1 It is given that $f(r)=\dfrac{1}{(r+1)(r+2)}$.

(a) Show that $f(r-1)-f(r)=\dfrac{2}{r(r+1)(r+2)}$ and find $\displaystyle\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)}$ in terms of n. [4]

(b) **(i)** Deduce the exact value of $\displaystyle\sum_{r=1}^{\infty}\frac{1}{r(r+1)(r+2)}$. [1]

(ii) For $n>3$, deduce an expression for $\displaystyle\sum_{r=3}^{N-1}\frac{1}{r(r-1)(r-2)}$ in terms of N. [2]

2 **(a)** A geometric progression G has positive first term a, common ratio r and sum to infinity S. The sum to infinity of the even-numbered terms of G, i.e. the second, fourth, sixth, … terms, is $-\frac{1}{2}S$.

(i) Find the value of r. [3]

(ii) In another geometric progression H, each term is the modulus of the corresponding term of G. Given that the third term of G is 2, show that the sum to infinity of H is 27. [3]

(b) An arithmetic progression has first term 1000 and common difference -1.4. Determine, with clear workings, the value of the first negative term of the sequence and the sum of all the positive terms. [4]

3 **(a)** ~~Points A and B lie on a parabola such that the line segment AB passes through the focus. Points R and S are the feet of perpendiculars from A and B to the directrix respectively. It is given that RS is m units and the perimeter of $ABSR$ is n units. Express the area of $ABSR$ in terms of m and n. [3]~~

~~**(b)** The conic C has foci at (5, 2) and (5, 10) and it passes through the point (2, 6).~~

~~**(i)** Find the cartesian equation of C in standard form. [3]~~

~~**(ii)** When the variable y in the cartesian equation of C is replaced with $5y$, the resultant equation represents another conic. Find the exact coordinates of the foci of this conic. [3]~~

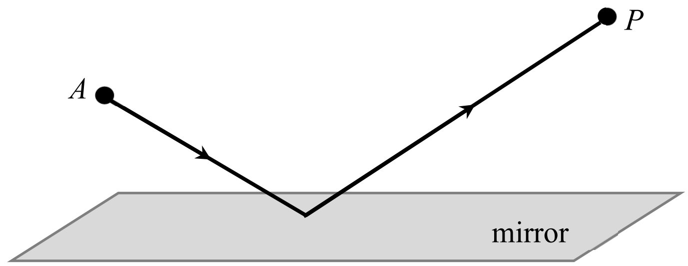
4 Referring to the pole O, the curve C has polar equation $r=\cot\theta$, where $\dfrac{\pi}{6}<\theta<\dfrac{\pi}{2}$.

(a) Sketch the curve C. [2]

(b) Show that $\dfrac{dy}{dx}=\dfrac{1}{r(r^2+2)}$. Determine the exact range of values of the gradient of C. [5]

(c) Obtain a cartesian equation of C in the form $y=f(x)$. [3]

5 Relative to an origin O, an object is placed at point P with coordinates $(-4, c, c)$, where c is a positive real constant, and there is a mirror plane with equation $x + y + z = 1$, as shown in the diagram (not drawn to scale). It is known that the shortest distance between P and the mirror is $3\sqrt{3}$.



(a) Show that $c = 7$. [3]

A point A has coordinates $(-15, 17, 5)$.

(b) Find the coordinates of A', the point of reflection of A in the mirror. [4]

A laser beam is directed from A towards a point on the mirror and is reflected to reach the object at P.

(c) Find the acute angle that the laser beam makes with the mirror. [3]

6 **Do not use a calculator in this question.**

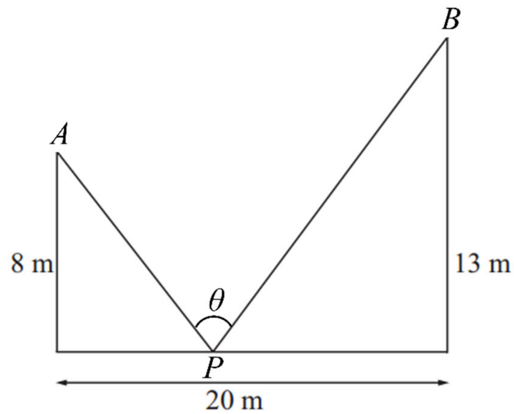
(a) **(i)** ~~It is given that $w = -2 + \mathrm{i}$ and $w^2 + \mathrm{i}w^* = \frac{\mathrm{i}}{z}$. Find the complex number z in the form $x + y\mathrm{i}$, $x, y \in \mathbb{R}$, showing your workings.~~ ~~[3]~~

~~**(ii)** Using the information in **(i)**, determine two values of u that satisfy $u^3 - i|u|^2 + \frac{\mathrm{i}u}{z^*} = 0$, justifying your answer clearly.~~ ~~[3]~~

~~**(b)** Given that $p = \sin\frac{2\pi}{5} - \mathrm{i}\cos\frac{2\pi}{5}$, determine the three smallest positive integers n such that p^n is a negative real number.~~ ~~[4]~~

7 A straight street of width 20 metres is bounded on its parallel sides by two vertical walls, one of height 13 metres, the other of height 8 metres. The intensity of light at point P at ground level on the street is proportional to the angle θ radians, where $\theta = \angle APB$ as shown in the cross-sectional diagram below.

diagram not to scale



(a) Given that the distance of P from the base of the wall of height 8 metres is x metres ($0 \le x \le 20$), show that

$$\theta = \tan^{-1}\frac{x}{8} + \tan^{-1}\frac{20-x}{13}.$$ [1]

(b) Find an expression for $\dfrac{\mathrm{d}\theta}{\mathrm{d}x}$. [3]

(c) Hence, determine the value of x corresponding to the maximum light intensity at P. Give your answer to four significant figures. You need not justify that the value of x obtained gives the maximum light intensity at P. [2]

(d) Find the minimum value of θ as x varies. [2]

(e) The point P moves across the street from the base of A to the base of B with speed 0.5 ms^{-1}. Determine the rate of change of θ with respect to time when P is at the midpoint of the street. [3]