1 The number of units, $D(x)$ of a particular product that people are willing to purchase per week in city A at a price \$$x$ is given by the function $D(x)=\dfrac{40320}{\mathrm{g}(x)}$, where $\mathrm{g}(x)$ is a quadratic polynomial in x. The following table shows the number of units people are willing to purchase at different prices.

x	5	8	10
$D(x)$	384	224	168

Find the number of units of the product that people are willing to purchase at a price of \$18. [4]

2 It is given that $p_n=\ln\dfrac{1+x^n}{1+x^{n+1}}$, where $-1<x<1$ and n is a positive integer.

(i) Find $\displaystyle\sum_{n=1}^{N} p_n$, giving your answer in terms of N and x. [3]

(ii) Hence find the sum to infinity of the series in part **(i)** in terms of x. [2]

3 In the triangle ABC, $AB=1$, $AC=2$ and angle $ABC=\left(\dfrac{\pi}{2}-x\right)$ radians. Given that x is sufficiently small for x^3 and higher powers of x to be ignored, show that $BC\approx p+qx+rx^2$, where p, q, r are constants to be determined in exact form. [5]

4 A part of a hyperbola has equation given by $\mathrm{f}(x)=\sqrt{\dfrac{(x+5)^2}{36}-1}$, $x\in D$, where $D\subseteq\mathbb{R}$.

(i) State the largest possible set D. [1]

(ii) State the equations of the asymptotes of $y=\mathrm{f}(x)$. [1]

(iii) Sketch the graph of $y=\mathrm{f}(x)$ for D in part **(i)**, showing clearly all the features of the curve. [2]

(iv) On separate diagrams, sketch the graphs of $y=\dfrac{1}{\mathrm{f}(x)}$ and $y=\mathrm{f}'(x)$, showing clearly all the features of the curves. [4]

5 Referred to the origin O, points A and B have position vectors **a** and **b** respectively. Point C lies on OB produced such that $\overrightarrow{OC} = \lambda\overrightarrow{OB}$ where $\lambda > 1$. Point D is such that $OCDA$ is a parallelogram. Point M lies on AD, between A and D, such that $AM : MD = 1 : 2$. Point N lies on OC, between O and C, such that $ON : NC = 4 : 3$.

(i) Find the position vectors of M and N, in terms of **a**, **b** and λ. [2]

(ii) Show that the area of triangle OMD is $k\lambda|\mathbf{a} \times \mathbf{b}|$, where k is a constant to be determined. [4]

(iii) The vector **p** is a unit vector in the direction of $\overrightarrow{OD}$. Give a geometrical meaning of $|\mathbf{p}.\mathbf{a}|$. [1]

6 A curve C is defined by the parametric equations

$$x = 25\sin^2 t, \quad y = 2\cos t$$

where $0 \leq t \leq \pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) The tangent to C at the point where $t = \dfrac{\pi}{4}$ cuts the x-axis at P and the y-axis at Q, find the exact area of the triangle OPQ, where O is the origin. [4]

(iii) State the equation of the normal to C where the normal is parallel to the x-axis. [1]

7 **(a)** **(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2\mathrm{e}^{\cos\frac{x}{2}}\right)$. [1]

(ii) Hence find $\displaystyle\int \frac{1}{2}\sin x\, \mathrm{e}^{\cos\frac{x}{2}}\, \mathrm{d}x$. [4]

(b) Using the substitution $u = 1 - \mathrm{e}^x$, find $\displaystyle\int \frac{1}{1-\mathrm{e}^x}\, \mathrm{d}x$. [4]

8 The function f is defined by $\mathrm{f}: x \mapsto \sqrt{a^2 - \frac{(x-a)^2}{4}}$ for $x \in \mathbb{R}$, $-a \le x \le 3a$, where a is a positive constant.

(i) Sketch the graph of $y = \mathrm{f}(x)$, giving the coordinates of any stationary points and the points where the graph meets the axes. [2]

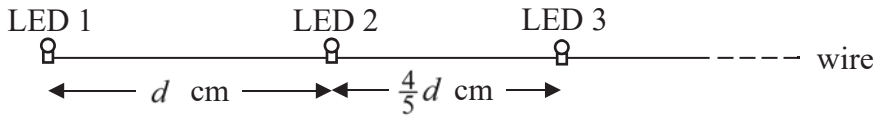
(ii) If the domain of f is further restricted to $-a \le x \le k$, state the greatest value of k for which the function f^{-1} exists. [1]

(iii) Using the restricted domain found in part **(ii)**, find f^{-1} in similar form. [3]

The function g is defined by $\mathrm{g}(x) = \mathrm{f}\left(\frac{3}{2}x\right)$ for $x \in \mathbb{R}$, $-\frac{2}{3}a \le x \le 2a$.

(iv) Explain why the composite function gf exists and find the range of gf. [3]

9 A company produces festive decorative Light Emitting Diode (LED) string lights, where micro LEDs are placed at intervals along a thin wire. In a particular design, the first LED (LED 1) is placed on one end of a wire with the second LED (LED 2) placed at a distance of d cm from LED 1, and each subsequent LED is placed at a distance $\frac{4}{5}$ times the preceding distance as shown.



(i) If the distance between LED 8 and LED 9 is 56.2 cm, find the value of d correct to 1 decimal place. [2]

(ii) Find the theoretical maximum length of the wire, giving your answer in centimetres correct to 1 decimal place. [2]

The LEDs consisting of three colours red, orange and yellow, are arranged in that order in a repeated manner, that is, LED 1 is red, LED 2 is orange, LED 3 is yellow, LED 4 is red, LED 5 is orange, LED 6 is yellow, and so on.

(iii) Find the colour of the LED nearest to a point on the wire 12.9 m from LED 1. [3]

(iv) If the minimum distance between any two consecutive LEDs is 1 cm so that they can be mounted on the wire, find the colour of the last LED. [3]

10 **(a)** Solve the simultaneous equations $v + \mathrm{i}u = 2$ and $av - 2u = 3\mathrm{i}$, where a is a real constant. Simplify your answers to cartesian form $x + \mathrm{i}y$, where x and y are in terms of a. [4]

(b) It is given that $(x + k)$ is a factor of the equation,

$$bx^3 + (12b + \mathrm{i})x^2 + (b + 12\mathrm{i})x + 12b = 0,$$

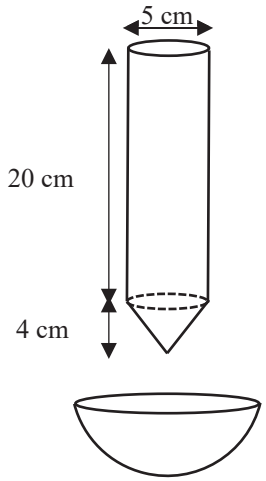
where k and b are non-zero real constants.

(i) Find the value of k. [2]

(ii) Show that the roots of the equation $bx^2 + \mathrm{i}x + b = 0$ are purely imaginary. [2]

(iii) Hence express $\mathrm{f}(x) = bx^3 + (12b + \mathrm{i})x^2 + (b + 12\mathrm{i})x + 12b$ as a product of three linear factors, leaving your answers in terms of b. [2]

11 A container is made up of a cylinder and an inverted right circular cone as shown in the diagram below. The height and the diameter of the cylinder are 20 cm and 5 cm respectively. The height of the cone is 4 cm. An external device ensures liquid flows out through a small hole at the vertex of the cone into a bowl below at a constant rate of 18 cm^3 per minute. The depth of the liquid and the radius of the liquid surface area in the container at the time t minutes are x cm and r cm respectively. The container is full of liquid initially.

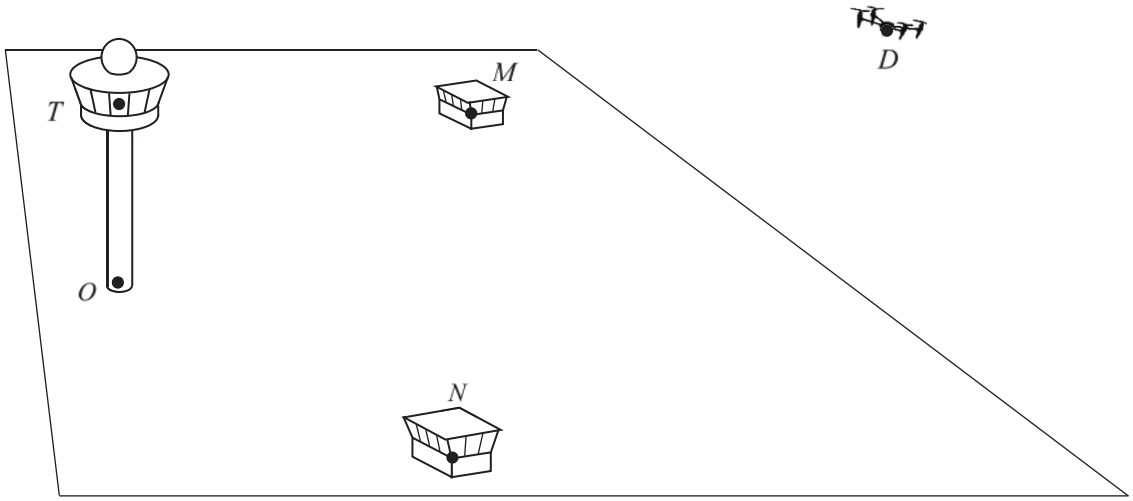


(a) When $x > 4$, find the rate of change of the depth of the liquid in the container. [4]

(b) Find the rate of change of r when $x = 2$. [5]

(c) The bowl as shown in the diagram in part **(a)** is generated by rotating part of the curve $\frac{x^2}{225} + \frac{y^2}{100} = 1$ which is below the x-axis through π radians about the y-axis. Assuming the bowl has negligible thickness, find the volume of the empty space in the bowl when the liquid has completely flowed from the container into the bowl, giving your answer correct to 3 decimal places. [3]

12 At an airport, an air traffic control room T is located in a vertical air traffic control tower, 70 m above ground level. Let $O(0,0,0)$ be the foot of the air traffic control tower and all points (x,y,z) are defined relative to O where the units are in kilometres. Two observation posts at the points $M(0.8,0.6,0)$ and $N(0.4,-0.9,0)$ are located within the perimeters of the airport as shown.



An air traffic controller on duty at T spots an errant drone in the vicinity of the airport. The two observation posts at M and N are alerted immediately. A laser rangefinder at M directs a laser beam in the direction $\begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix}$ at the errant drone to determine D, the position of the errant drone. The position D is confirmed using another laser beam from N, which passes through the point $(0.8, 0.75, 0.3)$, directed at the errant drone.

(i) Show that D has coordinates $(0.56,-0.24,0.12)$. [4]

A Drone Catcher, an anti-drone drone which uses a net to trap and capture errant drones, is deployed instantly from O and flies in a straight line directly to Γ intercept the errant drone.

(ii) Find the acute angle between the flight path of the Drone Catcher and the horizontal ground. [2]

At the same time, a Jammer Gun, which emits a signal to jam the control signals of the errant drone, is fired at the errant drone. The Jammer Gun is located at a point G on the plane p containing the points T, M and N.

(iii) Show that the equation of p is $\mathbf{r} \cdot \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = -6.72$. [3]

It is also known that the Jammer Gun is at the foot of the perpendicular from the errant drone to plane p.

(iv) Find the coordinates of G. [3]

(v) Hence, or otherwise, find the distance GD in metres. [2]

Section A: Pure Mathematics [40 marks]

1 On the same axes, sketch the graphs of $y = 2(x-a)^2$ and $y = 3a|x-a|$, where a is a positive constant, showing clearly all axial intercepts. [2]

(i) Solve the inequality $2(x-a)^2 \geq 3a|x-a|$. [4]

(ii) Hence solve $2\left(x-\frac{a}{2}\right)^2 \geq 3a\left|x-\frac{a}{2}\right|$. [2]

2 It is given that $y = \frac{e^{\sin x}}{\sqrt{1+2x}}$.

(i) Show that $\frac{1}{y}\frac{dy}{dx} + \frac{1}{1+2x} = \cos x$. [2]

(ii) By further differentiation of the result in part **(i)**, find the Maclaurin series for y in ascending powers of x, up to and including the term in x^3. [5]

(iii) Use your result from part **(ii)** to approximate the value of $\int_0^1 \frac{e^{\sin x}}{\sqrt{1+2x}}dx$. Explain why this approximation obtained is not good. [2]

(iv) Deduce the Maclaurin series for $\frac{1}{e^{\sin x}\sqrt{1-2x}}$ in ascending powers of x, up to and including the term in x^3. [1]

3 The complex numbers p and q are given by $\frac{a}{1+\sqrt{3}i}$ and $-\frac{a}{2}i$ respectively, where a is a positive real constant.

(i) Find the modulus and argument of p. [2]

(ii) Illustrate on an Argand diagram, the points P, Q and R representing the complex numbers p, q and $p+q$ respectively. State the shape of $OPRQ$.
Hence, find the argument of $p+q$ in terms of π and the modulus of $p+q$ in exact trigonometrical form. [6]

(iii) Find the smallest positive integer n such that $(p+q)^n$ is purely imaginary. [2]

4 **(a)** When studying a colony of bugs, a scientist found that the birth rate of the bugs is inversely proportional to its population and the death rate is proportional to its population. The population of the bugs (in thousands) at time t days after they were first observed is denoted by P. It was found that when the population is 2000, it remains constant.

(i) Assuming that P and t are continuous variables, show that $\frac{\mathrm{d}P}{\mathrm{d}t} = k\left(\frac{4}{P} - P\right)$, where k is a constant. [3]

(ii) Given that the initial population of the bugs was 4000, and that the population was decreasing at the rate of 3000 per day at that instant, find P in terms of t. [4]

(iii) Sketch the graph of P against t, giving the equation of any asymptote(s). State what happens to the population of the bugs in the long run. [2]

(b) Another population of bugs, N (in thousands) in time t days can be modelled by the differential equation $\frac{\mathrm{d}N}{\mathrm{d}t} = 4 + \frac{N}{t}$ for $t \geq 1$. Using the substitution $u = \frac{N}{t}$, solve this equation, given that the population was 1000 when $t = 1$. [3]

Section B: Statistics [60 marks]

5 The daily rainfall in a town follows a normal distribution with mean μ mm and standard deviation σ mm. Assume that the rainfall each day is independent of the rainfall on other days. It is given that there is a 10% chance that the rainfall on a randomly chosen day exceeds 9.8 mm, and there is a 10% chance that the mean daily rainfall in a randomly chosen 7-day week exceeds 8.2 mm.

(i) Show that $\sigma = 2.01$, correct to 2 decimal places. [4]

(ii) Find the maximum value of k such that there is a chance of at least 10% that the mean daily rainfall in a randomly chosen 30-day month exceeds k mm. Give your answer correct to 1 decimal place. [2]

6 Miss Tan carried out an investigation on whether there is a correlation between the amount of time spent on social media and exam scores. The average amount of time spent per month on social media, x hours, and the final exam score, y marks, of 6 randomly selected students from HCI were recorded. The data is shown below.

x	80	84	70	74	58	48
y	44	40	49	45	58	82

(i) Draw a scatter diagram to illustrate the data. [2]

(ii) It is found that the inclusion of a 7th point (x_7, y_7) will not affect the product moment correlation coefficient for the data. Find a possible point (x_7, y_7). [1]

Omit the 7th point (x_7, y_7) for the rest of this question.

(iii) State, with reason, which of the following equations, where a and b are constants, provides the most appropriate model for the relationship between x and y.

(A) $y = a + bx^2$,

(B) $e^y = ax^b$,

(C) $y = a + b\sqrt{x}$. [3]

(iv) Using the model chosen in part **(iii)**, estimate the score of a student who spent an average of 60 hours per month on social media, giving your answer correct to the nearest whole number. [2]

(v) Sam spends an average of 4 hours a day on social media. Assuming a 30-day month, suggest whether it is still reasonable to use the model in part **(iii)** to estimate his score. [1]

7 A cafe sells sandwiches in 2 sizes, "footlong" and "6-inch". The lengths in inches of "footlong" loaves have the distribution $\mathrm{N}(12.2, 0.04)$ and the lengths in inches of "6-inch" loaves have the distribution $\mathrm{N}(6.1, 0.02)$.

(i) Is a randomly chosen "footlong" loaf more likely to be less than 12 inches in length or a randomly chosen "6-inch" loaf more likely to be less than 6 inches in length? [2]

(ii) Find the probability that two randomly chosen "6-inch" loaves have total length more than one randomly chosen "footlong" loaf. [2]

Sue buys a "6-inch" sandwich 3 times a week.

(iii) Find the probability that Sue gets at most one sandwich that is less than 6 inches in length in a randomly chosen week. [2]

(iv) Given that Sue gets more than four sandwiches that are less than 6 inches in length in a randomly chosen 4-week period, find the probability that she gets exactly one such sandwich in the first week. [3]

8 The individual letters of the word PARALLEL are printed on identical cards and arranged in a straight line.

(a) Find the number of arrangements such that

(i) there are no restrictions, [1]

(ii) no L is next to any other L, [2]

(iii) the arrangements start and end with a consonant and all the vowels are together. [3]

(b) The cards are now placed in a bag and Tom draws the cards randomly from the bag one at a time.

(i) 4 cards are drawn without replacement. Find the probability that there is at least one vowel drawn. [2]

(ii) Tom decides to record the letter of the card drawn, on a piece of paper. If the letter on the card drawn is a vowel, Tom will put the drawn card back into the bag and continue with the next draw.

If the letter on the card drawn is a consonant, Tom will remove the card from subsequent draws. Find the probability that Tom records more consonants than vowels at the end of 3 draws. [3]

9 A company purchased a machine to pack shower gel into its bottles. The expected mean volume of shower gel in a bottle is 950 ml.

(a) The floor supervisor believes that the machine is packing less amount of shower gel than expected. A random sample of 80 bottles is taken and the data is as follows:

Volume of shower gel in a bottle (correct to nearest ml)	948	949	950	951	952	953	955
Number of bottles	9	22	36	6	4	1	2

(i) Find unbiased estimates of the population mean and variance, giving your answers correct to 2 decimal places. [2]

(ii) Write down the appropriate hypotheses to test the floor supervisor's belief. You should define any symbols used. [2]

(iii) Using the given data, find the *p*-value of the test. State what is meant by this *p*-value in the context of this question. [2]

(iv) It was concluded at α% level of significance that the machine is indeed packing less amount of shower gel than expected. State the set of values of α. [1]

(b) Due to a change in marketing policy, the machine is being recalibrated to pack smaller bottles of shower gel with mean volume of 250 ml. The volume of a recalibrated bottle of shower gel is denoted by Y ml. A random sample of 50 bottles of y ml each is taken and the data obtained is summarised by:

$$\sum (y - 250) = -25, \qquad \sum (y - 250)^2 = k.$$

Another test was conducted at the 1% significance level. The test concluded that the machine had been calibrated incorrectly. Find the range of values of k, correct to 1 decimal place. [4]

(c) Explain why there is no need for the floor supervisor to know anything about the population distribution of the volume of shower gel in a bottle for both parts **(a)** and **(b)**. [1]

10 In a game with a 4-sided fair die numbered 1 to 4 on each face, the score for a throw is the number on the bottom face of the die. A player gets to choose either option A or option B.

Option A: The player rolls the die once. The score x is the amount of money \$$x$ that the player wins.

Option B: The player rolls the die twice. The first score is x and the second score is y. If $y > x$, the player wins \$$2xy$, but if $y < x$, the player loses \$$(x - y)$. Otherwise, he neither wins nor loses any money.

(i) Find the expected amounts won by a player in one game when playing option A and when playing option B. Show that option B is a better option. [5]

(ii) Suggest why a risk averse player would still choose option A. [1]

(iii) Show that the variance of the amount won by a player in one game when playing option A is 1.25. [2]

In a competition, Abel and Benson each play the game 50 times. Abel chooses option A and Benson chooses option B.

It is given that the variance of the amount won by a player in one game when playing option B is $\frac{887}{16}$.

(iv) Find the distributions of the total amounts won by Abel and Benson respectively in the competition. [2]

(v) Show that the probability of the total amount won by Abel exceeding the total amount won by Benson in the competition is approximately 0.120. [3]

ANNEX

H2 MA 2019 JC2 Prelim (Paper 1 and Paper 2)

***Filename**: Change* SCHOOL *to your school name, e.g. NYJC*

Paper 1

Select topic from the dropdown list. If the question consists of multiple topics, choose 1 topic.

QN	TOPIC (H2) Paper 1	ANSWERS (Exclude graphs and text answers)
1	Equations & Inequalities	72 units
2	Sigma Notation & MOD	(i) $\ln\dfrac{1+x}{1+x^{N+1}}$ (ii) $\ln(1+x)$
3	Maclaurin & Binomial Series	$p=\sqrt{3}, q=1, r=\dfrac{1}{2\sqrt{3}}$
4	Graphs & Transformations	(i) $D=(-\infty,-11]\cup[1,\infty)$ (ii) $y=\pm\dfrac{x+5}{6}$
5	Vectors	(i) $\overrightarrow{OM}=\dfrac{1}{3}(3\mathbf{a}+\lambda\mathbf{b}), \overrightarrow{ON}=\dfrac{4}{7}\lambda\mathbf{b}$ (ii) $k=\dfrac{1}{3}$
6	Differentiation & Applications	(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-1}{25\cos t}$ (ii) $\dfrac{225}{4\sqrt{2}}$ (iii) $y=0$
7	Integration & Applications	(a)(i) $-\sin\dfrac{x}{2}\left(\mathrm{e}^{\cos\frac{x}{2}}\right)$ (ii) $-2\cos\dfrac{x}{2}\left(\mathrm{e}^{\cos\frac{x}{2}}\right)+2\mathrm{e}^{\cos\frac{x}{2}}+C$ (b) $-\ln\left\lvert 1-\mathrm{e}^{x}\right\rvert+x+C$
8	Functions	(ii) greatest value of k is a. (iii) $f^{-1}: x\mapsto a-2\sqrt{a^2-x^2},\ x\in\mathbb{R},\ 0\le x\le a$ (iv) $\mathrm{R}_{\mathrm{gf}}=\left[\dfrac{\sqrt{3}}{2}a, a\right]$
9	APGP	(i) $d=268.0$ (ii) 1339.9 cm (iii) red

		(iv) yellow
10	Complex Numbers	(a) $u=\frac{a}{4+a^2}-\frac{6+2a^2}{4+a^2}\text{i}, v=\frac{2}{4+a^2}-\frac{a}{4+a^2}\text{i}$ (b)(i) $k=12$ (iii) $b(x+12)(x+\frac{1+\sqrt{1+4b^2}}{2b}\text{i})(x+\frac{1-\sqrt{1+4b^2}}{2b}\text{i})$
11	Differentiation & Applications	(a) -0.916 cm/min (b) -2.29 cm/min (c) 4293.510 cm^3
12	Vectors	(ii) $11.1°$ (iv) $(0.547,-0.237,0.00325)$ (v) 117 m
13	H2 Prelim P1 Q13 Topic	
14	H2 Prelim P1 Q14 Topic	

Paper 2

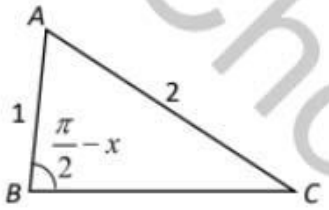
Select topic from the dropdown list. If the question consists of multiple topics, choose 1 topic.

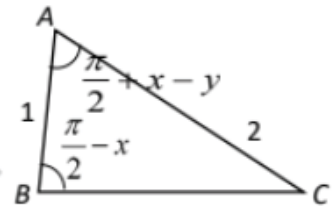
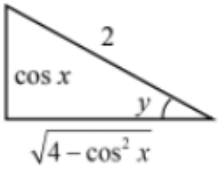
QN	TOPIC (H2) Paper 2	ANSWERS (Exclude graphs and text answers)
1	Equations & Inequalities	(i) $x\le-\frac{a}{2}$ or $x=a$ or $x\ge\frac{5a}{2}$ (ii) $x\le-a$ or $x=\frac{a}{2}$ or $x\ge 2a$
2	Maclaurin & Binomial Series	(ii) $y=1+x^2-\frac{3}{2}x^3+\ldots$ (iii) 0.985 (iv) $\frac{1}{e^{\sin x}\sqrt{1-2x}}\approx 1+x^2+\frac{3}{2}x^3$
3	Complex Numbers	(i) $\frac{a}{2}; -\frac{\pi}{3}$ (ii) $-\frac{5\pi}{12}; a\cos\frac{\pi}{12}$ (iii) 6
4	Differential Equations	(a) (ii) $P=2\sqrt{1+3e^{-2t}}$ (iii) The population of the bugs will decrease and approach 2000 in the long run. (b) $N=4t\ln t+t$
5	Normal Distribution	(ii) 7.6
6	Correlation & Regression	(ii) $(69,53)$ (iii) Since $\lvert r\rvert=0.96785$ for Model B is nearest to 1, Model B is the most accurate model (iv) 62 marks (v) unreasonable as 120 is not within data range

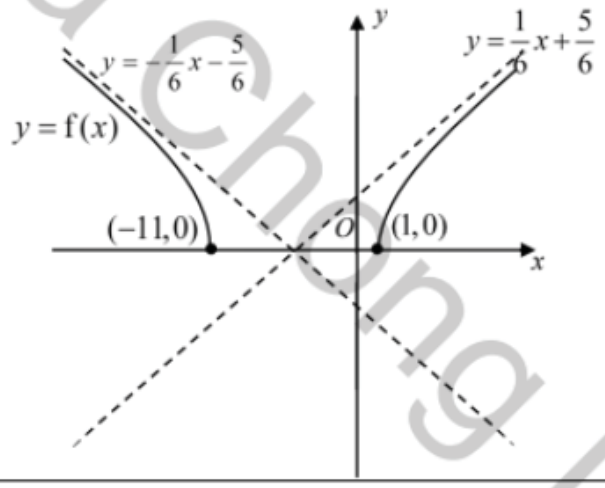
		where $48 \le x \le 84$
7	Normal Distribution	(i) a "6-inch" loaf more likely to be less than 6 inches (ii) $\frac{1}{2}$ (iii) 0.855 (iv) 0.446
8	PnC & Probability	(a) (i) 3360 (ii) 1200 (iii) 240 (b) (i) $\frac{13}{14}$ (ii) 0.644
9	Hypothesis Testing	(a) (i) 949.84 ; 1,73 (ii) H_0: $\mu = 950$ H_1: $\mu < 950$ (iii) 0.135 (iv) $\{\alpha \in \mathbb{R} : 13.5 \le \alpha \le 100\}$ (b) $12.5 \le k \le 104.8$ (c) There is no need for the floor supervisor to assume the volume of shower gel follow a normal distribution as the sample sizes in both part (a) and (b) are large, the sample mean volume of shower gel can be approximated to follow a normal distribution by Central Limit Theorem.
10	DRV	(i) \$2.50 ; \$3.75 (ii) Option A is a "sure win" option where the player would definitely gain a positive amount in all cases, whereas option B has a risk of losing money in some cases. (iv) $A \sim \mathrm{N}\left(125, \frac{125}{2}\right)$; $B \sim \mathrm{N}\left(\frac{375}{2}, \frac{22175}{8}\right)$
11	H2 Prelim P2 Q11 Topic	
12	H2 Prelim P2 Q12 Topic	
13	H2 Prelim P2 Q13 Topic	
14	H2 Prelim P2 Q14 Topic	

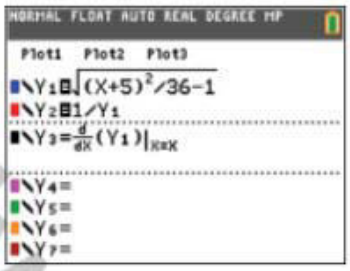
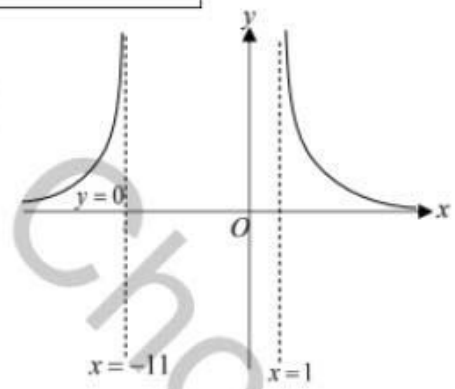
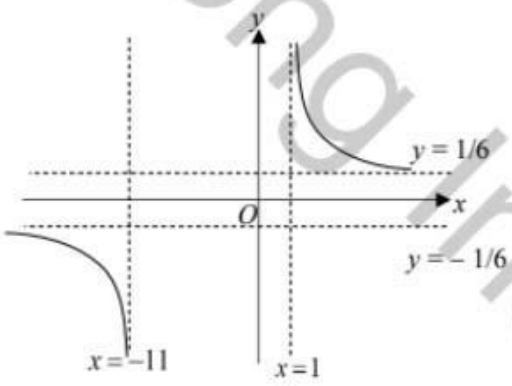
2019 C2 H2 Prelim P1 Markers Comments

Qn	Solutions	Comments
1	$D(x)=\dfrac{40320}{g(x)}$ $\Rightarrow g(x)=\dfrac{40320}{D(x)}$ $ax^2+bx+c=\dfrac{40320}{D(x)}$ Given $5^2a+5b+c=\dfrac{40320}{384}=105$ -- (1) $8^2a+8b+c=\dfrac{40320}{224}=180$ -- (2) $10^2a+10b+c=\dfrac{40320}{168}=240$ -- (3) Using GC, $a=1,\ b=12,\ c=20$ When $x=18$, $D(18)=\dfrac{40320}{18^2+12(18)+20}=72$ units	Generally, this question is well done. A few students mixed up $g(x)$ and $D(x)$ and let $5^2a+5b+c=384$ instead of $5^2a+5b+c=\dfrac{40320}{384}=105$. A few students let $g(x)$ be a degree 3 or 4 polynomial instead, of a quadratic polynomial.
2i	Method 1: (method of differences) $\sum_{n=1}^{N}p_n=\sum_{n=1}^{N}\ln\dfrac{1+x^n}{1+x^{n+1}}$ $=\sum_{n=1}^{N}\left(\ln(1+x^n)-\ln(1+x^{n+1})\right)$ $=\ln(1+x)-\ln(1+x^2)$ $+\ln(1+x^2)-\ln(1+x^3)$ $+\ln(1+x^3)-\ln(1+x^4)$ $\vdots$ $+\ln(1+x^N)-\ln(1+x^{N+1})$ $=\ln(1+x)-\ln(1+x^{N+1})$ $=\ln\dfrac{1+x}{1+x^{N+1}}$	This part is generally well done. Students must write down a few rows of the summation and show the diagonal cancellation clearly, instead of skipping the working. Some students listed out the entire summation all on 1 line, instead of 2 neat columns. It is difficult to see what terms to cancel. A few students gave the final answer as $\ln(1+x)-\ln(1+x^{n+1})$ instead of $\ln(1+x)-\ln(1+x^{N+1})$.

	Method 2: (using property of logarithm) $\sum_{n=1}^{N} p_n = \sum_{n=1}^{N} \ln \frac{1+x^n}{1+x^{n+1}}$ $= \ln \frac{1+x}{1+x^2} + \ln \frac{1+x^2}{1+x^3} + \ln \frac{1+x^3}{1+x^4} + \ldots + \ln \frac{1+x^N}{1+x^{N+1}}$ $= \ln \frac{1+x}{1+x^2} \cdot \frac{1+x^2}{1+x^3} \cdot \frac{1+x^3}{1+x^4} \cdot \ldots \cdot \frac{1+x^N}{1+x^{N+1}}$ $= \ln \frac{1+x}{1+x^{N+1}}$	
2ii	Since $-1 < x < 1$, as $N \to \infty$, $x^{N+1} \to 0$. $\therefore \sum_{n=1}^{\infty} p_n = \ln \frac{1+x}{1+0}$ $= \ln(1+x)$	Some students thought that as $N \to \infty$, $x^{N+1} \to \infty$.
3	 By Cosine Rule, $\cos\left(\frac{\pi}{2} - x\right)$ $2^2 = 1^2 + (BC)^2 - 2(BC)\cos\left(\frac{\pi}{2} - x\right)$ $4 = 1 + (BC)^2 - 2(BC)\sin x$ $(BC)^2 - 2(BC)\sin x - 3 = 0$ $BC = \frac{2\sin x \pm \sqrt{4\sin^2 x + 12}}{2}$ $= \sin x \pm \sqrt{\sin^2 x + 3}$ $\approx x \pm \sqrt{x^2 + 3}$ $= x + \sqrt{x^2 + 3}$, since $BC > 0$ $= x + \sqrt{3}\left[1 + \frac{x^2}{3}\right]^{\frac{1}{2}}$ $= x + \sqrt{3}\left[1 + \frac{1}{2}\frac{x^2}{3} + \ldots\right]$ $\approx \sqrt{3} + x + \frac{x^2}{2\sqrt{3}}$ $p = \sqrt{3}, q = 1, r = \frac{1}{2\sqrt{3}}$	Students recalled Cosine Rule Wrongly "$+2BC$" instead of "$-2BC$", or wrongly located the angle Two sets of solutions produced because students include $BC < 0$ Some students applied sine rule but mostly made mistakes at identifying the other angles with new unknowns. Therefore they should use the easiest method which is the Cosine Rule where there is only one unknown $\cos\left(\frac{\pi}{2} - x\right)$ involved.

	Alternative Solution  By sine rule, $\frac{BC}{\sin\left(\frac{\pi}{2}+x-y\right)}=\frac{2}{\sin\left(\frac{\pi}{2}-x\right)}=\frac{1}{\sin y}$ $\Rightarrow \frac{BC}{\cos(y-x)}=\frac{2}{\cos x}=\frac{1}{\sin y}$  $\Rightarrow BC=\frac{2\cos(y-x)}{\cos x}$ $\Rightarrow BC=\frac{2(\cos y\cos x+\sin y\sin x)}{\cos x}$ $\Rightarrow BC=\frac{2(\cos y\cos x+\sin y\sin x)}{\cos x}$ $\Rightarrow BC=\frac{2\left(\frac{\sqrt{4-\cos^2 x}}{2}\cos x+\frac{\cos x}{2}\sin x\right)}{\cos x}$ $\Rightarrow BC=\frac{\sqrt{4-\cos^2 x}\cos x+\cos x\sin x}{\cos x}$ $\Rightarrow BC=\sqrt{4-\cos^2 x}+\sin x$ $\Rightarrow BC=\sqrt{3+\sin^2 x}+\sin x$	
4i	Sketch the graph using GC, remember to set window appropriately. OR Notice that it is the upper half of a hyperbola. $y=\sqrt{\frac{(x+5)^2}{36}-1}$ $\frac{(x+5)^2}{6^2}-y^2=1$ We have $D=(-\infty,-11]\cup[1,\infty)$.	Common Mistakes are: Conclude with range of x instead of set of x Only one side of D concluded, e.g. $D=[1,\infty)$ or $D=(-\infty,-11]$ because students overlooked the two values to obtain 36 from $(x+5)^2$

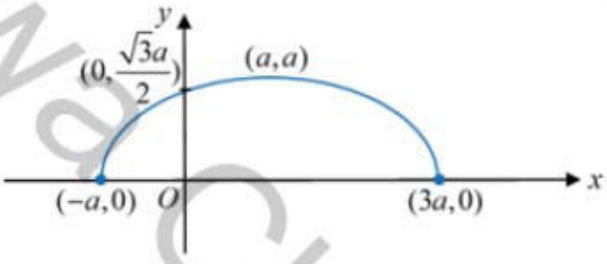
ii	Let $y = \sqrt{\frac{(x+5)^2}{36} - 1}$ $\frac{(x+5)^2}{6^2} - y^2 = 1$ To find asymptotes: $y^2 = \frac{(x+5)^2}{6^2}$ $y = \pm\frac{x+5}{6}$	Students wrongly included -1 when $x \to \pm\infty$ and thus ended up wrong asymptotes Only one asymptote is drawn as students omitted the negative square root
iii		Only half of the graph is drawn because of mistakes made at (i) and (ii) Drawn the whole hyperbola as students overlooked that only the positive square root is involved in the graph

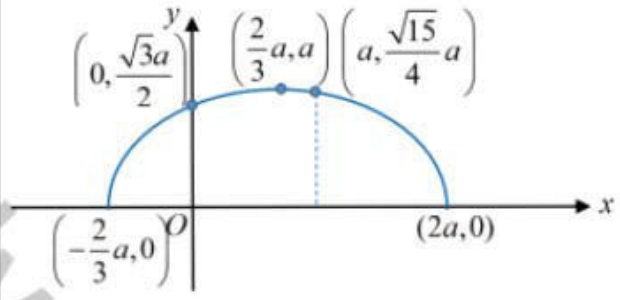
iv	Using GC to help if students is not sure  $y=\dfrac{1}{f(x)}$  $y=f'(x)$  	Some students did not label $y=0$ as the horizontal asymptote and thus lost marks Only half of graph is drawn because of mistakes made in (iii) Full graph was drawn because of mistakes made in (iii) Wrong asymptote concluded ($y=0$ instead of $y=\pm\dfrac{1}{6}$

5i	$\overrightarrow{OC} = \lambda\mathbf{b}$ $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{OC} = a + \lambda\mathbf{b}$ $\overrightarrow{ON} = \frac{4}{7}\lambda\mathbf{b}$ $\overrightarrow{OM} = \frac{\overrightarrow{OD} + 2\overrightarrow{OA}}{3}$ $= \frac{\mathbf{a} + \lambda\mathbf{b} + 2\mathbf{a}}{3}$ $= \frac{1}{3}(3\mathbf{a} + \lambda\mathbf{b})$	Generally very well done. Careless students tend to write $\frac{1}{3}(\mathbf{a} + \lambda\mathbf{b})$ instead of $\frac{1}{3}(3\mathbf{a} + \lambda\mathbf{b})$, which leads to wrong answers in part (ii). A handful of students tend to miss out the tilde sign for vectors.
ii	Area of triangle $OMD = \frac{1}{2}\left\|\overrightarrow{OM} \times \overrightarrow{OD}\right\|$ $= \frac{1}{2}\left\|\frac{1}{3}(\lambda\mathbf{b} + 3\mathbf{a}) \times (\lambda\mathbf{b} + \mathbf{a})\right\|$ $= \frac{1}{6}\left\|\lambda^2\mathbf{b} \times \mathbf{b} + \lambda\mathbf{b} \times \mathbf{a} + 3\lambda\mathbf{a} \times \mathbf{b} + 3\mathbf{a} \times \mathbf{a}\right\|$ $= \frac{1}{6}\left\|-\lambda\mathbf{a} \times \mathbf{b} + 3\lambda\mathbf{a} \times \mathbf{b}\right\|$ $(\because \mathbf{b} \times \mathbf{b} = \mathbf{0} = \mathbf{a} \times \mathbf{a})$ $= \frac{1}{3}\lambda\left\|\mathbf{a} \times \mathbf{b}\right\|$ 	Not very well done as quite a lot of students still cannot do cross product properly, where **order** matters. Algebraic simplification skills were weak where many students were confused by the λ, and yet could obtain the right answer. For example, the most common mistake was $\frac{1}{2}\left\|\frac{1}{3}(\lambda\mathbf{b} + 3\mathbf{a}) \times (\lambda\mathbf{b} + \mathbf{a})\right\|$ $= \frac{1}{2}\lambda\left\|\frac{1}{3}(\mathbf{b} + 3\mathbf{a}) \times (\mathbf{b} + \mathbf{a})\right\|$ instead of $\frac{1}{2}\left\|\frac{1}{3}(\lambda\mathbf{b} + 3\mathbf{a}) \times (\lambda\mathbf{b} + \mathbf{a})\right\|$ $= \frac{1}{2}\lambda^2\left\|\frac{1}{3}(\mathbf{b} + \frac{3}{\lambda}\mathbf{a}) \times (\mathbf{b} + \frac{1}{\lambda}\mathbf{a})\right\|$ Students were penalized for this even though their final answer was correct. Many missed out the tilde sign for the zero vector.
iii	$\left\|\mathbf{p}.\mathbf{a}\right\|$ is the length of projection of $\overrightarrow{OA}$ on $\overrightarrow{OD}$	Many varying wrong answers: - Some wrote d, which is not even given in the question.

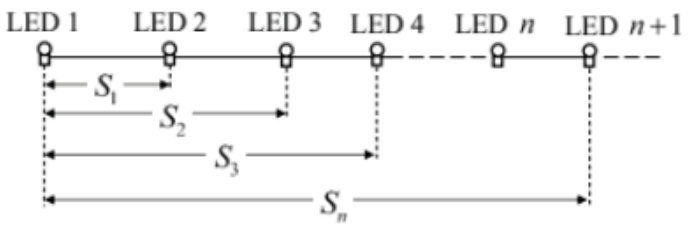
		- Length of projection of OA onto OD with no arrows above to indicate they are vectors. - Length of projection of $\overrightarrow{OA}$ onto **p** - Length of projection of **p** / $\overrightarrow{OD}$ onto $\overrightarrow{OA}$ - Length of projection of **line** OA onto **line** OD - Shortest distance from A to line OD - Projection **vector** of $\overrightarrow{OA}$ onto $\overrightarrow{OD}$ - Projection of **point** A onto **point** D
6i	$\frac{dx}{dt} = 50\sin t\cos t$, $\frac{dy}{dt} = -2\sin t$ $\frac{dy}{dx} = \frac{-1}{25\cos t}$	Generally well done
ii	When $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{-\sqrt{2}}{25}$, $x = \frac{25}{2}$, $y = \sqrt{2}$ $y = -\frac{\sqrt{2}}{25}x + c$ $c = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$ Equation of tangent: $y = -\frac{\sqrt{2}}{25}x + \frac{3}{\sqrt{2}}$ When $x = 0$, $y = \frac{3}{\sqrt{2}}$ When $y = 0$, $x = \frac{75}{2}$ Area of triangle $OPQ = \frac{1}{2}\left(\frac{75}{2}\right)\left(\frac{3}{\sqrt{2}}\right) = \frac{75}{4}\left(\frac{3}{\sqrt{2}}\right) = \frac{225}{4\sqrt{2}}$	Generally well done A handful of careless mistakes due to the simplification of surds.

iii	$y = 0$	Easy question but not very well done. Many thought it is $\frac{dy}{dx} = 0$ but question asked for the NORMAL, which is parallel to the x-axis, not the tangent.
7ai	Let $y = 2e^{\cos\frac{x}{2}}$ $\frac{dy}{dx} = -\frac{1}{2}\sin\frac{x}{2}\left(2e^{\cos\frac{x}{2}}\right)$ $= -\sin\frac{x}{2}\left(e^{\cos\frac{x}{2}}\right)$	This is an easy question, with most students doing well for it.
aii	$\int \frac{1}{2}\sin x\left(e^{\cos\frac{x}{2}}\right)dx$ $= \int \frac{1}{2}\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)\left(e^{\cos\frac{x}{2}}\right)dx$ $= -\int\left(\cos\frac{x}{2}\right)\left(-\sin\frac{x}{2}e^{\cos\frac{x}{2}}\right)dx$ $= \left(-\cos\frac{x}{2}\right)\left(2e^{\cos\frac{x}{2}}\right) - \int\left(\frac{1}{2}\sin\frac{x}{2}\right)\left(2e^{\cos\frac{x}{2}}\right)dx$ $= -2\cos\frac{x}{2}\left(e^{\cos\frac{x}{2}}\right) + 2e^{\cos\frac{x}{2}} + C$	It should be $\int -\sin\frac{x}{2}e^{\cos\frac{x}{2}}dx = 2e^{\cos\frac{x}{2}} + C$, instead of $\int -\sin x\, e^{\cos\frac{x}{2}}dx = 2e^{\cos\frac{x}{2}} + C$.
b	$\int \frac{1}{1-e^x}dx$ Let $u = 1 - e^x \Rightarrow \frac{du}{dx} = -e^x \Rightarrow \frac{dx}{du} = \frac{1}{u-1}$ $= \int \frac{1}{u(u-1)}du$ $= \int -\frac{1}{u} + \frac{1}{u-1}du$ $= -\ln\lvert u\rvert + \ln\lvert u-1\rvert + C$	Some students wrote $\int \frac{u-1}{u}du$ instead of $\int \frac{1}{u(u-1)}du$. Some students wrote $\int \frac{1}{\left(u-\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}du$ as

	$= -\ln\lvert 1-e^x\rvert + \ln\lvert -e^x\rvert + C$ $= -\ln\lvert 1-e^x\rvert + x + C$	$= \frac{1}{2\left(\frac{1}{2}\right)} \ln\left\lvert \frac{u-\frac{1}{4}}{u} \right\rvert$ instead of $\frac{1}{2\left(\frac{1}{2}\right)} \ln\left\lvert \frac{u-1}{u} \right\rvert$.
8i		A number of students did not read the question carefully, which asks for **coordinates** of (1) stationary points (2) x- and y- axial intercepts Students are required to give a rounded curve where it cuts the x axis (the curve is supposed to be half of an ellipse). If it is too sharp, 1 mark will be deducted.
ii	For f^{-1} to exist, f must be one-one. i.e. every horizontal line $y=h$, $h \in \mathbb{R}$ can only cut the graph of $y=f(x)$ at most once. Hence, the greatest value of k is a.	Some students gave a range of values. Question asks for the maximum value of k.
iii	Let $y = \sqrt{a^2 - \frac{(x-a)^2}{4}}$ $\Rightarrow y^2 = a^2 - \frac{(x-a)^2}{4}$ $\Rightarrow (x-a)^2 = 4(a^2-y^2)$ $\Rightarrow x = a \pm 2\sqrt{a^2-y^2}$ $\Rightarrow x = a - 2\sqrt{a^2-y^2}$ $(\because -a \le x \le a)$ $f^{-1}: x \mapsto a - 2\sqrt{a^2-x^2}$, $x \in \mathbb{R}$, $0 \le x \le a$	Many students either did not give $\pm$ when taking square root or did not give the correct reason for rejecting the **positive** square root. Many students also did not give the correct form when the question asks for the inverse function to be given in **similar form**.
wiv	$g(x) = f\left(\frac{3}{2}x\right)$ for $x \in \mathbb{R}$, $-\frac{2}{3}a \le x \le 2a$. Since $R_f = [0,a] \subseteq \left[-\frac{2}{3}a, 2a\right] = D_g$, the composite function gf exists.	Some students did not give the correct notation when writing R_f and D_g and hence 1 mark was deducted.

		Some students were confused between $R_f \subseteq D_g$ and $D_g \subseteq R_f$. Some students also mixed up the symbols $\subseteq$ with $\in$, which means belongs to or an element of.
	$D_{gf} = D_f \xrightarrow{f} R_f = [0,a] \xrightarrow{g} \left[\frac{\sqrt{3}}{2}a,a\right] = R_{gf}$ $R_{gf} = \left[\frac{\sqrt{3}}{2}a,a\right]$	Many students gave $R_{gf} = \left[\frac{\sqrt{3}}{2}a,\frac{\sqrt{15}}{4}a\right]$ as the answer, failing to see that a is actually the max point.
9i	LED 1 LED 2 LED 3 LED 4 LED 8 LED 9 $u_1 = d$ $u_2 = d(\frac{4}{5})$ $u_3 = d(\frac{4}{5})^2$ $u_8 = d(\frac{4}{5})^7$ $d(\frac{4}{5})^7 = 56.2$ $\therefore\ d = 267.9824829 = 268.0$ (1 d.p.)	• A number of candidates used $d(\frac{4}{5})^8$ instead of $d(\frac{4}{5})^7$. Such an error can easily be avoided by drawing a simple sketch as shown to determine the required distance as $u_8 = d(\frac{4}{5})^7$. • Many candidates took the tedious approach of $S_8 - S_7 = 56.2$ when $u_8 = 56.2$ suffices.
ii	Since $\lvert r\rvert = \lvert\frac{4}{5}\rvert < 1$, sum to infinity exists. Hence maximum theoretical length of wire $= \frac{d}{1-r}$ $= \frac{267.9824829}{1-\frac{4}{5}}$ $= 1339.912415$ $= 1339.9$ cm (1 d.p.)	• Since $\lvert r\rvert = \lvert\frac{4}{5}\rvert < 1$, candidates should apply $S_\infty = \frac{d}{1-r}$ directly. • A large number of candidates rounded off the answer to 1340.0 cm instead of 1339.9 cm.

iii



Let S_n be distance from LED 1 to LED $n+1$.

Method 1: (using GC table)

Using sum of GP, $S_n = \dfrac{267.9824829[1-(\frac{4}{5})^n]}{1-\frac{4}{5}}$

Using GC,

n	S_n	$\lvert S_n - 1290\rvert$
14	1281.0	9.0
15	1292.8	2.8

$\therefore$ closest LED to 1290 is when $n=15$.

$\therefore$ required LED is LED $(15+1)$ = LED 16.

Hence colour of LED 16 is red.

Method 2: (using algebraic manipulation)

Let $S_n = \dfrac{267.9824829[1-(\frac{4}{5})^n]}{1-\frac{4}{5}} = 1290$

$1-(\frac{4}{5})^n = 0.9627494943$

$n\ln(\frac{4}{5}) = \ln 0.0372505057$

$\therefore\ n = 14.74427443$

Using GC,

n	S_n	$\lvert S_n - 1290\rvert$
14	1281.0	9.0
15	1292.8	2.8

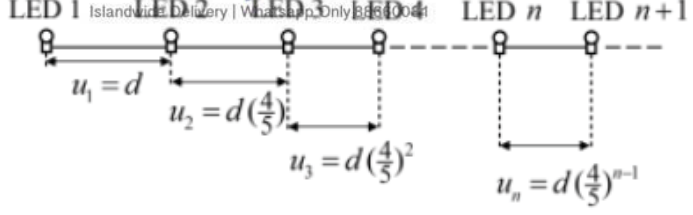
$\therefore$ closest LED to 1290 is when $n=15$.

$\therefore$ required LED is LED $(15+1)$ = LED 16.

Hence colour of LED 16 is red.

- Many candidates mistook n as the LED number, when S_n is the distance from LED 1 to LED $(n+1)$, not LED 1 to LED n.
- The majority of candidates who used method 2 to obtain $n=14.7$ concluded immediately that $n=15$ since $n=14.7$ is 'nearer' to $n=15$ than $n=14$. This is incorrect because n takes only integer values and thus $n=14.7$ has no meaning. The relative distances of S_{14} and S_{15} from 1290 would have to be compared to determine the nearest LED.
- Most candidates were able to use the cyclical nature of the order of the colours of the LEDs to obtain the colour of the LED.

iv



- Many candidates took the tedious approach of $S_n - S_{n-1} \geq 1$ when $u_n = d(\frac{4}{5})^{n-1} \geq 1$ suffices.
- Candidates tended to disregard the inequality and solve

	Let $u_n = d(\frac{4}{5})^{n-1} = 267.9824829(\frac{4}{5})^{n-1} \geq 1$ $(\frac{4}{5})^{n-1} \geq \frac{1}{267.9824829}$ $\ln(\frac{4}{5})^{n-1} \geq \ln\frac{1}{267.9824829}$ $(n-1)\ln(\frac{4}{5}) \geq \ln\frac{1}{267.9824829}$ $n-1 \leq 25.05526859$ $\therefore \quad n \leq 26.05526859$ Since largest integer $n=26$, last LED is LED $(26+1)=$ LED 27 Hence colour of last LED 27 is yellow.	$267.9824829(\frac{4}{5})^{n-1}=1$ instead. This is inappropriate as an answer of $n=26.05526859$ gives no indication of whether $n=26$ or $n=27$. Solving as an inequality on the other hand provides a clear and convincing argument why $n=26$ eventually.
10a	$v+\mathrm{i}u=2$ ----(1) $av-2u=3\mathrm{i}$ ----(2) $(1)\times a-(2)$: $\mathrm{i}au+2u=2a-3\mathrm{i}$ $u=\frac{2a-3\mathrm{i}}{2+\mathrm{i}a}\times\frac{2-\mathrm{i}a}{2-\mathrm{i}a}$ $u=\frac{4a-6\mathrm{i}-2a^2\mathrm{i}-3a}{4+a^2}$ $u=\frac{a-(6+2a^2)\mathrm{i}}{4+a^2}$ $=\frac{a}{4+a^2}-\frac{6+2a^2}{4+a^2}\mathrm{i}$ Sub u into (1) $v=2-\mathrm{i}u$ $=2-\mathrm{i}(\frac{a}{4+a^2}-\frac{6+2a^2}{4+a^2}\mathrm{i})$ $=2-\frac{6+2a^2}{4+a^2}-\frac{a}{4+a^2}\mathrm{i}$ $=\frac{2}{4+a^2}-\frac{a}{4+a^2}\mathrm{i}$	• Many students have tried using the approach of letting $u=x+y\mathrm{i}$ for form equations involving x and y. This is a longer method. • Several students have wrong concept of equating the real and imaginary part. E.g. they simply obtain $au=-3$ and $2u=2a$ when they have this equation: $\mathrm{i}au+2u=2a-3\mathrm{i}$. Real and imaginary part can only be equated when the equation has solely real and imaginary numbers in the equation (not when there is complex number term) • Students should not stop as x and y expression, and should write out u and v in the form of $x+y\mathrm{i}$. • Considerable number of students did not follow the instruction to **simplify** their answers.

bi	Sub $x=-k$ into $bx^3+(12b+\mathrm{i})x^2+(12\mathrm{i}+b)x+12b=0$ We have: $-bk^3+(12b+\mathrm{i})k^2-(12\mathrm{i}+b)k+12b=0$ $-bk^3+12bk^2-bk+12b=0 \quad ---(1)$ and $k^2-12k=0----(2)$ (2) implies $k(k-12)=0$ $k=0,12$ Reject $k=0$ as $b\neq 0$.	• Several students are careless in omitting having $x=-k$ in the term $(12\mathrm{i}+b)x$ when they do substitution of $x=-k$ into the cubic equation. • Several students tried the longer method of factorisation instead of using Factor Theorem to solve. • Some students yet understand the mean of $(x+k)$ is a factor and substituted $(x+k)$ into the equation instead. Likely they yet understood the meaning of root and factor. • Students should not simply "cancel" k when solving $k^2-12k=0$. They should factorise completely and justify the rejection of $k=0$.
bii	$bx^2+\mathrm{i}x+b=0$ $x=\dfrac{-\mathrm{i}\pm\sqrt{-1-4b^2}}{2b}$ $x=\dfrac{-1\pm\sqrt{1+4b^2}}{2b}\mathrm{i}$ Hence roots are purely imaginary 	• Many students work out the discriminant to be < 0 and then no real roots exist. The use of discriminant is applicable only for equation with real coefficients. • Many students are unaware that $\sqrt{-1-4b^2}$ is simply $\sqrt{1+4b^2}\,\mathrm{i}$. • Students should not waste time trying to complete the square to find x, and be aware of usage of formula to get x.
biii	Hence $bx^3+(12b+\mathrm{i})x^2+(12\mathrm{i}+b)x+12b=(x+12)(bx^2+cx+b)$ By compare coeff of x, $c=\mathrm{i}$, using (i):	• Many students worked out the discriminant to be < 0 and then claimed no

	$bx^3+(12b+\mathrm{i})x^2+(12\mathrm{i}+b)x+12b=(x+12)(bx^2+\mathrm{i}x+b)$ $=b(x+12)(x+\frac{1+\sqrt{1+4b^2}}{2b}\mathrm{i})(x+\frac{1-\sqrt{1+4b^2}}{2b}\mathrm{i})$	real roots exist. The use of discriminant is applicable only for equation with real coefficients. It can't be used in this context. • Many students are unaware that $\sqrt{-1-4b^2}$ is simply $\sqrt{1+4b^2}\,\mathrm{i}$. • Students should not waste time trying to complete the square to find x, and be aware of usage of formula to get x.
11 a	Height of liquid in the cylinder $=x-4$ $V=\pi(2.5)^2(x-4)+\frac{1}{3}\pi(2.5)^2(4)$ $=6.25\pi(x-4)+\frac{25}{3}\pi$ $\frac{dV}{dx}=6.25\pi$ and $\frac{dV}{dt}=-18$ $\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}$ $=\frac{-18}{6.25\pi}$ $=-0.916$ cm/min	• Many students took the height of the liquid in the cylinder as x instead of $x-4$ as they forgot to consider the height of the cone. • Many students did not include the negative sign for $\frac{dx}{dt}$ because they did not consider the volume of the liquid flowing out of the cone to be $\frac{dV}{dt}=-18$ cm^3/min
b	$\frac{r}{x}=\frac{2.5}{4}\Rightarrow x=1.6r$ When $x=2$, $r=\frac{5}{4}$ $V_c=\frac{1}{3}\pi r^2 x$ $=\frac{1}{3}\pi r^2(1.6r)$ $=\frac{1.6}{3}\pi r^3$ $\frac{dV}{dr}=1.6\pi r^2$	• Several students did not realise that $x=1.6r$ and replaced x as 2 in the volume expression $V_c=\frac{1}{3}\pi r^2 x$. • Similar to 11(a) many students did not include the negative sign for $\frac{dr}{dt}$ because they did not consider the volume of

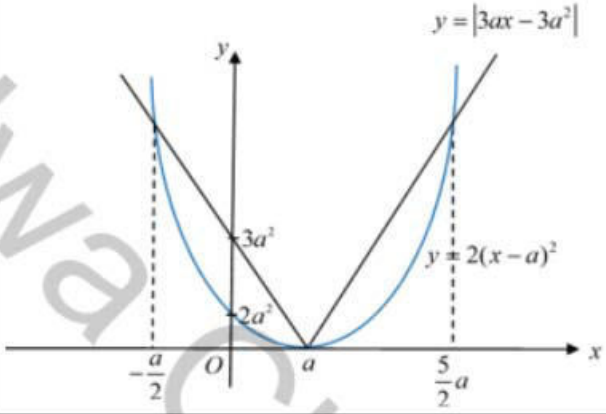
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $= \frac{1}{1.6\pi\left(\frac{5}{4}\right)^2} \times (-18)$ $= -2.29$ cm/min	the liquid flowing out of the cone to be $\frac{dV}{dt} = -18$ cm^3/min
c	Vol of bowl $= \pi\int_{-10}^{0} 225\left(1 - \frac{y^2}{100}\right) dy$ Vol. of empty space $= \pi\int_{-10}^{0} 225\left(1 - \frac{y^2}{100}\right) dy - \pi(2.5)^2(20) - \frac{1}{3}\pi(2.5)^2(4)$ A B $= 4293.510$ cm^3 (3 d.p.)	• Many students did not obtain the correct expression for the volume of the bowl, $\pi\int_{-10}^{0} x^2\, dy$. A handful of students omitted π in the volume expression. • Some students placed the modulus in the volume expression e.g. $\pi\left\|\int_{-10}^{0} x^2\, dy\right\|$, not realising that the volume expression did not require the modulus. • A handful of students did not read the question carefully and left the final answer either in exact form or 3 significant figures.
12i	Equation of line through MD: $\mathbf{r} = \begin{pmatrix}0.8\\0.6\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\7\\-1\end{pmatrix}$, $\lambda \in \mathbb{R}$ $\begin{pmatrix}0.8\\0.75\\0.3\end{pmatrix} - \begin{pmatrix}0.4\\-0.9\\0\end{pmatrix} = \begin{pmatrix}0.4\\1.65\\0.3\end{pmatrix}$ Equation of line through ND: $\mathbf{r} = \begin{pmatrix}0.4\\-0.9\\0\end{pmatrix} + \mu\begin{pmatrix}0.4\\1.65\\0.3\end{pmatrix}$, $\lambda \in \mathbb{R}$. Since lines through MD and ND intersect at D,	Students need to take note of the format of an equation of line that it is of the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$. $\overrightarrow{MD} = \begin{pmatrix}0.8\\0.6\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\7\\-1\end{pmatrix}$ is WRONG! In fact, $\overrightarrow{OD}$ is of the form $\begin{pmatrix}0.8\\0.6\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\7\\-1\end{pmatrix}$ instead.

	$\begin{cases} 0.8+2\lambda = 0.4+0.4\mu \\ 0.6+7\lambda = -0.9+1.65\mu \\ -\lambda = 0.3\mu \end{cases}$ Solving using GC, $\lambda = -\frac{3}{25},\ \mu = \frac{2}{5}$. Substitute $\lambda = -1.2$ into equation of line through MD, $\begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} - \frac{3}{25}\begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix} = \begin{pmatrix} 0.56 \\ -0.24 \\ 0.12 \end{pmatrix}$ $\therefore$ coordinates of D is $(0.56, -0.24, 0.12)$. (shown)	Many did not leave final answer in coordinates form, or write the equation to be shown. Some missed out curly sign for vectors.
ii	Equation of horizontal ground: $\mathbf{r} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$ Required angle $= 90° - \cos^{-1}\left\lvert \frac{\begin{pmatrix} 0.56 \\ -0.24 \\ 0.12 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{0.56^2 + (-0.24)^2 + 0.12^2}\sqrt{1}} \right\rvert$ $= 11.14233567°$ $= 11.1°$ (1 d.p.)	Many used the cosine formula, failed to use sin formula or link to $90°$. Many did not realize that the normal vector for the horizontal floor is $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Some gave $\tan\theta = \frac{0.12}{0.56}$ or equivalent. Note that this is wrong, as the point below D is not on x-axis.
iii	Given $T(0,0,0.07)$. $\overrightarrow{NM} = \begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0.4 \\ -0.9 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 1.5 \\ 0 \end{pmatrix}$ $\overrightarrow{TM} = \begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0.07 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.6 \\ -0.07 \end{pmatrix}$ $\begin{pmatrix} 0.4 \\ 1.5 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0.8 \\ 0.6 \\ -0.07 \end{pmatrix} = \begin{pmatrix} 0.028 \\ -0.96 \\ -0.105 \end{pmatrix} = 0.01\begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix}$ $\therefore$ equation of p: $\mathbf{r} \cdot \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = -6.72$ (Shown)	Many wrote 70 instead of 0.07, forgot to convert to km. $\begin{pmatrix} -0.105 \\ 0.028 \\ -0.96 \end{pmatrix} \neq \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix}$!! Students conveniently equated the two, which is wrong. Please take note for presentation.

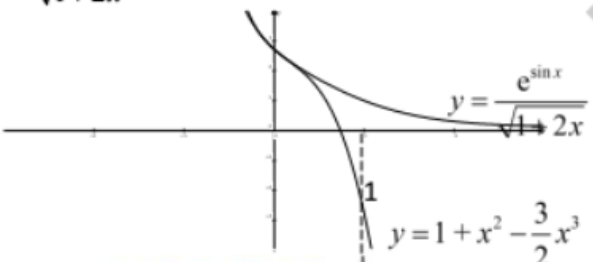
iv	Equation of line perpendicular to p passing through D is $\mathbf{r} = \begin{pmatrix} 0.56 \\ -0.24 \\ 0.12 \end{pmatrix} + t \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix}, t \in \mathbb{R}$. Substitute equation of line into p, $\begin{pmatrix} 0.56 - 10.5t \\ -0.24 + 2.8t \\ 0.12 - 96t \end{pmatrix} \cdot \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = -6.72$ $\therefore t = 0.00121618712$ Hence $\begin{pmatrix} 0.56 \\ -0.24 \\ 0.12 \end{pmatrix} + 0.00121618712 \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = \begin{pmatrix} 0.5472300352 \\ -0.2365946761 \\ 0.00324603648 \end{pmatrix}$ $= \begin{pmatrix} 0.547 \\ -0.237 \\ 0.00325 \end{pmatrix}$ $\therefore$ coordinates of G is $(0.547, -0.237, 0.00325)$.	Students using projection vector method have to take note of the direction of the resultant vector. $\overrightarrow{DG} = \left(\overrightarrow{DT} . \mathbf{n}\right)\mathbf{n}$. There should NOT be any modulus for the formula when students use the projection vector method. Quite a number of students have errors in accuracy. Answers are to be given in 3 s.f., for the 3rd coordinate, it's of 5 dec places, thus more decimal places have to taken into consideration for the value of t. Answer should be left in coordinates form.
v	<u>Method 1</u>: [Hence using $(0.547, -0.237, 0.00325)$] Required distance DG $= \sqrt{(0.56 - 0.54723)^2 + (-0.24 - (-0.23659))^2 + (0.12 - 0.003246)^2}$ $= 0.1174998$ $= 0.117$ (3 s.f.) $\therefore$ required distance $= 117$ m <u>Method 2</u>: [Otherwise using dot product] $\overrightarrow{MD} = \begin{pmatrix} 0.56 \\ -0.24 \\ 0.12 \end{pmatrix} - \begin{pmatrix} 0.8 \\ 0.6 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.24 \\ -0.84 \\ 0.12 \end{pmatrix}$ Required distance $= \left\| \begin{pmatrix} -0.24 \\ -0.84 \\ 0.12 \end{pmatrix} \cdot \frac{\begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix}}{\sqrt{(-10.5)^2 + 2.8^2 + (-96)^2}} \right\|$ $= 0.1174996006$ $= 0.117$ (3 s.f.) $\therefore$ required distance $= 117$ m	Students have error in accuracy for answer due to answer in part (iv). Some students did not know how to convert to metres correctly, or failed to convert to metres.

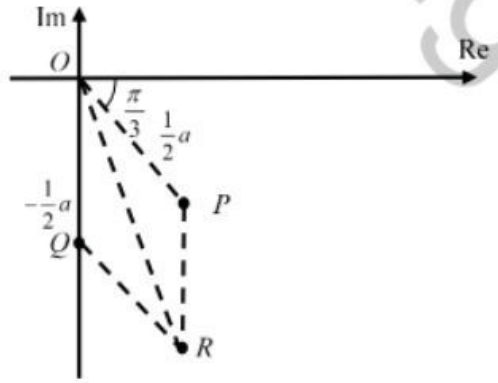
	Method 3 $\left\|\overrightarrow{DG}\right\| = \beta \left\| \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} \right\|$ $= 0.00121618712\sqrt{9334.09}$ $= 0.1174996006$ $= 0.117$ (3 s.f.) $\therefore$ required distance $= 117$ m	

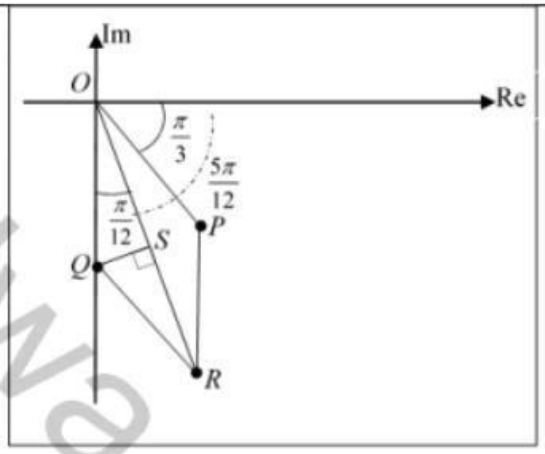
2019 C2 H2 Prelim P2 Solutions

Qn	Solutions	Comments
1		In general, students did well for this part. Common mistakes: 1. Forgot to label y-axis intercept 2. The shape of parabola, some students curved back at end of the graph 3. Some students even label the numerical values on the graph
1i	To find points of intersection, we solve $y = \lvert 3ax - 3a^2 \rvert$ $y = 2(x-a)^2$ i.e. $2(x-a)^2 = \lvert 3ax - 3a^2 \rvert$ $2(x-a)^2 = 3ax - 3a^2 \text{ -----(1)}$ $\Rightarrow 2x^2 - 7ax + 5a^2 = 0$ $\Rightarrow (2x-5a)(x-a) = 0$ $\Rightarrow x = \frac{5}{2}a \text{ or } x = a$ Or $2(x-a)^2 = -(3ax - 3a^2) \text{ ------(2)}$ $\Rightarrow 2x^2 - ax - a^2 = 0$ $\Rightarrow (2x+a)(x-a) = 0$ $\Rightarrow x = -\frac{1}{2}a \quad \text{or} \quad x = a$ For $2(x-a)^2 \geq \lvert 3ax - 3a^2 \rvert$, $x \leq -\frac{a}{2}$ or $x = a$ or $x \geq \frac{5a}{2}$	A lot of students failed to utilize the previous graph. Majority of students failed to spot that $x = a$ is also part of the solution The easiest way is to find out the intercepts between two graphs and based on the actual graph to draw the final conclusion. A lot of students use algebraic method to solve the inequality directly which leads to chaotic manipulations. Common mistake: 1. $\lvert a \rvert \leq b \Rightarrow -b \leq a \leq b$ which means $-b \leq a$ **AND** $a \leq b$ Many students use **OR** very loosely. 2. Some students try analytical ways of solving the inequality, but they are not familiar with AND, OR. The proper way should be: (i) If $x \geq a$,

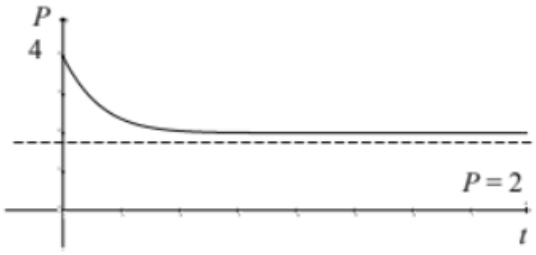
		$3ax-3a^2 \le 2(x-a)^2$, solve the inequality, the solution should intercept with $x \ge a$. (ii) If $x < a$, $-(3ax-3a^2) \le 2(x-a)^2$, solve the inequality, the solution should intercept with $x < a$. The final solution should be the union of the (i) and (ii) solutions.
1ii	Replace x with $x+\frac{a}{2}$, $2(x+\frac{a}{2}-a)^2 \ge \left\|3a\left(x+\frac{a}{2}\right)-3a^2\right\|$ $2\left(x-\frac{a}{2}\right)^2 \ge \left\|3ax-\frac{3a^2}{2}\right\|$ $\therefore\ x+\frac{a}{2} \le -\frac{a}{2}$ or $x+\frac{a}{2}=a$ or $x+\frac{a}{2} \ge \frac{5a}{2}$ $\Rightarrow x \le -a$ or $x=\frac{a}{2}$ or $x \ge 2a$	A lot of students failed to identify the replacement of x with $x+\frac{a}{2}$
2i	$y=\frac{e^{\sin x}}{\sqrt{1+2x}}$ $y\sqrt{1+2x}=e^{\sin x}$ $\sqrt{1+2x}\frac{dy}{dx}+\frac{1}{2}\frac{2}{\sqrt{1+2x}}y=\cos x\, e^{\sin x}$ $\frac{dy}{dx}+\frac{y}{1+2x}=\frac{e^{\sin x}}{\sqrt{1+2x}}\cos x$ $=y\cos x$ $\frac{1}{y}\frac{dy}{dx}+\frac{1}{1+2x}=\cos x$ Alternatively, $\ln\left[y\sqrt{1+2x}\right]=\sin x$ $\frac{\sqrt{1+2x}\left(\frac{dy}{dx}\right)+(1+2x)^{-\frac{1}{2}}}{y\sqrt{1+2x}}=\cos x$ $\frac{1}{y}\frac{dy}{dx}+\frac{1}{1+2x}=\cos x$ (Shown)	This question is in general well done. Students are reminded to show detailed working for a "show" question in order to get the full credit.

ii	Differentiating again w.r.t. x, $\frac{1}{y}\frac{d^2y}{dx^2}-\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2-\frac{2}{(1+2x)^2}=-\sin x$ $\frac{1}{y}\frac{d^3y}{dx^3}-\frac{1}{y^2}\frac{dy}{dx}\frac{d^2y}{dx^2}-\frac{2}{y^2}\frac{dy}{dx}\frac{d^2y}{dx^2}+\frac{2}{y^3}\left(\frac{dy}{dx}\right)^3+\frac{8}{(1+2x)^3}=-\cos x$ $\frac{1}{y}\frac{d^3y}{dx^3}-\frac{3}{y^2}\frac{dy}{dx}\frac{d^2y}{dx^2}+\frac{2}{y^3}\left(\frac{dy}{dx}\right)^3+\frac{8}{(1+2x)^3}=-\cos x$ When $x=0, y=1, \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=2, \frac{d^3y}{dx^3}=-9$. The Maclaurin's series for y is $y=1+\frac{2}{2!}x^2-\frac{9}{3!}x^3+\ldots$ $=1+x^2-\frac{3}{2}x^3+\ldots$	The most common mistake for this question is the implicit differentiation of expressions such as $\frac{1}{y}\frac{dy}{dx}$. Many students forgot to put $\frac{dy}{dx}$ after differentiating an expression in y. The correct differentiation is shown as following: $\frac{d}{dx}\left(\frac{1}{y}\frac{dy}{dx}\right)=\frac{1}{y}\frac{d^2y}{dx^2}-\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2$
iii	$\int_0^1 \frac{e^{\sin x}}{\sqrt{1+2x}}dx \approx \int_0^1 1+x^2-\frac{3}{2}x^3\ dx$ $=\frac{23}{24}$ $=0.985$ (3 s.f.)	A lot of students did not realise that they can use G.C. to evaluate the integral and they made mistake while evaluating the integral.
iii	$\int_0^1 \frac{e^{\sin x}}{\sqrt{1+2x}}dx=0.058$  The graphs of $y=\frac{e^{\sin x}}{\sqrt{1+2x}}$ and $y=1+x^2-\frac{3}{2}x^3$ differ significantly near to $x=1$. Hence the approximation is not good.	1. Many students just mentioned that the Maclaurin series ignore higher power of x. Ignoring the higher power of x is the reason why it is an approximation rather than the exact answer. However, this does not explain why the approximation is not good. 2. Many students just commented that x is too big or x is not small without further explaining how this affects the approximation. 3. Some students mentioned the valid range for expansion. However, valid range for expansion is not relevant for approximation. x being in the valid range of expansion does not guaranteed a good approximation either.

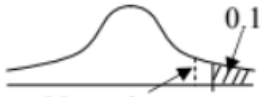
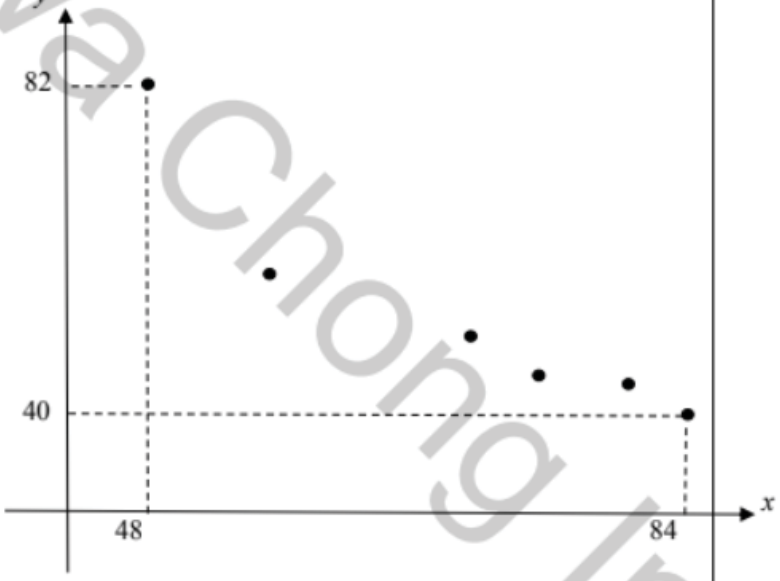
		4. Some students mentioned that x is not smaller than 1. However, being less than 1 does not guaranteed a good approximation.
iv	Replace x with $-x$ $y = \frac{e^{\sin(-x)}}{\sqrt{1-2x}} = \frac{e^{-\sin x}}{\sqrt{1-2x}} = \frac{1}{e^{\sin x}\sqrt{1-2x}}$ $\frac{1}{e^{\sin x}\sqrt{1-2x}} \approx 1 + x^2 + \frac{3}{2}x^3$	Many students tried to use standard series. They need to see the connection between this part and part (ii).
3i	$\lvert p\rvert = \left\lvert \frac{a}{1+\sqrt{3}\mathrm{i}} \right\rvert = \frac{a}{2}$ $\arg(p) = \arg\left(\frac{a}{1+\sqrt{3}\mathrm{i}}\right)$ $= \arg(a) - \arg\left(1+\sqrt{3}\mathrm{i}\right)$ $= 0 - \frac{\pi}{3}$ $= -\frac{\pi}{3}$	• Instead of using the properties of arg and mod, almost all went thru multiply by conjugate to simplify p. and some made careless mistake • A few didn't know that p is in the 4th quadrant • A few gave $\lvert p\rvert = \frac{2a}{4}$
ii	 The shape is a rhombus.	• Almost all find the cartersian form of $p + q$, instead of use the vector addition method to find $p + q$, and hence didn't notice the shape is a rhombus • Some know that $\lvert p\rvert = \lvert q\rvert$, but diagram didn't show same length • A few mentioned that it is a quadrilateral, trapezium, and even triangle.
ii	$\angle QOR = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{12}$ $\arg(p+q) = -(\frac{\pi}{3} + \frac{\pi}{12}) = -\frac{5\pi}{12}$	• Some use GC to find the $\arg(p+q) = \tan^{-1}(-2-\sqrt{3}) = -\frac{5\pi}{12}$ once GC is used, answer may be rounded off, hence no full credit • Some didn't know that leaving arg in π means exact, and gave ans like 0.9411π

	 $\|p+q\| = 2\times OS = 2\times\frac{a}{2}\cos\frac{\pi}{12} = a\cos\frac{\pi}{12}$	• Many cant find $\|p+q\|$ in trigo form • Some use $\|p+q\|=\|p\|+\|q\|$, $\arg(p+q)=\arg(p)+\arg(q)$
iii	For $(p+q)^n$ to be purely imaginary, $(p+q)^n = (a\cos\frac{\pi}{12})^n[\cos(-\frac{5n\pi}{12})+i\sin(-\frac{5n\pi}{12})]$ $\cos(-\frac{5n\pi}{12})=0$ Hence the smallest positive integer n is 6, as $\cos(-\frac{5\pi}{2})=0$.	• Some use cartesian form to do such question and get stuck • a few leave it blank

4ai	$\frac{dP}{dt} = \frac{a}{P} - bP$, where a, b are constants When $P = 2$, $\frac{dP}{dt} = \frac{a}{2} - 2b = 0 \Rightarrow a = 4b$ $\frac{dP}{dt} = \frac{4b}{P} - bP$ $= b\left(\frac{4}{P} - P\right)$ $= k\left(\frac{4}{P} - P\right)$, where $k = b$	Some students made the following mistakes in the formation of DE: 1. $\frac{dP}{dt} = \frac{a}{P} - aP$ (using the same constants for in birth rate and death rate) 2. Take birth rate as $\frac{P}{a}$ (P should be in the denominator) Note that $P = 2$, as P denotes population in thousands.
4aii	When $P = 4$, $\frac{dP}{dt} = -3$. $\frac{dP}{dt} = k\left(\frac{4}{4} - 4\right) = -3$ $\Rightarrow k = 1$ $\frac{dP}{dt} = \frac{4}{P} - P$ $= \frac{4 - P^2}{P}$ $\frac{dt}{dP} = \frac{P}{4 - P^2}$ $= -\frac{1}{2}\frac{-2P}{4 - P^2}$ $t = -\frac{1}{2}\ln\lvert 4 - P^2\rvert + C$ $-2t = \ln\lvert 4 - P^2\rvert - 2C$ $\ln\lvert 4 - P^2\rvert = 2C - 2t$ $4 - P^2 = \pm e^{2C}e^{-2t}$ $= Ae^{-2t}$, where $A = \pm e^{2C}$ When $t = 0$, $P = 4$ $A = -12$ $4 - P^2 = -12e^{-2t}$ $P^2 = 4 + 12e^{-2t}$ $P = 2\sqrt{1 + 3e^{-2t}}$	The population is decreasing, hence, $\frac{dP}{dt} = -3$ (note the negative sign). Some students made the mistake $\frac{dt}{dP} = \frac{P}{4 - P^2} = \frac{P}{4} - \frac{P}{P^2}$ Some students did not notice that P is in thousands. Students should substitute the initial condition $t = 0$, $P = 4$ only after they have correctly resolved the modulus in making $4 - P^2$ the subject. It is not explicitly mentioned in the question that $P \geq 2$. Resolve the modulus step by step instead of changing to brackets straightaway. As the final answer, students should make P the subject.

4a iii	 The population of the bugs will decrease and approach 2000 in the long run.	Students should label the vertical intercept in sketching the graph. Many students omitted "decreasing" in the description of the trend in the long term. Note that the graph continues to decrease as it approaches the asymptote. The graph should not overlap the asymptote. Students should sketch for $t \geq 0$.
4b	**Method 1:** $u = \frac{N}{t}$ $\frac{du}{dt} = \frac{t\frac{dN}{dt} - N}{t^2}$ $t\frac{du}{dt} = \frac{dN}{dt} - \frac{N}{t}$ Substitute into $\frac{dN}{dt} = 4 + \frac{N}{t}$: $t\frac{du}{dt} = 4$ $\frac{du}{dt} = \frac{4}{t}$ $u = 4\ln t + C$ $\frac{N}{t} = 4\ln t + C$ When $t = 1$, $N = 1 \Rightarrow C = 1$. $N = 4t\ln t + t$ **Method 2:** $u = \frac{N}{t}$ $N = ut$ $\frac{dN}{dt} = \frac{du}{dt}t + u$ Substitute into $\frac{dN}{dt} = 4 + \frac{N}{t}$:	This part of the question was poorly attempted as many students assumed either N or t is constant in their differentiation. Note that all 3 (u, N, t) are variables. A number of students made the mistake: $\frac{N}{t} = 4\ln t + C$ $\Rightarrow N = 4t\ln t + B$ The final answer should be in terms of the variables given in the original DE (N and t in this case).

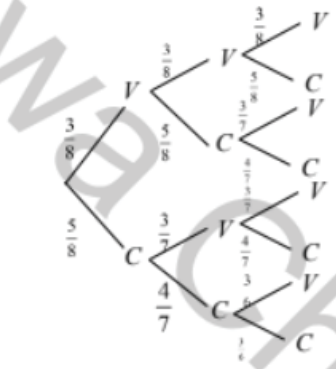
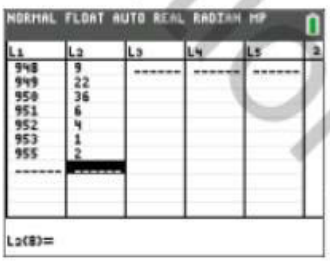
	$\frac{du}{dt}t+u=4+u$ $\frac{du}{dt}t=4$ $\frac{du}{dt}=\frac{4}{t}$ $u=4\ln t+C$ $\frac{N}{t}=4\ln t+C$ When $t=1,\ N=1 \Rightarrow C=1$. $N=4t\ln t+t$	
5 **(i)**	Let X denote the daily rainfall in mm. $X\sim \mathrm{N}(\mu,\sigma^2)$ In 7 days, $\bar{X}\sim \mathrm{N}\left(\mu,\frac{\sigma^2}{7}\right)$ $\mathrm{P}(X>9.8)=0.1$ $\mathrm{P}\left(Z>\frac{9.8-\mu}{\sigma}\right)=0.1$ $\frac{9.8-\mu}{\sigma}=1.2815516$ $9.8=\mu+1.2815516\sigma$ --- (1) $\mathrm{P}(\bar{X}>8.2)=0.1$ $\mathrm{P}\left(Z>\frac{8.2-\mu}{\sigma/\sqrt{7}}\right)=0.1$ $\frac{8.2-\mu}{\sigma/\sqrt{7}}=1.2815516$ $8.2=\mu+0.4843810\sigma$ --- (2) Solving (1) and (2), $\mu=7.22780, \sigma=2.00710$ $\therefore \sigma=2.01$ (2dp) (shown)	Some students compute the value of μ only in part (ii). Should do it in part (i).
(ii)	In 30 days, $\bar{X}\sim \mathrm{N}\left(7.22780,\frac{2.01^2}{30}\right)$ $\mathrm{P}(\bar{X}>k)\geq 0.1$ $k\leq 7.6981(5\text{sf})$ $\therefore \max k=7.6(1\text{dp})$	No need to use CLT as part (i) already stated normal distribution. A few students forget to write down the distribution of $\bar{X}$. Some students wrongly round up the answer to 7.7.

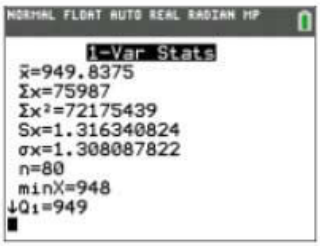
	Note: change in inequality sign because for area to be greater than or equal to 0.1, the value of k can be at 7.6974 or to its left, i.e. less than 7.6974 (see diagram below).  Possible actual boundary 7.6974 to get area larger than 0.1	
6 **(i)**	 	A few students did not label the coordinates of the extreme points.
(ii)	(x_7, y_7) is $(\bar{x}, \bar{y})$ Using G.C, $(\bar{x}, \bar{y}) = (69, 53)$ OR $(x_7, y_7) =$ $\left(\frac{80+84+70+74+58+48}{6} \quad \frac{44+40+49+45+58+82}{6}\right)$ $= (69, 53)$	Most students get this correct.
(iii)	Model A: $y = a + bx$ r-value $= -0.92386$ Model B: $y = \ln a + b \ln x$ r-value $= -0.96785$ Model C: $y = a + b\sqrt{x}$ r-value $= -0.95855$ Since $\lvert r \rvert$ for Model B is nearest to 1, Model B is the most accurate model	Some students use the scatter diagrams or the shape of the curves to choose the Model. This is wrong because the curve for Model B and C can be similar (values of a and b are unknown). Students need to compare the r values to decide.

(iv)	$y = \ln a + b \ln x$ By GC, $\ln a = 350.11$ $b = -70.469$ $y = 350.11 - 70.469 \ln x$ When $x = 60$, $y = 62$ marks. The student is estimated to score 62 marks.	Note: **Marks are not awarded for using Model C**. Only correct answer of 62 using Model B gets the full marks. A few students did not round up to whole number. A few students get the wrong equation $\ln y = \ln a + b \ln x$.
(v)	$4 \times 30 = 120$ unreasonable as 120 is not within data range where $48 \le x \le 84$.	Some students wrongly stated that 4 hours is not within the data range. They should multiply by 30 to get the total hours (120) for the month and explain that 120 is not within the data range of $48 \le x \le 84$ (for a month).
7 **(i)**	Let X and Y denote the length (in inches) of a "footlong" and a "6-inch" loaf respectively. $X \sim \mathrm{N}(12.2, 0.2^2)$, $Y \sim \mathrm{N}\left(6.1, \left(0.1\sqrt{2}\right)^2\right)$ $\mathrm{P}(Y < 6) = 0.239750 = 0.240 (3\text{sf})$ $\mathrm{P}(X < 12) = 0.158655 = 0.159 (3\text{sf}) < \mathrm{P}(Y < 6)$ $\therefore$ a "6-inch" loaf more likely to be less than 6 inches.	A handful of students attempted to conclude from either just the variance/standard deviation or just the gap of 12.2 from 12 vs 6.1 from 6, but incomplete explanations gained no credit. It is much simpler to calculate probabilities and compare them. Students are advised to conclude in full sentences clearly, with phrasing as close to the original one in the question as possible.
(ii)	$\mathrm{E}(Y_1 + Y_2 - X) = 2(6.1) - 12.2 = 0$ $\mathrm{Var}(Y_1 + Y_2 - X) = 2(0.1\sqrt{2})^2 + 0.2^2 = 0.08$ $Y_1 + Y_2 - X \sim \mathrm{N}(0, 0.08)$ $\mathrm{P}(Y_1 + Y_2 > X) = \mathrm{P}(Y_1 + Y_2 - X > 0) = \frac{1}{2}$	It is unacceptable to write 2Y instead of $Y_1 + Y_2$. Most students seemed unaware that the probability is exactly ½ as 0 is the mean. Some students calculated variance wrongly but were not penalised here as only the mean is required to arrive at the answer.

(iii)	Let A denote the number of "6-inch" sandwiches less than 6-inches in length in a week. $A \sim \mathrm{B}(3, 0.239750)$ $\mathrm{P}(A \le 1) = 0.855122 = 0.855(3\text{sf})$ Alternative required probability $= \mathrm{P}(Y<6)\left[\mathrm{P}(Y>6)\right]^2 \times 3 + \left[\mathrm{P}(Y>6)\right]^3$ $= 0.23975(1-0.23975)^2 \times 3 + (1-0.23975)^3$ $= 0.855(3\text{sf})$	Students must define the random variable properly. Some students wrongly defined A [e.g. "the event" or "a sandwich"] when all binomial distributions should be "the number of", while some wrongly used the same symbol as part (i), which is "length" and ~N(6.1, 0.02). Some students attempted this part without binomial distribution, but many missed out on the multiplier "x3", or only calculated for 1 sandwich shorter than 6 inches instead of <u>at most 1</u> such sandwich.
(iv)	Let B and C denote the number of "6-inch" sandwiches less than 6-inches in length in a 4-week and 3-week period respectively. $B \sim \mathrm{B}(12, 0.239750)$ and $C \sim \mathrm{B}(9, 0.239750)$ $\mathrm{P}(A=1 \mid B>4) = \dfrac{\mathrm{P}(A=1)\mathrm{P}(C>3)}{\mathrm{P}(B>4)}$ $= \dfrac{\mathrm{P}(A=1)\left[1-\mathrm{P}(C\le 3)\right]}{1-\mathrm{P}(B\le 4)}$ $= \dfrac{0.41571 \times 0.14710}{0.13721}$ $= 0.445674$ $= 0.446(3\text{sf})$	This part proved to be more challenging for many students, fraught with poor notation. Some students did not notice that a conditional probability was required. Some students were unable to decipher that they had to break the numerator up into 1st week and 2nd-4th week, i.e. $\mathrm{P}\left[(A=1)\cap(B>4)\right]$ $= \mathrm{P}(A=1)\mathrm{P}(C>3)$ Some students wrongly assumed the 2 events are independent and ended up cancelling $\mathrm{P}(B>4)$ to get $\dfrac{\mathrm{P}(A=1)\mathrm{P}(B>4)}{\mathrm{P}(B>4)}$ $= \mathrm{P}(A=1)$
8 (a)(i)	Number of arrangements $= \dfrac{8!}{3!2!} = 3360$	Some students did not divide by repetition.
8 (a)(ii)	Number of arrangements $= \dfrac{5!}{2!} \times {}^{6}\mathrm{C}_{3} = 1200$	Many students who did by complement cases did the P(exactly 2 Ls tgt) case wrongly
8 (a)(iii)	Arrange 3 vowels (into a group) $\dfrac{3!}{2!} = 3$	Many students have missing cases, especially the case of starting and ending with L.

	Arrange 5 consonants (for slotting) $\frac{5!}{3!}=20$ Since the ends must be consonants, only the middle 4 slots can be used for the group to slot. (ie. ${}^4C_1$) Number of arrangements$=3\times 20\times 4=240$ OR Case 1: First and Last 'L' $4!\times\frac{3!}{2!}=72$ Case 2: First and last 'L' and non-L $2\times 2\times\frac{4!}{2!}\times\frac{3!}{2!}=144$ Case 3: First and last non- L $2\times\frac{4!}{3!}\times\frac{3!}{2!}=24$ Total= 240 ways	
(b)(i)	P(1V) + P(2V) + P(3V) $=\frac{{}^5C_3\,{}^3C_1+{}^5C_2\,{}^3C_2+{}^5C_1\,{}^3C_3}{{}^8C_4}$ $=\frac{65}{70}=\frac{13}{14}$ Probability = 1 – P(all consonants) $=1-\left(\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\right)=\frac{13}{14}$ OR 1 – P(no vowels) $=1-\frac{{}^5C_4}{{}^8C_4}$ $=1-\frac{5}{70}$ $=\frac{13}{14}$	Generally well done.

	OR P(1 vowel) + P(2 vowels) + P(3 vowels) $=\frac{3\times5\times4\times3}{8\times7\times6\times5}\times\frac{4!}{3!}+\frac{3\times2\times5\times4}{8\times7\times6\times5}\times\frac{4!}{2!2!}+\frac{3\times2\times1\times5}{8\times7\times6\times5}\times\frac{4!}{3!}$ $=\frac{13}{14}$	
8 **(b)(ii)**	 Probability = P(VCC)+P(CVC)+P(CCV)+P(CCC) $=\frac{3}{8}\times\frac{5}{8}\times\frac{4}{7}+\frac{5}{8}\times\frac{3}{7}\times\frac{4}{7}+\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}+\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}$ $=\frac{505}{784}$ $=0.644$	Most students who drew the tree diagram was able to get the answer.
9(i)	Using GC, Unbiased estimate of population mean is $\bar{x}=949.84$ (2 d.p.) Unbiased estimate of population variance is $s^2=1.31634^2=1.73$ (2 d.p.) Note: If student write $\mu=949.84$ or $\sigma^2=1.31634^2=1.73$, annotate but do not deduct.	Generally well done for this part. Students need to be reminded to *answer to the question*, leaving final answer to *2 decimal places*, **NOT** to give exact value nor to 3 s.f. To cut down possible careless mistakes, should try the following steps: 1. create list for x (volume of shower gel, L_1) and f (no. of bottles, L_2),  2. Calculate (1- Var Stats)

		  **Note** that value of s is given by s_x = 1.316340824, **NOT** σ_x = 1.308087822 (which gives the sample variance). Unbiased estimate of population variance is given by s^2, which is $(1.316340824)^2$, **NOT** $(1.308087822)^2$ nor 316340824 – forgotten to square s. **Note** also that $\mu \neq \bar{x},\ \sigma^2 \neq s^2$
9 (b)(ii)	Let X be the volume of shower gel dispensed by the machine. Let μ denote the population mean volume of shower gel dispensed by the machine. $\bar{X} \sim \mathrm{N}\left(\mu, \frac{s^2}{n}\right)$ approx. by Central Limit Theorem. $\bar{X} \sim \mathrm{N}\left(950, \frac{s^2}{n}\right)$ H_0: $\mu = 950$ H_1: $\mu < 950$	Part (ii) was generally well answered. Most common mistake was found in the definition of μ. Some students defined μ as 'mean volume' or even 'sample mean volume'. In fact, μ denotes the *population mean volume* of the shower gel – students are expected to bring in the context of the question. As such, just stating μ as the population mean is not good enough to be awarded any credit.
9 (a)(iii)	Under H_0, using GC, $p-\text{value} = 0.13474 \approx 0.135$	Relatively more students made mistakes in finding the p-value. In fact, p-value is likely to be wrong if students made any mistake in the computation of $\bar{x}$ or s^2 in (i).

	The p - value is the probability of the sample mean volume of shower gel in a bottle is as extreme as 949.84 when the mean is actually 950.	There were few students who wrongly took *p*-value as $\frac{9+22}{80}=0.3875$, showing a misconception of what *p*-value is. Most students were unable to give the meaning of *p*-value in the <u>*context of the question*</u> correctly. Some wrongly took '*p*-value' to be the 'level of significance' and some wrote '*p*-value' as the 'smallest level of significance for the null hypothesis to be rejected'. Few students gave a generic meaning of *p*-value, without bringing in the context of the question.
9 (a)(iv)	Since we reject H_0 , $p-\text{value} < \alpha/100$ $\alpha \geq 13.476$ $\{\alpha \in \mathbb{R} : 13.5 \leq \alpha \leq 100\}$	Badly done by most students as they did not give the answer in <u>*set notation*</u>. Many did not include the upper bound of 100 in their answer. Students should be reminded not to use pre-mature rounding off answer in the computation of α, and should round off the final answer 'within the boundaries'.
9(b)	Let Y be the volume of shower gel dispensed by the machine after recalibration. Let μ denote the population mean volume of refill dispensed by the machine. $\bar{Y} \sim \text{N}\left(\mu, \frac{s^2}{n}\right)$ approx. by Central Limit Theorem H_0: $\mu = 250$ H_1: $\mu \neq 250$ Test statistic: $Z = \frac{\bar{Y}-\mu}{\frac{s}{\sqrt{n}}}$ Level of significance: 1% Reject H_0 if $p-\text{value} \leq 0.01$	Common mistakes students made in their presentation: 1. Finding $\bar{y}$ and s^2 wrongly. 2. Applying Central Limit Theorem to the distribution of Y. 3. Writing the distribution wrongly as $\bar{Y} \sim \text{N}(\mu, s^2)$ instead of the correct one $\bar{Y} \sim \text{N}\left(\mu, \frac{s^2}{n}\right)$. 4. Wrong critical values.

	$\frac{249.5-250}{s/\sqrt{50}} \le -2.5758293$ or $\frac{249.5-250}{s/\sqrt{50}} \ge 2.5758293$ (NA) $0 < \frac{s}{\sqrt{50}} \le 0.19411$ $0 < s \le 1.3726$ $0 < \frac{1}{49}\left(k - \frac{(-25)^2}{50}\right) \le 1.3726^2$ $12.5 < k \le 104.8$	Majority of the students left out the lower bound of k (i.e. $12.5 \le k$), without realizing that $s > 0$ is implied. Many students attempted to solve the inequality $\frac{249.5-250}{s/\sqrt{50}} \ge 2.5758293$, without realizing that there is no solutions as the LHS is negative.
9(c)	There is no need for the floor supervisor to assume the volume of shower gel follow a normal distribution as the sample sizes in both part (a) and (b) are large. The sample mean volume of shower gel can be approximated to follow a normal distribution by Central Limit Theorem.	Answer to (c) is not satisfactory. The correct statement should be 'the *sample mean volume* of shower gel can be approximated to follow a normal distribution by Central Limit Theorem', but most students either wrote 'mean volume', 'volume' or 'sample', in their statements. Students should be specific in their answers, avoid using 'it' or 'they' follows a normal distribution.
10 **(i)**	Let $\$W$ denote the amount won by a player in one game. Under option A, $E(W_A) = 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{4}\right)$ $= (1+2+3+4)\left(\frac{1}{4}\right)$ $= \frac{5}{2} = \$2.50$ Under option B, the winning amount for each combination is listed below:	Students should always define the random variable involved if question did not do so and also ensure that the notations used to represent expected amount for each option is also properly defined for clarity of solutions. Generally well done in finding expected winning amount for Option *A* However there was a significant number of students who did not know how to calculate the expected winning amount for Option *B*. It is recommended to list out the associated winning amount for each possible outcomes from rolling 2 dice and then multiply with the corresponding probabilities for each

y / x	1	2	3	4
1	0	4	6	8
2	-1	0	12	16
3	-2	-1	0	24
4	-3	-2	-1	0

	$\mathrm{E}(W_B)=\left(\frac{1}{16}\right)[4+6+12+8+16+24$ $-1-2-3-1-2-1]$ $=\left(\frac{1}{16}\right)[70-10]$ $=\frac{15}{4}=\$3.75$ Since $\mathrm{E}(W_B)>\mathrm{E}(W_A)$, option B is better. (shown)	case to compute expected winning amount for Option *B*. It is necessary to compare the 2 expected winning amounts before arriving at the conclusion that Option *B* is better.
(ii)	Option *A* is a "sure win" option where the player would definitely gain a positive amount in all cases, whereas option *B* has a risk of losing money in some cases.	Generally well done.
(iii)	$\mathrm{E}(W_A^2)=(1^2+2^2+3^2+4^2)\left(\frac{1}{4}\right)=\frac{15}{2}$ $\mathrm{Var}(W_A)=\mathrm{E}(W_A^2)-[\mathrm{E}(W_A)]^2=\frac{15}{2}-\left(\frac{5}{2}\right)^2=\frac{5}{4}=1.25(\text{shown})$	Generally well done. There was only a very small handful students who did not know the correct formula for finding variance.
(iv)	Let *A* and *B* denote the total amount won by Abel and Benson respectively. $A=W_{A1}+W_{A2}+\cdots+W_{A50}$ Since $n=50$ is large, by Central Limit Theorem, $A\sim \mathrm{N}\left(50\left(\frac{5}{2}\right),50\left(\frac{5}{4}\right)\right)$ approximately i.e. $A\sim \mathrm{N}\left(125,\frac{125}{2}\right)$ $B=W_{B1}+W_{B2}+\cdots+W_{B50}$ $B\sim \mathrm{N}\left(50\left(\frac{15}{4}\right),50\left(\frac{887}{16}\right)\right)$ approximately by Central Limit Theorem since $n=50$ is large $B\sim \mathrm{N}\left(\frac{375}{2},\frac{22175}{8}\right)$	A significant number of students were confused about the random variables involved in this question and many of them did not know what is meant by 'distributions' in the context of the question. The biggest misconception is to apply CLT for the winning amount of 1 round instead for the winning amounts of 50 rounds. Other common errors include - misuse the random variable that represents winning amount for 1 round as the same as that for 50 rounds - mistook 'distributions' as range of possible values of the random variables - mistook 'distributions' as expectations and variances - assume that the winning amount of 1 round for each options follow normal distributions - did not use CLT or use CLT without justification

		- Did not evaluate the means and variances of the distributions involved
(v)	$A-B \sim \mathrm{N}\left(125-\frac{375}{2}, \frac{125}{2}+\frac{22175}{8}\right)$ $A-B \sim \mathrm{N}\left(-\frac{125}{2}, \frac{22675}{8}\right)$ $\mathrm{P}(A>B)=\mathrm{P}(A-B>0)$ $=0.120207$ $=0.120(3\mathrm{sf})$	Majority of the students who attempted this part know what was required of them to find the required probability. A significant number of students could not find the correct final answer or get full 3 marks due to the following: - error from finding the mean or variance for wining amounts of 1 round/50 round for option B in earlier parts - did not specify the distribution of $A-B$