13 ELECTRIC FIELD

Content

- Concept of an electric field
- Electric force between point charges
- Electric field of a point charge
- Uniform electric fields
- Electric potential

ask Mr Cheong for rev stuff

CT1 . T2 W1
3 hours
P1 30 mcq (30 marks, 1h)
P2 : structured, DA (80 m, 2h)

Learning Outcomes

Candidates should be able to:

(a) show an understanding of the concept of an electric field as an example of a field of force and define electric field strength at a point as the electric force exerted per unit positive charge placed at that point.

(b) represent an electric field by means of field lines.

(c) recognise the analogy between certain qualitative and quantitative aspects of electric field and gravitational field.

(d) recall and use Coulomb's law in the form $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ for the electric force between two point charges in free space or air.

(e) recall and use $E = Q/4\pi\varepsilon_0 r^2$ for the electric field strength of a point charge in free space or air.

(f) calculate the electric field strength of the uniform field between charged parallel plates in terms of potential difference and plate separation.

(g) calculate the forces on charges in uniform electric fields.

(h) describe the effect of a uniform electric field on the motion of charged particles.

(i) define electric potential at a point as the work done per unit positive charge in bringing a small test charge from infinity to that point.

(j) state that the field strength of the electric field at a point is numerically equal to the potential gradient at that point.

(k) use the equation $V = Q/4\pi\varepsilon_0 r$ for the electric potential in the field of a point charge in free space or air.

13.1 Introduction

Links Between Sections and Topics

There are four fundamental forces in physics: the gravitational, electromagnetic, strong and weak interactions. While the strong and weak interactions explain phenomena at the sub-atomic level, a large number of daily human experiences can be explained by gravitational and electromagnetic interactions.

Electromagnetic interactions involve particles that have a property called electric charge, an attribute that appears to be just as fundamental as mass, or even more so – charge seems to be precisely quantised, while it is not clear if mass is quantised. An object with mass experiences a force in a gravitational field, and electrically-charged objects experience forces in electric and magnetic fields. Like mass-energy, charge obeys a conservation law as well. There are important analogies and distinctions between concepts in the gravitational field and in the electrical field topics.

A charge produces an electric field in the space around it, and a second charge placed in this field experiences a force due to this field. The electric force between two isolated point charges is governed by Coulomb's law, which is mathematically similar to Newton's law of gravitation for isolated point masses. We can use the concepts of work done and energy in the context of electrical interactions as well, and these ideas provide another route to solving problems that can in certain cases bring out the simplicity of the situation. Terms like electric potential and electric potential energy are defined similarly as with gravitational potential and gravitational potential energy.

Practical use of electricity often occurs in circuits rather than in free space. Circuits provide a means of conveying energy and information from one place to another. Within a circuit, the complicated effects of forces and electric fields at the microscopic level result in a macroscopic description where consideration of energy and electric potentials mostly suffices. The collective movement of charges results in electrical current, driven by potential differences (also known as voltages). Both current and potential difference can be experimentally measured. Applying the principles of charge and energy conservation provide powerful tools to analyse a variety of electrical circuits.

The mystery of magnetism was first discovered in magnetic stones by the ancients. Today, we understand magnetism as an effect inseparable from electricity, summarised by Maxwell's laws of electromagnetism. Unlike electric forces, which act on electric charges whether moving or stationary, magnetic forces act only on moving charges. Moving charges produce a magnetic field, and another moving charge or current placed in this magnetic field experiences a force. This apparent asymmetry in electromagnetic phenomena contributed to the development of the theory of relativity.

Applications and Relevance to Daily Life

Technologies harnessing electrical and magnetic properties pervade modern society. Converting energy into electrical energy traditionally involves the induced electromotive force and current produced by a changing magnetic flux or a time-varying magnetic field. Transmitting electrical energy over long distances is made feasible by the use of alternating current and voltage transformers. Semiconductor devices in computers and smartphones are the product of our deep understanding of the physics of electricity and magnetism in solid state materials. Innovations are also pushing on the quantum frontier.

Even more fundamentally, elastic forces in springs and contact forces between surfaces arise from electrical forces at the atomic level. In biology, electricity is also important in signalling and control. The heart rhythms are maintained by waves of electrical excitation, from nerve impulses that spread through special tissue in the heart muscles.

Links to Core Ideas

Systems and Interactions	Models and Representations	Conservation Laws
• F_E as the interaction between a charge and an external *E*-field • F_B as the interaction between a moving charge and an external *B*-field	• Microscopic model of the flow of charges • Ohm's law (for ohmic conductors) • Faraday's law • Common representations: diagrams of electric circuits, field lines and equipotential lines, magnetic flux density patterns, etc • Simplifying assumptions: e.g. point charges, negligible internal resistance, infinitely extended planes	• Conservation of charges in circuits • Conservation of energy in circuits • Lenz's law as conservation of energy

13.2 Electric Force

Coulomb's Law

Definition

Coulomb's law states that ***the force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them.***

Mathematically, it is written as $F \propto \dfrac{Qq}{r^2}$.

Formula

Or,

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2},}$$

where Q and q are the charges (in coulomb, C), r the distance between them and ε_0 the *permittivity of free space*, which has a value of 8.85×10^{-12} F m^{-1}.

like charges repel, i.e. sign of F is positive. unlike charges attract i.e. sign of F is negative.

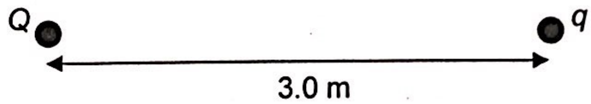
Note

Note:

- Both Coulombs' law and Newton's law of gravitation are sometimes referred to as *inverse-square laws*.
- The major difference between gravitational force and **electric (or electrostatic) force** is that the former is always an attractive force while the latter can be attractive or repulsive. This is because there are two types of charges – positive and negative. Like charges repel, while unlike charges attract.

Example 1

Two charges Q (+2.0 μC) and q (+1.0 μC) are separated by a distance of 3.0 m.



(a) Determine the magnitude and direction of the force acting on q by Q.

(b) If q is negatively charged instead, but with the same magnitude, determine the magnitude and direction of the force on q by Q.

Solution

(a) Magnitude of force,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{1}{4\pi(8.85\times10^{-12})}\frac{(+2.0\times10^{-6})(+1.0\times10^{-6})}{(3.0)^2} = 2.0\times10^{-3}\text{ N}$$

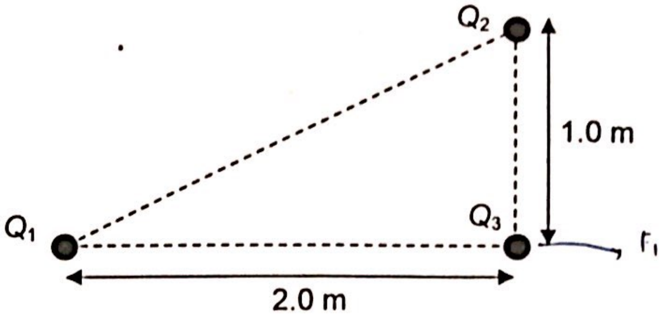
Since both charges are positive, the force acting on q due to Q is directed away from Q or to the right.

(b) Magnitude of force,

$$F = 2.0\times10^{-3}\text{ N}$$

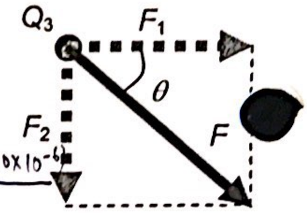
Since both charges are oppositely charged, the force acting on q due to Q is directed towards Q or to the left.

Example 2 Three charges $Q_1 = +2.0\ \mu C$, $Q_2 = +1.0\ \mu C$ and $Q_3 = +3.0\ \mu C$ are arranged as shown.



Determine the magnitude and direction of the resultant force acting on Q_3.

Solution Let F_1 and F_2 be the force acting on Q_3 due to Q_1 and Q_2 respectively.



$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r_{13}^{\ 2}} = \frac{1}{4\pi(8.85\times10^{-12})}\frac{(Q_1Q_3)}{r_1^{\ 2}} = \frac{1}{4\pi\varepsilon_0}\frac{(2.0\times10^{-6})(3.0\times10^{-6})}{(2.0)^2}$$

$$= 13.5\times10^{-3}\ N$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r_{23}^{\ 2}} = \frac{1\,(1.0\times10^{-6})(3.0\times10^{-6})}{4\pi\varepsilon_0(1.0)^2} = 27.0\times10^{-3}\ N$$

$$|F| = \sqrt{F_1^{\ 2}+F_2^{\ 2}} = \sqrt{(13.5\times10^{-3})^2+(27.0\times10^{-3})^2} = 0.030\ N\ (2\ s.f.)$$

The magnitude of the resultant force, $F = 0.030\ N$

and it is acting at an angle θ below the horizontal given by

$$\tan\theta = \frac{F_2}{F_1} = \frac{27.0}{13.5}$$

$$\theta = 63^\circ\ (2\ s.f.)$$

13.3 Electric Field Strength

Electric Field When one charge is first introduced, it creates a *field* of influence around itself that permeates the entire universe. Any other charge that is introduced thereafter will interact with this field, and hence experience a force.

Definition

An electric field is a region of space in which a charge placed in that region experiences an electric force

Electric Field Strength

Definition

The electric field strength at a point is defined as the electric force exerted per unit positive charge placed at that point.

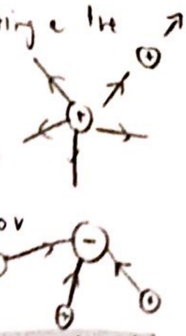
Formula

Mathematically, it is written as

$F = \frac{q_1 q_2}{4\pi \varepsilon r^2} = q_2 E_1$

$$\boxed{E = \frac{F}{q}}$$

decide direction by putting a +ve test charge there



where E is the electric field strength and F is the force acting on the charge q.

10 V $\xrightarrow{E}$ -10 V

Note

Note:

- Electric field strength is a **vector** quantity.
- Its direction is that of the force acting on a **positive** charge.
- It points from a region of higher electric potential to a region of lower electric potential (see later section 13.4).
- Its unit is N C^{-1} or V m^{-1}.
- Some A-Level questions refer to it as just "electric field".

Example 3

A charge of -2.0 μC experiences a force of 8.0 N when placed at a point in an electric field. Determine the electric field strength at the point.

doesn't matter if uniform or not

Solution

The magnitude of the electric field strength is $E = \frac{F}{q} = \frac{8.0}{2.0 \times 10^{-6}} = 4.0 \times 10^{6}$ N C^{-1}

The direction of the electric field strength is opposite to that of the force acting on the negative charge.

"e field opposite to force"

positive ⟶ negative

Electric Field Strength due to a Point Charge

A positive point charge q is placed a distance r from another positive point charge Q.

The charge q will experience a force F given by Coulomb's law: $F = \frac{1}{4\pi\varepsilon_0}\frac{Q \times q}{r^2}$

From the definition of electric field strength $E = \frac{F}{q}$, the electric field strength E due to charge Q at a distance r from it is

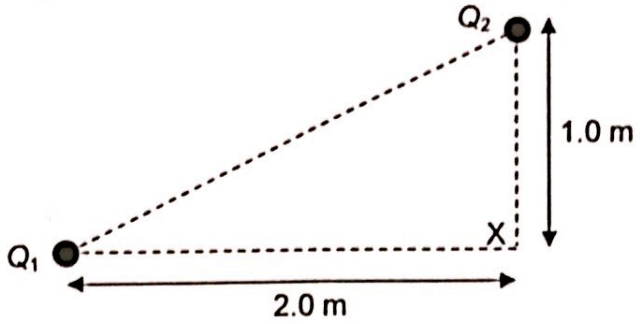
Formula

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

only applicable to point charges

Note that an electric field exists around any charged object and its field strength does not require the presence of another charge such as q, which can be viewed as a test charge.

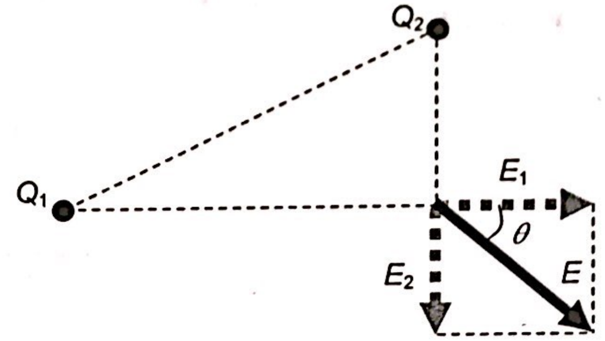
Example 4 Two charges Q_1 = +2.0 μC and Q_2 = +1.0 μC are placed at the corners of a triangle with dimensions as shown.



(a) Determine the electric field strength at the third corner X.

(b) If a charge Q_3 of +3.0 μC is placed at corner X, determine the force it experiences.

Solution (a)



Let E_1 and E_2 be the electric field strength at corner X due to Q_1 and Q_2, respectively.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2} = \frac{1}{4\pi(8.85\times10^{-12})}\frac{(2.0)\times10^{-6}}{(2.0)^2} = 4.496\times10^3\ \text{NC}^{-1}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2^2} = \frac{1}{4\pi(8.85\times10^{-12})}\frac{(1.0)\times10^{-6}}{(1.0)^2} = 8.992\times10^3\ \text{NC}^{-1}$$

just write as $\frac{1}{4\pi\varepsilon_0}$

The magnitude of the resultant field strength at corner X is

$$E = \sqrt{E_1^2 + E_2^2} = \sqrt{(4.496\times10^3)^2 + (8.992\times10^3)^2} = 10000\ \text{NC}^{-1}$$

The direction of the resultant field strength is at an angle θ below the horizontal where

$$\theta = \tan^{-1}\left(\frac{E_2}{E_1}\right) = \left(\frac{8.992}{4.496}\right) = 63°$$

(b) $F = q_3E$ = (3.0 x 10^{-6})(10053) = 0.030 N (2 s.f.)

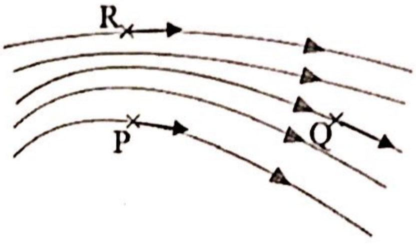
Since charge Q_3 is a positive charge, the force acting on it is in the direction of the electric field strength at the point.

Notice that the answer to (b) is identical to that of Example 2. This is not surprising as the 2 scenarios are identical. Only the methods used to solve them were different.

Electric Field Lines

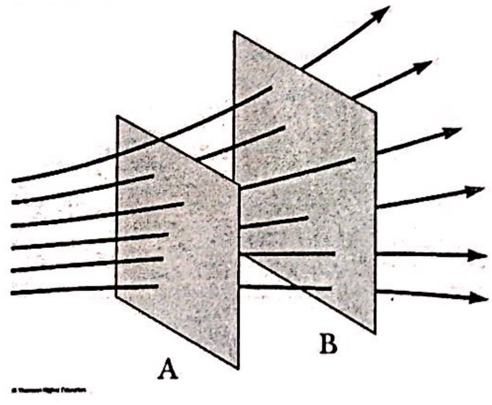
The concept of a field is useful in understanding gravitational, electric and magnetic forces. The field lines we draw around masses, charges and magnets are used to denote the direction of the force that will be experienced by a body (that interacts with the field) placed within the region. It must be emphasised that the electric field is a three-dimensional pattern, but on paper it is represented by a two-dimensional slice of the pattern. Below is a qualitative description of electric field.

1) The **direction of an electric field line** indicates the direction of the electric force acting on a small positive test charge placed at that point. The direction of the electric field strength at any point is obtained by drawing a tangent to the electric field line at that point.



where the electric force moves, the field points

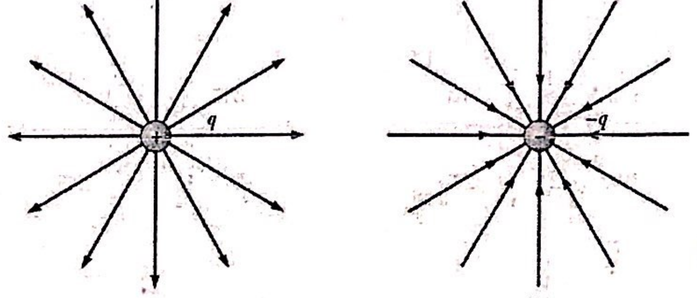
2) The **density of the field lines** (represented by the spacing between the field lines) of a region indicates the relative magnitude of the field strength. The closer the spacing of the field lines, the stronger the field at that region, and vice versa. Also, the field lines do not cross one another.



Since the density of field lines at A is higher than at B, the electric field strength at A has a larger magnitude than that at B.

3) Electric field lines **originate from positive charges and terminate on negative charges** (these lines will reach infinity in the absence of another charge).

field lines never cross



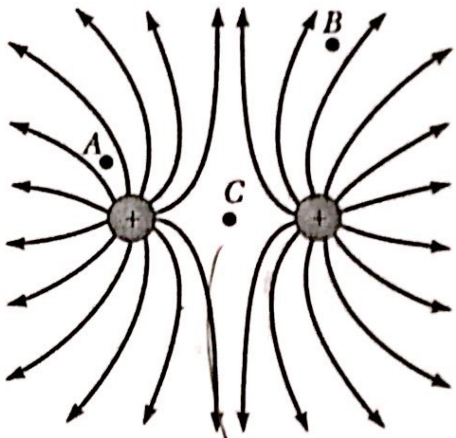
(a) (b)

imagine invisible hand pushing charges together

attractive → F
⊕ ← ⊖
similar to grav: both are attractive ← u is negative

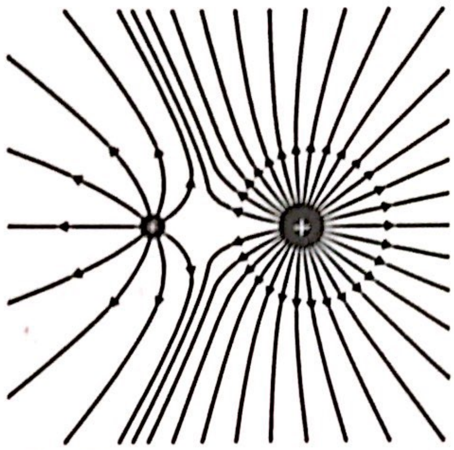
repel
⊕ ← ⊕
u is positive cause must do work to bring together

Electric Field Lines due to 2 Point Charges

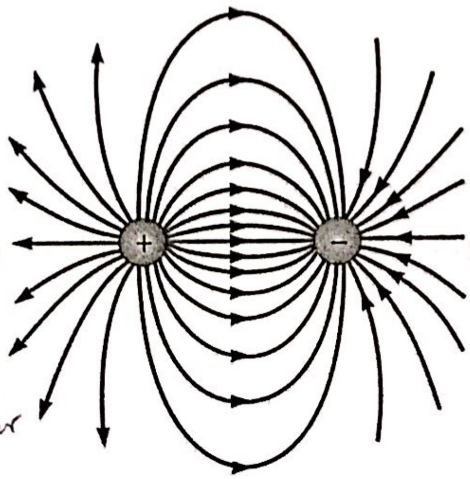


(a) Positive charges of the same magnitude

neutral point

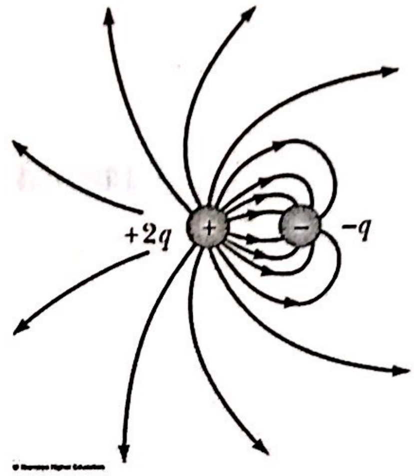


(b) Positive charges of different magnitudes



same size = same number of lines

(c) Opposite charges of the same magnitude



(d) Opposite charges of different magnitudes

neutral point is further away from stronger charge

explanation for not crossing
① undefined potential at that point
② at that point, where would a +ve charge go?

transfer to page 10

Electric Field Lines in the Presence of a Conductor

Any charges added to an **isolated solid or hollow conductor** will redistribute themselves uniformly onto the surface of the conductor until no charge is present within the conductor. The presence of an electric field causes the free electrons in a conductor to move until equilibrium is achieved. An internal electric field is set up within the conductor which cancels the external electric field.

Note

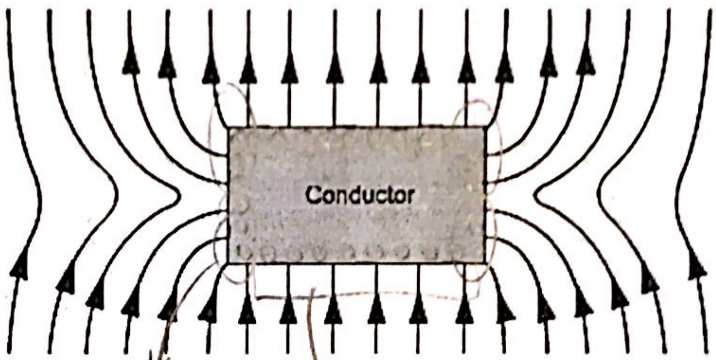
Therefore,

(a) there is no electric field within a solid or hollow conductor;

(b) electric field lines must start and end at the surface of the conductor;

(c) electric field lines are perpendicular to the surface of the conductor – otherwise, there will be a component tangential to the surface of the conductor which will cause the charges to move and redistribute until the net force on them is zero.

d) the potential within the conductor is constant – i.e. the p.d. between any points is zero (see 13.4)

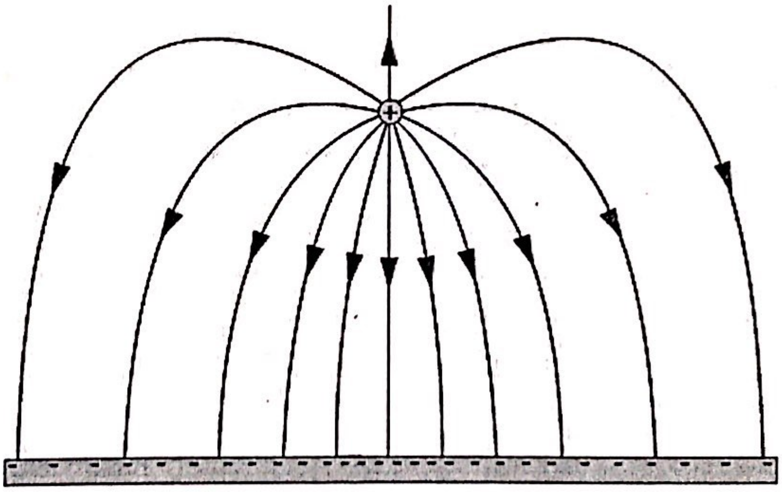


lightning conductor

high, strong electric field

uniform electric field strength

Electric field lines at the surfaces of an isolated conductor placed within an electric field



Electric field lines between a point charge and plane conductor

Faraday

Graph of Electric Field Strength vs Distance for a Charged Conducting Sphere

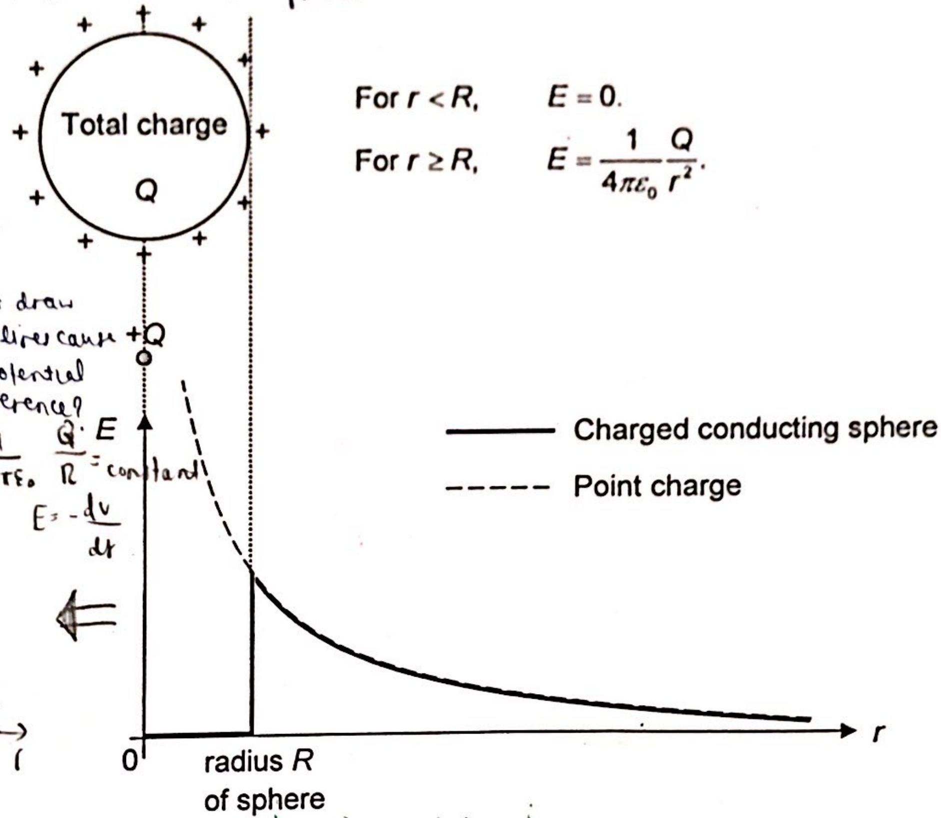
It can be shown that for a charged solid or hollow conducting sphere, the electric field outside the sphere behaves as though all its charges are concentrated at its centre. Therefore, we can use the equation for point charges to determine the electric field strength outside the sphere.

Consider a point charge +Q and a sphere with charge +Q. The graphs of electric field strength E vs distance r are as shown below.

Inside the conducting sphere, the electric field is zero because all the charges are uniformly distributed on the surface of the sphere. the potential within the sphere is the same as that of the surface of the sphere.

For $r < R$, $E = 0$.

For $r \geq R$, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$.

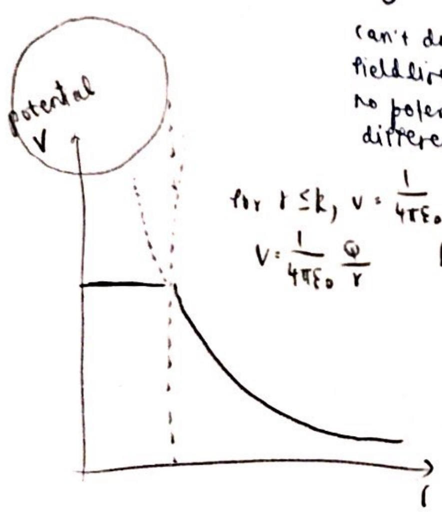


can't draw field lines cause no potential difference?

for $r \leq R$, $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R}$ = constant

$V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$

$E = -\dfrac{dV}{dr}$



how would electric potential look like?

perpendicular

$E = \dfrac{-dV}{dr}$

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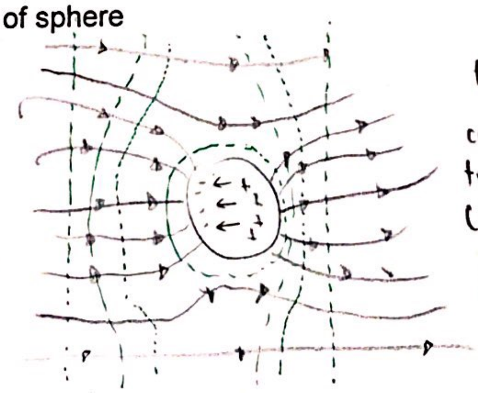


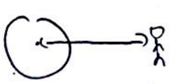
the presence of an electric field causes the free e⁻ in the conductor to move until equilibrium is achieved (ie no current flows)

(within the conductor, no electric field)

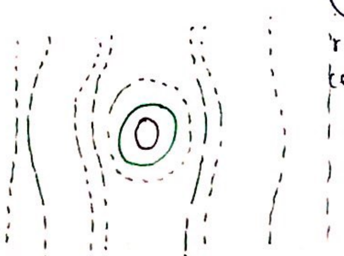
at this point, an internal electric field is set up within the conductor which cancels the external electric field.

hence, electric field inside a conductor is zero. (i.e. a constant).

everything in notes: solid/hollow metal conductor



r measured from centre of planet



what is the PE of the system? How much work required to bring the 2 charges from ∞ to this location?

13.4 Electric Potential and Electric Potential Energy

Relationship between Electric Potential & Potential Energy

Definition

The electric potential at a point is defined as the work done per unit positive charge by an external force in bringing a small test charge from infinity to that point.

: electric potential energy

Like gravitation, there is no electrical influence at infinity, and the potential and potential energy there is zero.

If W is the work done by an external force in moving a small positive test charge q from infinity to a point in the electric field, then the potential V at that point is given by

Formula

$$\boxed{V = \frac{W}{q}}$$

if there's no other charges, W = 0

The unit for electric potential is **the volt (V)**, or **J C^{-1}**. Note that potential is a scalar quantity. However, unlike gravitation, the electric potential can take both positive and negative values because the work done by the external force can be both negative and positive due to attractive and repulsive electric forces, respectively.

We can imagine the external force moving the test charge from rest at infinity and stopping it at the intended position. The work done by the external force is entirely converted to the change in potential energy of the system (in the absence of dissipative forces). Hence, the electric potential energy U of a charge placed at a point of electric potential V in an electric field is given by

Formula

$$\boxed{U = W = qV}$$

energy can be defined as this

Caution! *Charge q, potential V and potential energy U can be positive or negative. The signs must be of all quantities used in the equations must be correct.*

The electron-volt (eV)

In many situations, the energy gained by a charged particle is small and hence it is convenient to introduce a new unit of energy called the electron-volt (eV).

The electron-volt is the energy gained by an electron when it is accelerated through a p.d. of one volt. $1\text{ eV} = 1.60 \times 10^{-19}$ J.

Electric Potential due to a Point Charge

The electric potential V at a point a distance r from a point charge Q can be derived from the definition of electric potential.

Not in syllabus, for interest only: From the definition,

$$V = \frac{W}{q} = \frac{1}{q}\int_{\infty}^{r} F_{ext}\,dr = \frac{1}{q}\int_{\infty}^{r}(-F)\,dr = \frac{1}{q}\int_{\infty}^{r}\left(-\frac{qQ}{4\pi\varepsilon_o r^2}\right)dr = \frac{1}{q}\left[\frac{qQ}{4\pi\varepsilon_o r}\right]_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_o r}$$

Formula Hence, **for a point charge,**

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

Since electric potential energy is $U = qV$, the electric potential energy of a system of **two point charges is**

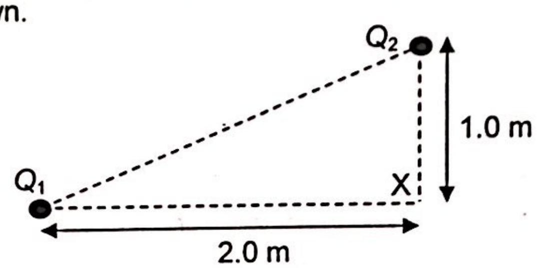
Formula

$$\boxed{U = \frac{Qq}{4\pi\varepsilon_0 r}}$$

energy = potential X q

Unlike gravitation, there is no negative signs in the expression for both V and U; using the correct signs for Q and q take care of the signs for both V and U.

Example 5 Two charges Q_1 = +2.0 μC and Q_2 = −2.0 μC are placed at the corners of a triangle with dimensions as shown.



(a) Determine the electric potential at the third corner X.

(b) If a charge Q_3 of +1.0 μC is placed at corner X, determine the charge in the electric potential energy of this new configuration.

"work done by an external agent to place a third charge at X" same thing

Solution (a) As electric potential is a scalar quantity, to find the potential at corner X, we add the potentials at that point due to the two charges algebraically.

V_1 (due to Q_1) = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{(2.0\times10^{-6})}{2.0} = 8992\text{ V}$

V_2 (due to Q_2) = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{(-2.0\times10^{-6})}{1.0} = -17984\text{ V}$

Therefore, the net electric potential V at the third corner is $V_1 + V_2 = 8992 - 17984 = -9.0\text{ kV}$

~~$\Delta U = q\Delta V$~~ ...

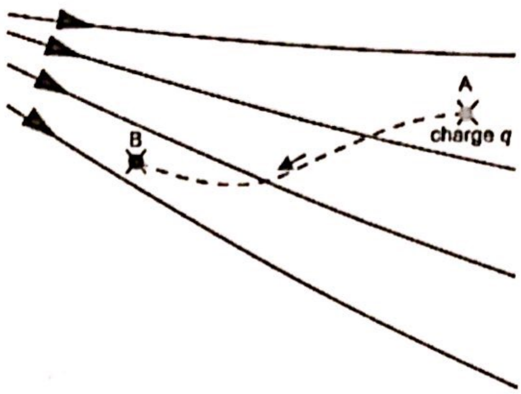
(b) The electric potential energy U of the new system is $U = \frac{(+2.0\times10^{-6})(-2.0\times10^{-6})}{4\pi\varepsilon_0(\sqrt{4-1})}$

=

$\Delta U = q\Delta V = (1.0\times10^{-6})(-8992-0)$
$= -9.0\times10^{-4}\text{ J}$

Potential Difference

We often encounter situations where charged bodies are moved from one point to another within an electric field, and we want to determine the energy required to move them. Consider a charge q at point A being moved to a new point B in an arbitrary electric field through an arbitrary path,



The change in potential ΔV (or the **potential difference**) between two points is given by

Formula

$$\boxed{\Delta V = V_{final} - V_{initial}} = V_B - V_A$$

Caution! *The positive or negative sign of the potential must be substituted into V_{final} and $V_{initial}$.*

The work done W by an external force (or the change in potential energy of the system) in moving the charge q is

Formula

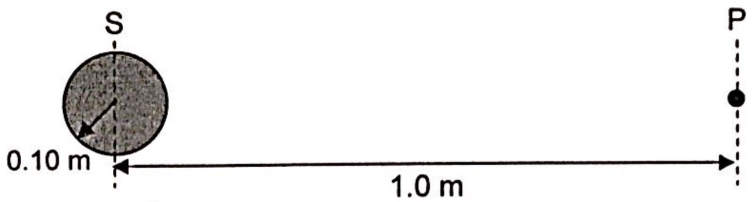
$$\boxed{W = q\Delta V} \quad \text{or} \quad \boxed{\Delta U = q\Delta V}$$

path-independent

Caution! *The positive or negative sign of the charge and potential difference must be substituted into q and ΔV.*

Example 6 A fixed, positively charged sphere S of radius 0.10 m carries a charge of 1.0×10^{-4} C. A particle P of mass 2.0×10^{-5} kg and charge -1.5×10^{-10} C is released from rest at a distance of 1.0 m from the centre of the charged body. Calculate the velocity of the particle when it strikes the surface of the sphere. Neglect any gravitational effect.

Solution



gain in KE due to loss of PE

Because the particle moves from one point to another,
- we will first calculate the change in potential,
- then the change in potential energy
- and finally use the principle of conservation of energy to determine its speed.

Electric potential at the initial position of P due to S:

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_i} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(1.0\times10^{-4})}{1.0} = 8.9918\times10^5 \text{ V}$$

Electric potential at the final position of P due to S:

try do deal in potential & PE first cause no direction

$$V_f = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_f} = \frac{1}{4\pi\varepsilon_0}\frac{(1.0\times10^{-4})}{0.1} = 8.9918\times10^6 \text{ V}$$

$$\Delta U_E = q(V_f - V_i)$$
$$= (-1.5\times10^{-10})(8.9918\times10^6 - 8.9918\times10^5)$$
$$= -1.2139\times10^{-3} \text{ J}$$

This means that the electric potential energy of the system decreases as P moves towards S, which leads to an increase in the kinetic energy of P.

By the principle of conservation of energy,

~~$W = q\Delta V = \frac{1}{2}mv^2$~~

increase in KE = decrease in PE

$\frac{1}{2}mv^2 - 0 = 1.2139\times10^{-3}$

$v = 11 \text{ ms}^{-1}$

13.5 Equipotential Lines and Surfaces

Analogous to Contour Lines in Maps

Equipotential lines are like contour lines on a map which trace lines of equal altitude. In this case, the "altitude" is the electric potential.

An equipotential line joins points with the same potential. **Equipotential lines are perpendicular to electric field lines**. (In 3D space, equipotential lines form equipotential surfaces.)

Movement along an equipotential line or surface requires no work because there is no component of electric force along such path (which is always perpendicular to the electric field lines). It can also be explained by a lack of change in electric potential.

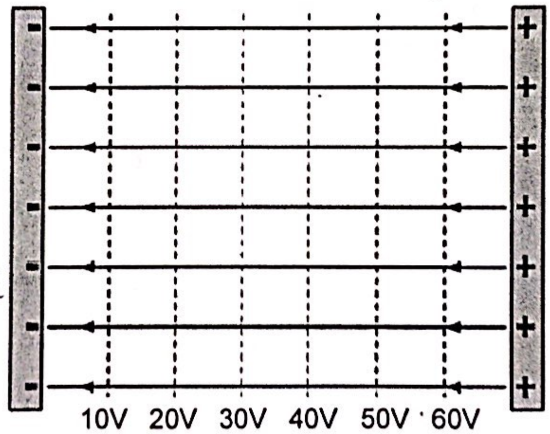
Dashed lines are usually used to represent equipotential lines, while solid lines are electric field lines.

Uniform Electric Field

For equally charged parallel conducting plates, like those in a capacitor, the electric field lines are perpendicular to the plates and the equipotential lines are parallel to the plates.

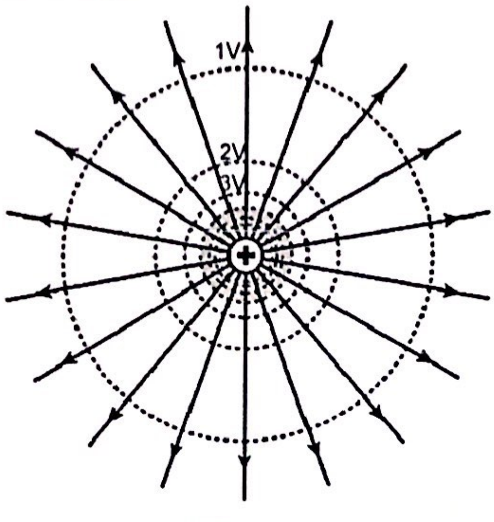
close together
= stronger field

not necessarily charged, just lower potential



Point Charge

In 2D, the equipotential lines around a point charge are circles centred on the charge. In 3D, spherical equipotential surfaces are centred on the charge. The dashed lines represent voltage at equal increments – the equipotential lines get further apart with increasing distance from the charge.



13.6 Important Relationships

Between *E* & *V*

From the definition of potential,

$$V = \frac{W}{q} = \frac{1}{q}\int_{\infty}^{r} F_{ext}\,dr = \frac{1}{q}\int_{\infty}^{r}(-F)\ dr$$

Since the electric field strength $E = \frac{F}{q}$, the expression becomes

$$V = \int_{\infty}^{r}(-E)\ dr.$$

This implies that electric field strength E can be obtained from the derivative of electric potential V with respect to r. Mathematically, this is expressed as

Formula

$$\boxed{E = -\frac{dV}{dr}}$$

Therefore:

Definition

The **electric field strength at a point is numerically equal to the potential gradient at that point.**

The negative sign indicates that the electric field points in the direction of decreasing potential. (The gradient of a V–r graph gives the magnitude of the electric field strength.)

Between *F* & *U*

Similarly, from the definition of potential energy,

$$U = \int_{\infty}^{r} F_{ext}\,dr = \int_{\infty}^{r}(-F)\ dr, \quad \text{or} \quad qE = -q\frac{dV}{dr},$$

we have

Formula

$$\boxed{F = -\frac{dU}{dr}}$$

13.7 Comparison between Electric Field and Gravitational Field

For point charges or point masses	Electric field	Gravitational field
Force	$F_E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r^2}$	$F_G = -G\dfrac{Mm}{r^2}$
Field strength	$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$	$g = -G\dfrac{M}{r^2}$
Potential energy	$U_E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{r}$	$U_G = -G\dfrac{Mm}{r}$
Potential	$V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$	$\phi = -G\dfrac{M}{r}$
General relationships	$F_E = -\dfrac{dU_E}{dr}$ $E = -\dfrac{dV}{dr}$ $E = \dfrac{F_E}{q}$ $V = \dfrac{U_E}{q}$	$F_G = -\dfrac{dU_G}{dr}$ $g = -\dfrac{d\phi}{dr}$ $g = \dfrac{F_G}{m}$ $\phi = \dfrac{U_G}{m}$

From the table, it is evident that there are similarities between the expressions for electric and gravitational fields.

* point charges

13.8 Uniform Electric Field

Within a region of uniform electric field:

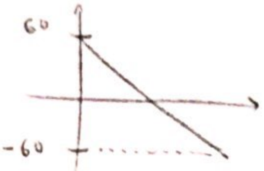
(a) the electric field strength at any point has the same magnitude and direction;

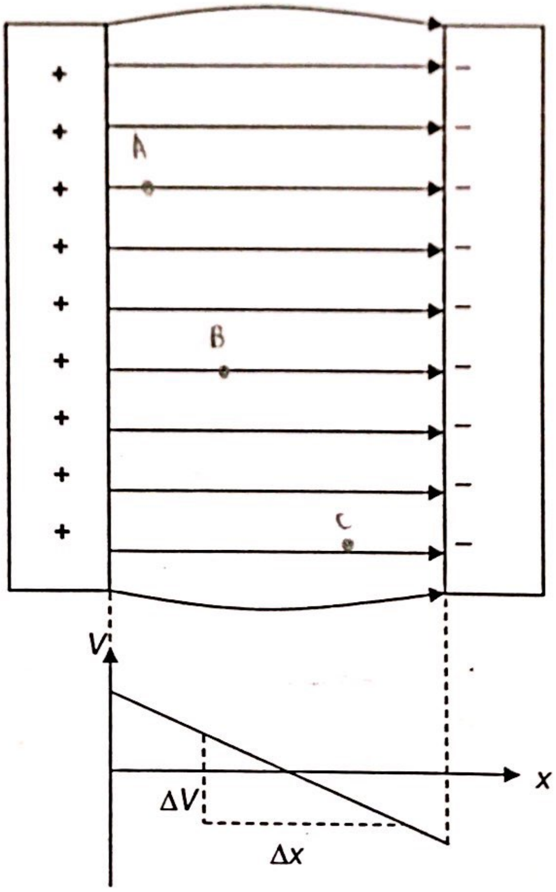
(b) the electric field lines are parallel and equally spaced.

Two equally but oppositely charged parallel plates of infinite dimensions can produce a uniform electric field between them. However in the laboratory, the plates used have finite dimensions and the field at the ends of the plates will not be uniform due to fringing effects. In the subsequent discussion, we will only consider the region near the centre of the two plates where the field is uniform.

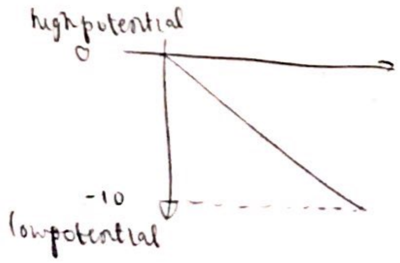
60V

-60V





force on A, B, C (any charged particle) is the same



Since the electric field between the parallel plates is uniform, the potential gradient at any point within the plates is given by

$$\frac{dV}{dx} = \frac{\Delta V}{\Delta x} = -E = \text{constant}$$

where x is the perpendicular distance from the plate with the higher potential.

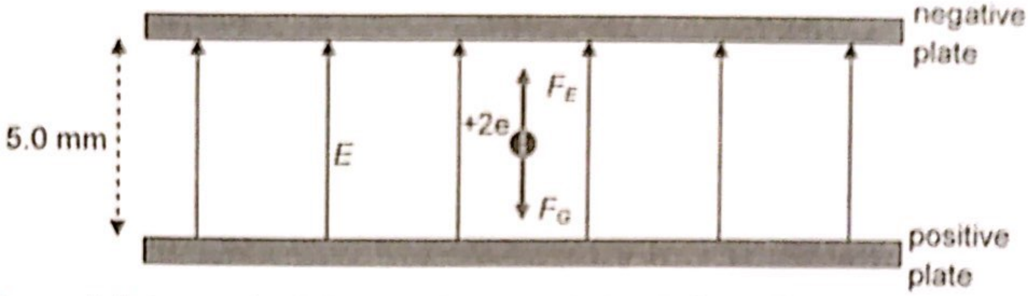
Therefore, if we know the potential difference ΔV between the two plates and the separation d of the plates, the **magnitude of the electric field strength E between the parallel plates is**

Formula

$$\boxed{E = \left|\frac{\Delta V}{d}\right|}$$

Example 7

A particle of mass 2.0×10^{-15} kg and charge $+2e$ remains stationary within two parallel plates maintained at a potential difference V and separated by a distance of 5.0 mm. By considering the forces acting on the particle, determine V. ($e = 1.60 \times 10^{-19}$ C)



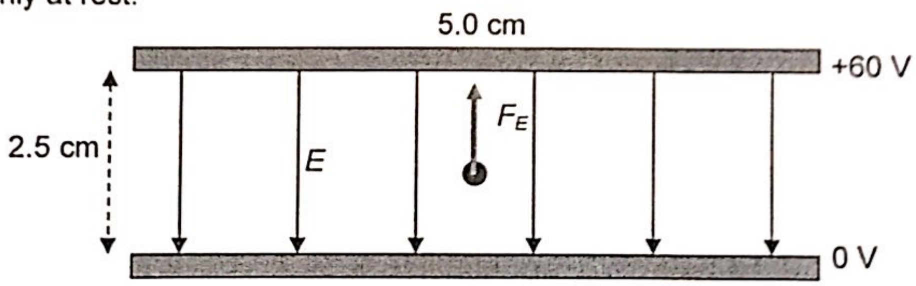
Solution

For the particle to remain stationary, the upward electric force F_E balances the downward weight F_G.

$$\left.\begin{aligned} F_E &= F_G \\ qE &= mg \\ q\frac{V}{d} &= mg \end{aligned}\right\} \quad V = \frac{mgd}{q} = \frac{(2.0\times10^{-15})(9.81)(5.0\times10^{-3})}{2\times1.60\times10^{-19}} = 307 \text{ V}$$

Example 8

Two horizontal plates of length 5.0 cm are placed 2.5 cm apart in a vacuum. The upper plate is maintained at a potential of +60 V relative to the lower plate. An electron is injected through a hole in the upper plate with a speed of 2.9×10^6 m s^{-1} towards the lower plate. Determine the distance s from the upper plate where the electron is momentarily at rest.



Solution

Since the plates are parallel, the electric field between them is uniform and the electron will experience a constant electric force F_E in the direction opposite to its motion.

$$F_E = qE = q\left|\frac{\Delta V}{d}\right| = (1.60\times10^{-19})\left(\frac{60}{2.5\times10^{-2}}\right) = 3.84\times10^{-16} \text{ N}$$

As the electron moves away from the upper plate, its kinetic energy decreases until it is momentarily at rest.

By the principle of conservation of energy,

$$\begin{aligned} \text{loss in KE} &= \text{gain in EPE} \\ &= \text{work done against electric field} \end{aligned}$$

$$\frac{1}{2}mu^2 - 0 = F_E \times s$$

$$s = \frac{mu^2}{2F_E} = \frac{(9.11\times10^{-31})(2.9\times10^6)^2}{2(3.84\times10^{-16})} = 1.0\times10^{-2} \text{ m}$$

Note: As the electron is very light, its weight is negligible compared to the electric force it experiences. Hence, weight is usually ignored in such calculations.

↳ gravitational force

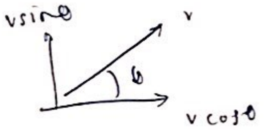
Example 9 An electron is projected horizontally between the plates in Example 8 with an initial speed of 7.5×10^6 m s^{-1}. Neglecting any gravitational and edge effects, (fringing)

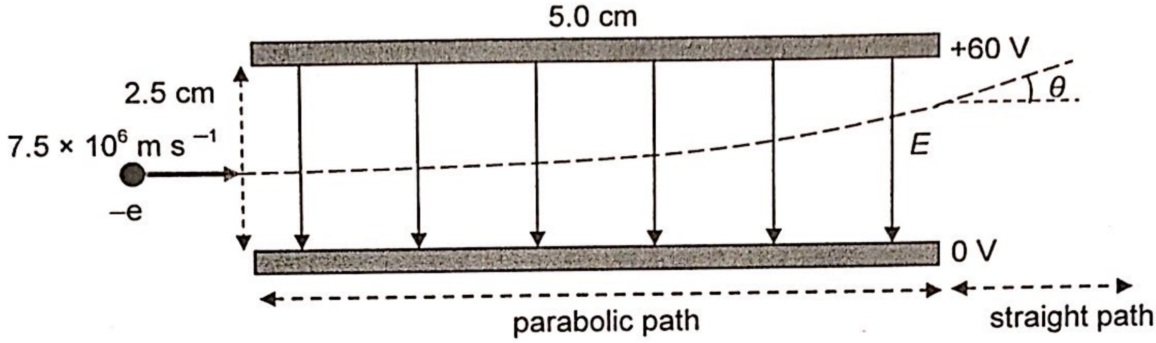
(a) describe and explain the path taken by the electron while moving between the plates,

(b) determine the angle the electron makes with the horizontal upon leaving the plates.

(rest mass of electron $m_e = 9.11 \times 10^{-31}$ kg, elementary charge $e = 1.60 \times 10^{-19}$ C)

Solution





(a) Since the plates are parallel, the electric field between them is uniform. The electron will experience a constant acceleration (due to the constant electric force) towards the upper positive plate, while its horizontal velocity remains constant. Hence, the electron will move in a parabolic path towards the upper positive plate.

Beyond the parallel plates, the electron moves along a straight path.

(b) Horizontally:

$$t = \frac{s_x}{u_x} = \frac{5.0 \times 10^{-2}}{7.5 \times 10^{6}} = 6.667 \times 10^{-9}\ \text{s} \quad - (1)$$

Vertically:

$$a_y = \frac{F_E}{m_e} = \left(q\frac{\Delta V}{d}\right)/m_e = \frac{(1.60 \times 10^{-19})(60)}{(2.5 \times 10^{-2})(9.11 \times 10^{-31})}$$

$$= 4.626 \times 10^{13}\ \text{m s}^{-2} \quad - (2)$$

sub (1) & (2) into $v_y = u_y + a_y t$

$$v\sin\theta = 0 + \frac{(1.60 \times 10^{-19})(60)}{(2.5 \times 10^{-2})}\left(\frac{1}{9.11 \times 10^{-31}}\right)(6.6667 \times 10^{-9})$$

$$= 2.810 \times 10^{6}\ \text{m s}^{-1}$$

$$\tan\theta = \frac{v\sin\theta}{v\cos\theta} = \frac{2.8101 \times 10^{6}}{7.5 \times 10^{6}}$$

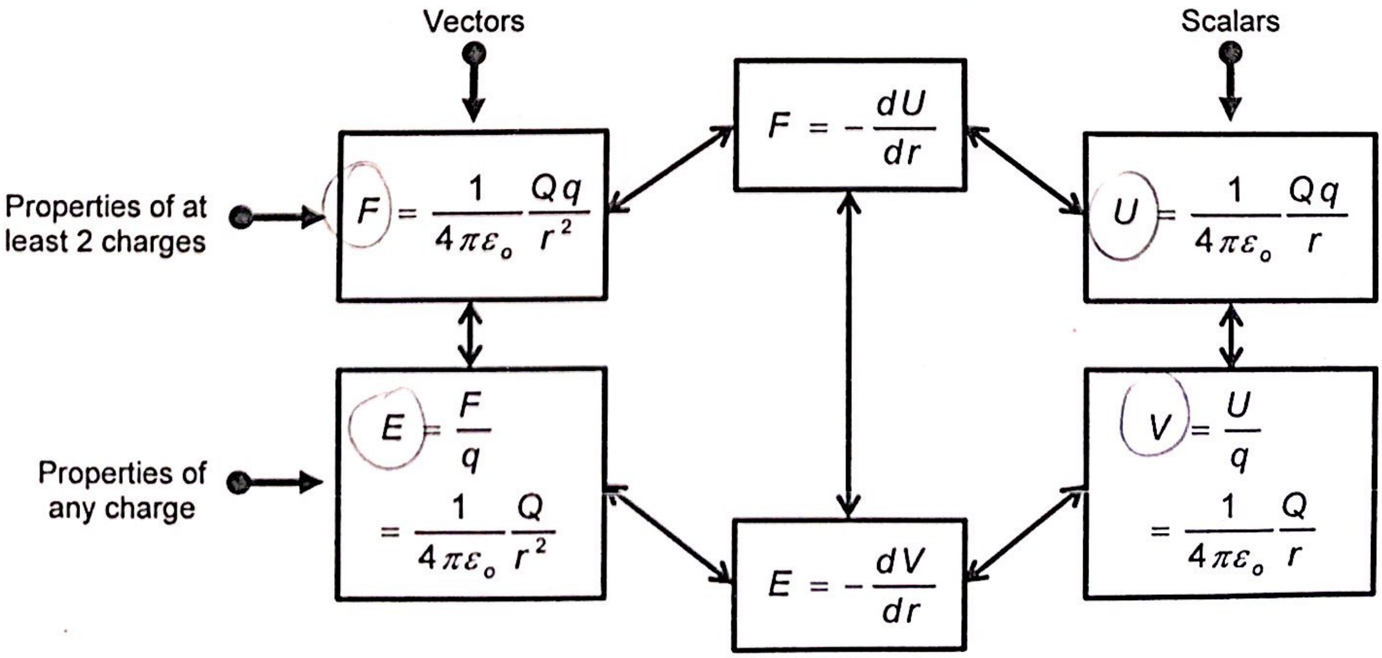
Hence: $\theta = 20.54 = 21^\circ$

It can be seen from this example that the effect of electric field on a charged particle is similar to that of gravitational field on a mass.

Summary

Quantity	Expression	Unit	Remarks
Force **(Vector)**	$F_E = -\frac{dU_E}{dr}$ $F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$	N	• Attractive or repulsive. • Property of a system of charges.
Field Strength **(Vector)**	$E = -\frac{dV}{dr}$ $E = \frac{F_E}{q}$ $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$	$V\ m^{-1}$ or $N\ C^{-1}$	• Points away from positive charge, or into negative charge. • Property of any charge. • The electric field strength at a point is defined as the electric force exerted per unit positive charge placed at that point.
Potential Energy **(Scalar)**	$U_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$	J	• Positive or negative, depends on product Qq. • Property of a system of charges.
Potential **(Scalar)**	$V = \frac{U_E}{q}$ $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$	V	• Positive or negative, depends on charge Q. • Property of any charge. • The electric potential at a point is defined as the work done per unit positive charge by an external force in bringing a small test charge from infinity to that point.

F, E, U, V: point charges only!



Appendix

(A) Milikan's Oil-Drop Experiment

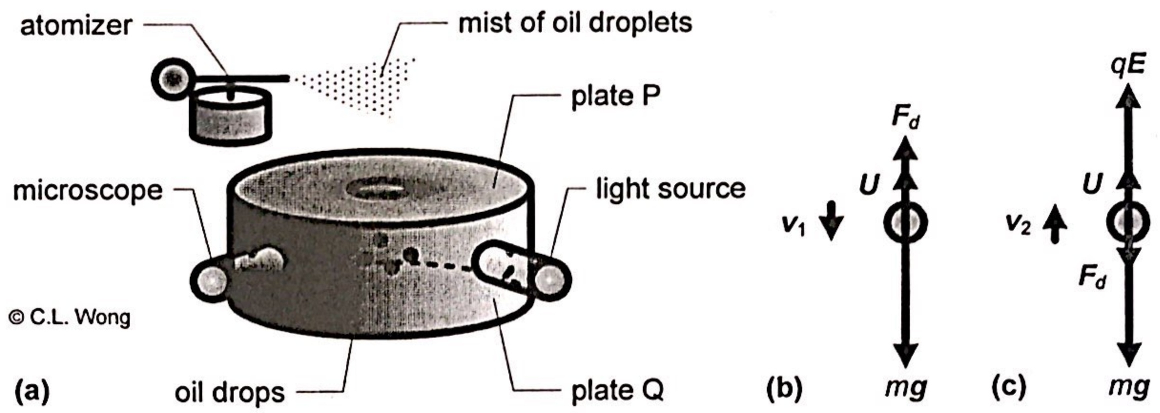


Fig. (a) shows a **simplified scheme** of the Millikan's oil-drop experiment, first used by Robert Millikan and Harvey Fletcher to determine the electric charge of the electron in 1909. By measuring the terminal speeds of the oil droplet between the plates, its electric charge can be measured. Robert Millikan was awarded the Nobel Prize in physics in 1923.

Fig. (b): **Without an electric field**, the oil droplet falls with terminal speed v_1.

An atomizer is used to produce tiny oil droplets which fall through a hole in the upper plate P. Frictional effects in the atomizer caused the oil droplets to be charged; some positively and others negatively. The droplets are illuminated by a light source and observed using a microscope.

First, we can determine the radius r and hence the weight W of an observed oil droplet by allowing it to fall with terminal speed. The drag force F_d acting on the oil droplet is given by Stokes' law: $F_d = 6\pi\eta rv$, where v is the terminal speed and η is the viscosity of air.

When the oil droplet is falling with terminal speed v_1, $\frac{4}{3}\pi r^3\rho_{oil}g = \frac{4}{3}\pi r^3\rho_{air}g + 6\pi\eta rv_1$

$$\therefore r = \sqrt{\frac{9\eta v_1}{2g(\rho_{oil} - \rho_{air})}}.$$

Fig. (c): **With an electric field**, the oil droplet rises with terminal speed v_2.

A uniform electric field is set up between plates P and Q by adjusting the potential difference V between the plates using a potentiometer circuit (not shown). The electric field strength E between the plates is $E = \frac{V}{d}$, where d is the plate separation.

The potential difference V causes the oil droplet to rise with a terminal speed v_2, such that

$$\frac{4}{3}\pi r^3\rho_{oil}g + 6\pi\eta rv_2 = \frac{4}{3}\pi r^3\rho_{air}g + qE$$

The electric charge q on the oil droplet can then be found by measuring v_2.

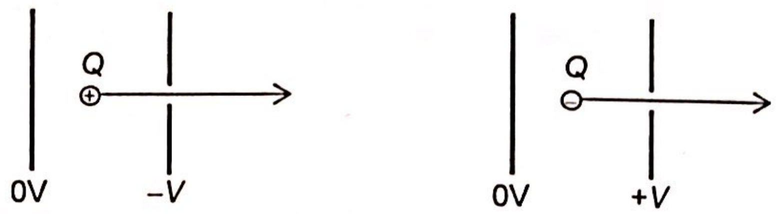
Note

By repeating the procedure for different oil droplets carrying different charges, Robert Millikan was able to prove that

- the fundamental unit of electric charge e is 1.60×10^{-19} C;
- any charge q is always an integer multiple of e, i.e., electric charge is quantized;
- an electron is negatively charged.

(B) Linear Accelerator (LINAC)

The most common method of accelerating a charged particle from rest is to use a uniform electric field.

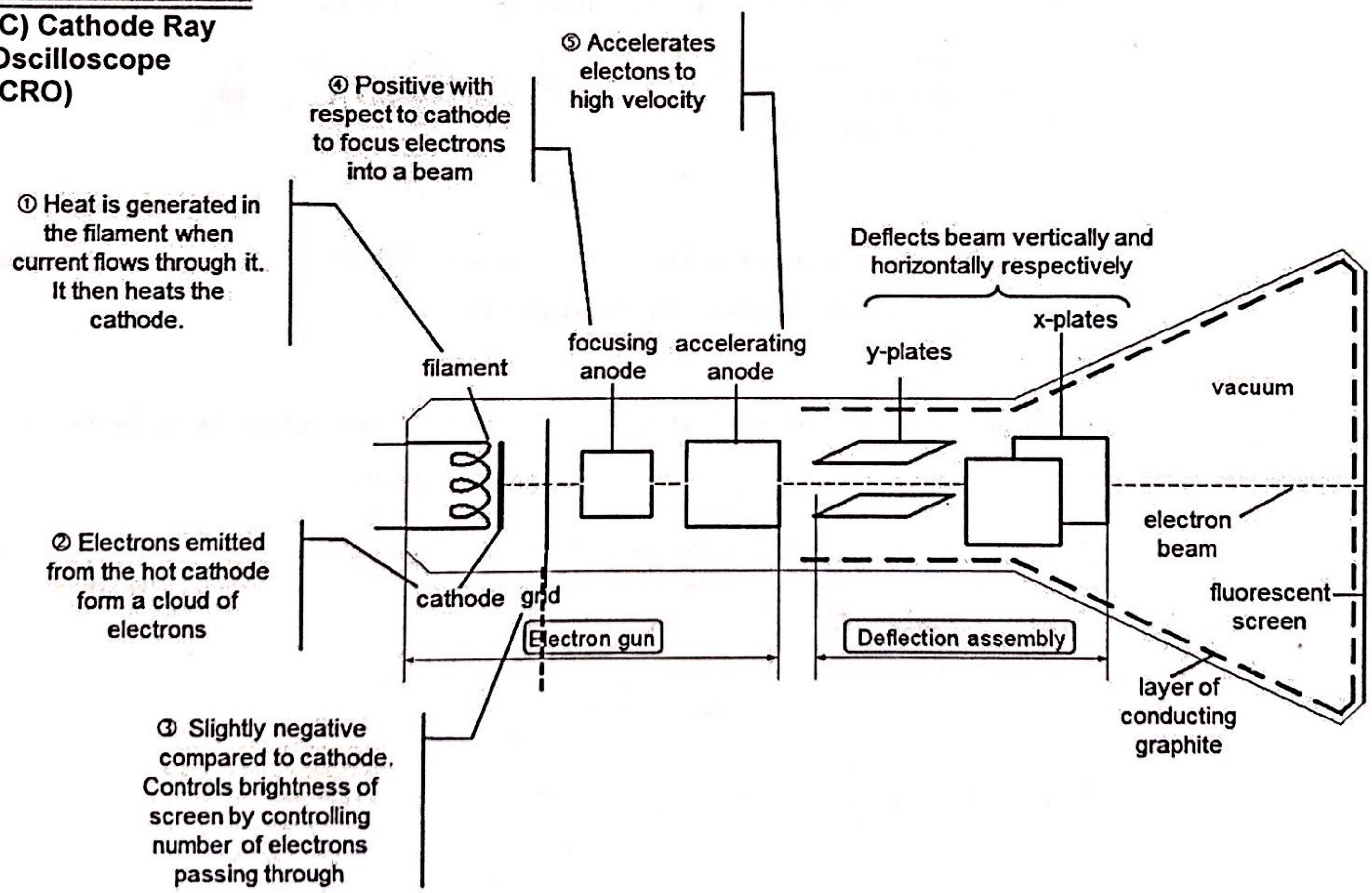


When a particle of charge Q and mass m is accelerated from rest through a potential difference V and distance d, its acceleration is $a = \frac{F_E}{m} = \frac{QE}{m} = \frac{Q}{m}\frac{V}{d}$.

The kinetic energy it gained is $\Delta E_K = QV$,

and its final velocity is $\frac{1}{2}m\left(v_f^2 - v_i^2\right) = QV \Rightarrow v_f = \sqrt{\frac{2QV}{m}}$, if $v_i = 0$.

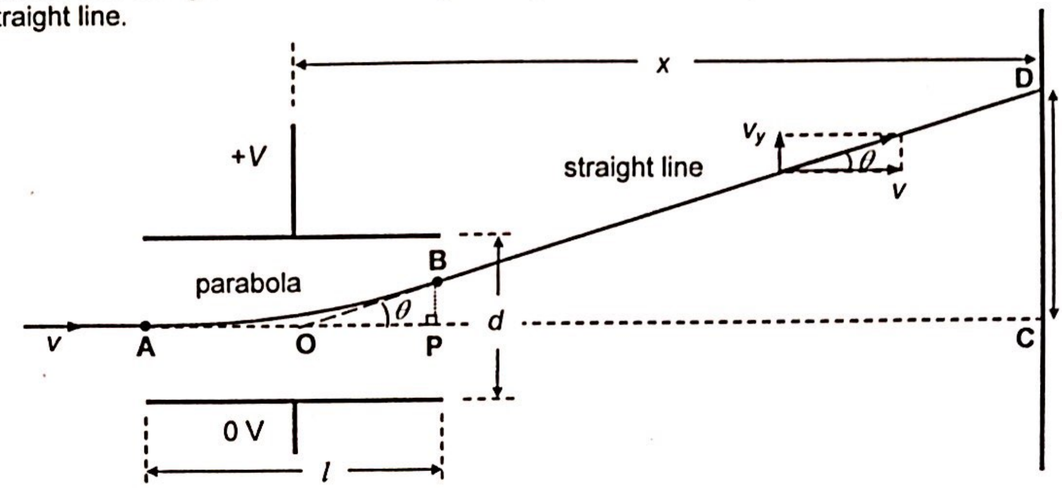
(C) Cathode Ray Oscilloscope (CRO)



The main features of a cathode ray sscilloscope

Deflection by parallel plates

Consider a beam of electrons travelling between two horizontal parallel plates as shown below. The electrons experience a constant upward acceleration between the plates, causing them to move in a parabolic path with the component of the velocity perpendicular to the electric field being constant. On exiting the region between the plates, the electrons travel in a straight line.



Deflection of an electron beam through parallel plates

Within the parallel plates, an electron will experience an upward electrostatic force. This is the net force acting on the electron (as the weight of the electron is negligible compared to the electrostatic force acting on it).

$$F_E = eE = m_e a \quad \therefore a = \frac{eE}{m_e}$$

Horizontally, there is no acceleration, hence $u_x = v_x = v$. Using $s_x = u_x t + \frac{1}{2}a_x t^2$ the time taken for the electrons to traverse the length of the plates is

$$l = vt + 0 \quad \Rightarrow \quad t = \frac{l}{v}$$

Considering its vertical motion, its vertical velocity v_y after emerging from the electric field is

$$v_y = u_y + a_y t = 0 + \left(\frac{eE}{m_e}\right)\left(\frac{l}{v}\right) = \frac{eEl}{m_e v}$$

The electron traverses a parabolic path AB. When it emerges from the electric field, it no longer experiences an electrostatic force and hence travels in a straight line at an angle θ to the horizontal given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{\left(eEl/m_e v\right)}{v} = \frac{eEl}{m_e v^2}$$

By using $s_y = u_y t + \frac{1}{2}a_y t^2$, the vertical displacement of the electron is

$$\text{BP} = 0 + \frac{1}{2}\left(\frac{eE}{m_e}\right)\left(\frac{l}{v}\right)^2 = \frac{eEl^2}{2m_e v^2}$$

Since $\tan\theta = \frac{\text{BP}}{\text{OP}} = \frac{eEl}{m_e v^2}$ and $\text{BP} = \frac{eEl^2}{2m_e v^2} \Rightarrow \text{OP} = \frac{eEl^2}{2m_e v^2} \div \frac{eEl}{m_e v^2} = \frac{l}{2}$

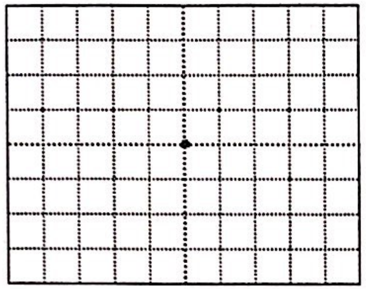
The deflected electron will eventually strike the screen at point D. ΔOBP and ΔODC are similar triangles, hence

$$\frac{y}{x} = \frac{\text{BP}}{\text{OP}} = \tan\theta \quad \Rightarrow \quad y = x\left(\frac{eEl}{m_e v^2}\right) = \frac{xe\left(V/d\right)l}{m_e v^2} = \left(\frac{xel}{m_e v^2 d}\right)V$$

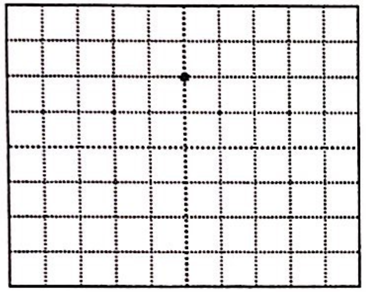
Hence, deflection y of the electrons is proportional to the p.d. V between the parallel plates.

How to read a CRO *(with the time-base off)*

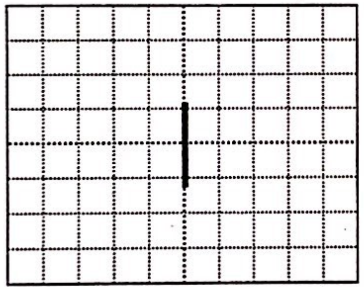
When the CRO is switched on, a spot of light appears at the centre of the screen if no voltage is connected to the *y*-input and the time base is switched off.



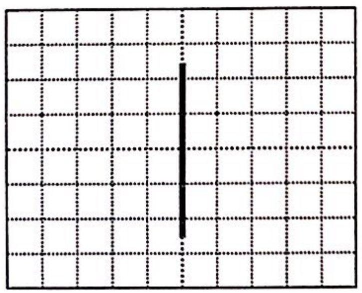
When a d.c. voltage is connected to the *y*-input, the spot of light is deflected as shown.
The CRO commonly found in the laboratory are already calibrated.
y-sensitivity = 2.0 V div^{-1}
$V_{d.c.}$ = 4.0 V



The voltage of an a.c. supply varies from $+V_0$ to $-V_0$ where V_0 is the amplitude.
When applied to the y-input, the spot oscillates with frequency equals to the frequency of a.c. supply. A vertical line is seen on the screen.
y-sensitivity = 2.0 V div^{-1}
V_0 = 2.5 V

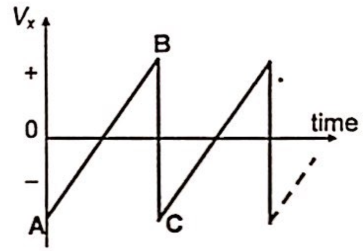


The screen on the right is obtained when the *y*-sensitivity is adjusted.
y-sensitivity = 1.0 V div^{-1}
V_0 = 2.5 V

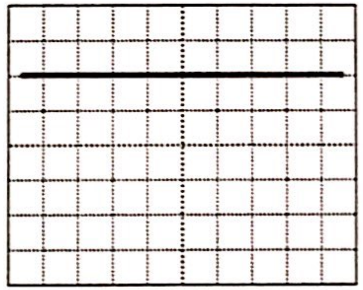


How to read a CRO *(with the time-base on)*

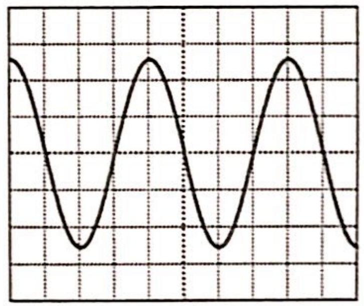
When the time-base is switched on, an internally generated saw-tooth voltage shown on the right is applied to the *x*-plates of the CRO.
As voltage V_x varies linearly with time from A to B, the spot moves at a constant speed from left to right of the screen. When the voltage changes suddenly from B to C, the spot returns to the left almost instantaneously.



When a d.c. voltage is connected to the *y*-input and the time base is switched on, the spot of light will be deflected from left to right.



When an a.c. voltage is applied to the *y*-plates with the time-base switched on, the spot moves up and down as well as across the screen.
time-base = 1 ms div^{-1}
period = 4 ms
frequency = 250 Hz



The same a.c. voltage is applied to the *y*-input but the time base is adjusted.
time-base = 2 ms div^{-1}
period = 4 ms