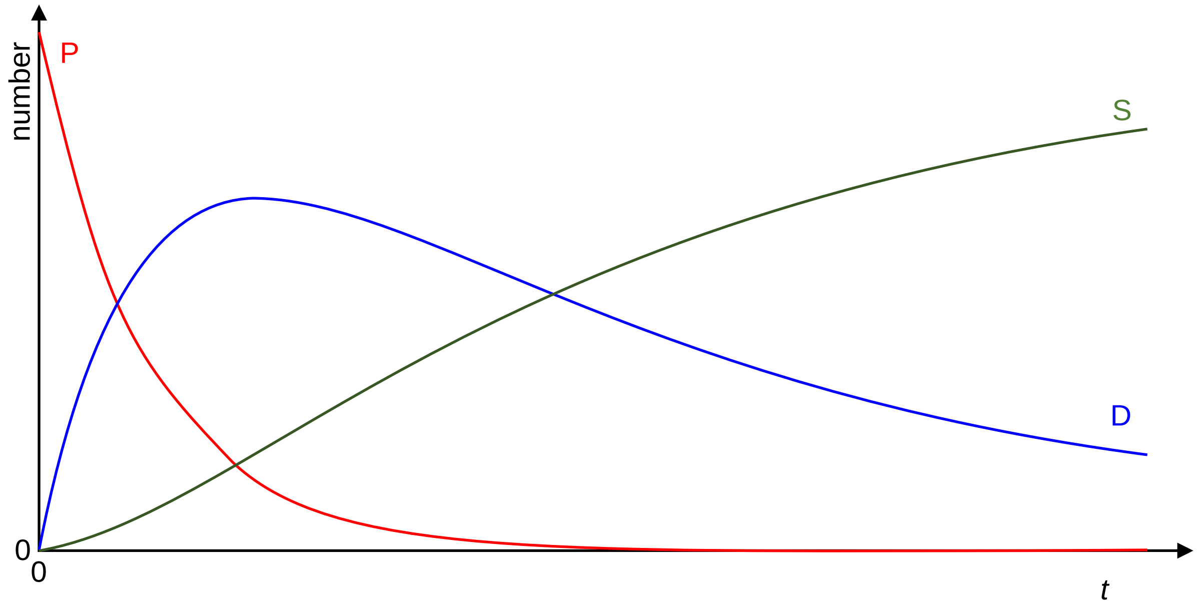
$$C - C_{\text{bg}} = C_0\left(\frac{1}{2}\right)^{\frac{t_{\text{elapsed}}}{t_{1/2}}}$$

$$70 - 20 = 760\left(\frac{1}{2}\right)^{\frac{t_{\text{elapsed}}}{5600}}$$

$$t_{\text{elapsed}} = 21986\text{ years}$$

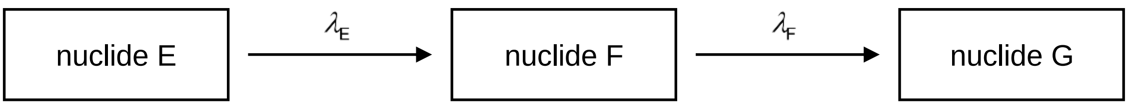
Example 21

An unstable nuclide P has decay constant λ_P and decays to form nuclide D, which is unstable and decays with decay constant λ_D to form a stable nuclide S. A radioactive sample contains only P initially. The variation with time *t* of the number of nuclei of each of the three nuclides is shown.



(a) Use the symbols P, D and S to identify the curve for each of the three nuclides.

(b) In the decay chain for P → D → S, λ_P is approximately $5\lambda_D$. The decay chain of a different nuclide E is shown below:



The decay constant λ_F is very much larger than the decay constant λ_E.

(i) By reference to the half-life of nuclide F, explain why the number of nuclei of nuclide F is always small.

(ii) Sketch the variation with time *t* of the number of nuclei of each of the three nuclides.

Solution:

(b) half-life of F is much shorter than half-life of E
nuclei of F decay almost as soon as they are produced and so does not accumulate in number