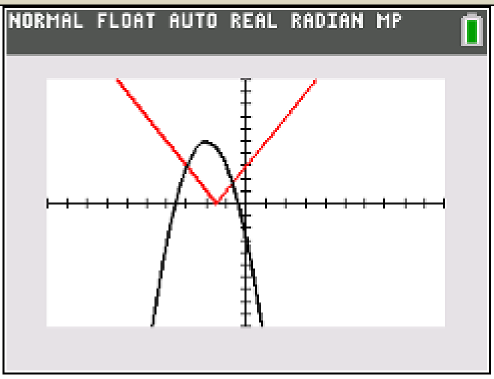
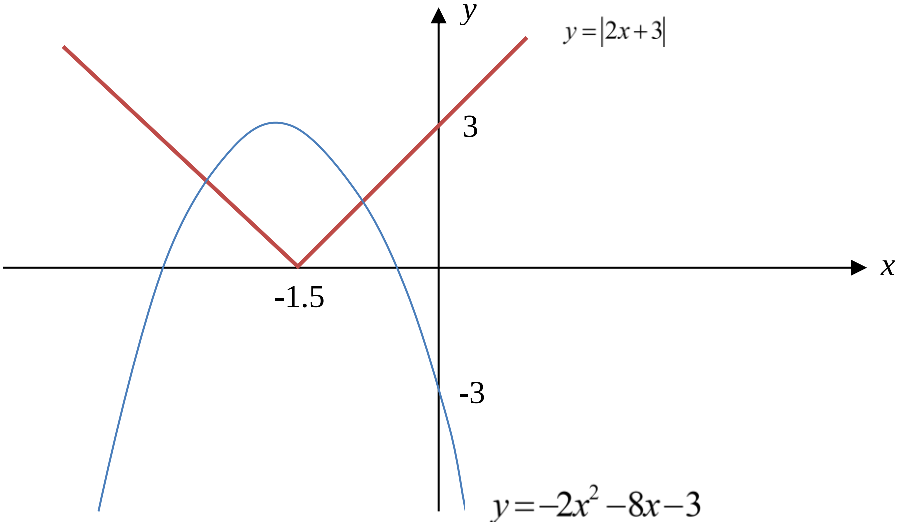
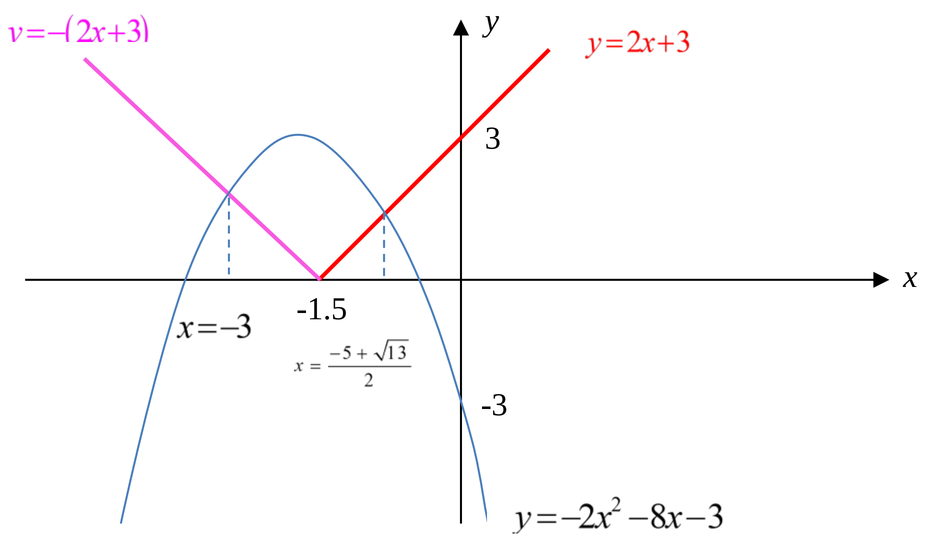
Q1 Solution	
(i)	 

(ii)

<u>Method ①:</u> Hence (Graphical Method)



For exact points of intersection, let

$2x+3=-2x^2-8x-3$ $2x^2+10x+6=0$ $x^2+5x+3=0$ $x=\frac{-5\pm\sqrt{5^2-4(1)(3)}}{2(1)}$ $x=\frac{-5+\sqrt{13}}{2}$ or $x=\frac{-5-\sqrt{13}}{2}$ $\therefore x=\frac{-5+\sqrt{13}}{2}$ (reject $x=\frac{-5-\sqrt{13}}{2}$ since from graph, $x>-1.5$)	$-(2x+3)=-2x^2-8x-3$ $-2x-3=-2x^2-8x-3$ $2x^2+6x=0$ $x(x+3)=0$ $x=0$ or $x=-3$ $\therefore x=-3$ (reject $x=0$, since from graph, $x<-1.5$)

Therefore, for $|2x+3|>-2x^2-8x-3$,

$x>\frac{-5+\sqrt{13}}{2}$ or $x<-3$

<u>Method ②:</u> Otherwise (Definition of Modulus)

$|2x+3|>-2x^2-8x-3$

$\Rightarrow 2x+3>-2x^2-8x-3$ (for $x>-1.5$) OR $2x+3<-(-2x^2-8x-3)$ (for $x<-1.5$)

$\Rightarrow 2x+3+2x^2+8x+3>0$

$\Rightarrow 2x^2+10x+6>0$

$\Rightarrow x^2+5x+3>0$

$2x+3<2x^2+8x+3$

$0<2x^2+8x+3-2x-3$

$0<2x^2+6x$

$0<x^2+3x$

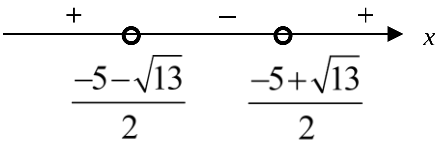
$0<x(x+3)$

Let $x^2 + 5x + 3 = 0$,

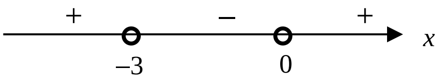
$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

Critical Points: $x = \frac{-5+\sqrt{13}}{2}$, $x = \frac{-5-\sqrt{13}}{2}$

Test Point Method:



$\therefore x < \frac{-5-\sqrt{13}}{2}$ (rejected) or $x > \frac{-5+\sqrt{13}}{2}$



$\therefore x < -3$ or $x > 0$ (rejected)

Let $x(x+3) = 0$,

Critical Points: $x = 0, x = -3$

Q2 Solution

(i) Let a and d be the first term and common difference of the arithmetic series respectively.
Let b and r be the first term and common ratio of the geometric series respectively.

A.P.	G.P.
$T_2 = a+d$ $T_5 = a+4d$ $T_{10} = a+9d$	$u_2 = b$ $u_3 = br$ $u_4 = br^2$

<u>Method ①:</u>

$$\frac{a+9d}{a+4d} = \frac{a+4d}{a+d}$$

$$(a+9d)(a+d) = (a+4d)^2$$

$$a^2 + 10ad + 9d^2 = a^2 + 8ad + 16d^2$$

$$2ad - 7d^2 = 0$$

$$d(2a-7d) = 0$$

$$d = 0 \quad \text{or} \quad a = 3.5d$$

$$(\text{rejected } \because d \neq 0)$$

$$r = \frac{3.5d+4d}{3.5d+d}$$

$$r = \frac{5}{3}$$

<u>Method ②:</u>

$$br - b = a+4d-(a+d)$$

$$b(r-1) = 3d \quad \text{------(1)}$$

$$br^2 - br = a+9d-(a+4d)$$

$$br(r-1) = 5d \quad \text{------(2)}$$

$$\frac{(2)}{(1)}: \frac{br(r-1)}{b(r-1)} = \frac{5d}{3d} \quad (\text{since } r \neq 1)$$

$$r = \frac{5}{3}$$

Method ③:

$$br - b = a + 4d - (a + d)$$
$$br - b = 3d$$
$$d = \frac{br - b}{3} \text{ -------(1)}$$

$$br^2 - br = a + 9d - (a + 4d)$$
$$br^2 - br = 5d \text{ ------(2)}$$

Substitute (1) into (2):

$$br^2 - br = \frac{5}{3}(br - b)$$
$$3r^2 - 3r = 5r - 5 \qquad \text{(since } b \neq 0\text{)}$$
$$3r^2 - 8r + 5 = 0$$
$$(3r - 5)(r - 1) = 0$$
$$r = 1 \quad \text{or} \quad r = \frac{5}{3}$$

$(\text{rejected } \because r \neq 1)$

(ii)

Using $a = 3.5d \Rightarrow d = \frac{7}{3.5} = 2$

Using $b = a + d = 7 + 2 = 9$

$$br^{n-1} - [a + (n-1)d] \geq 1000$$
$$9r^{n-1} - [7 + (n-1)(2)] \geq 1000$$
$$9\left(\frac{5}{3}\right)^{n-1} - [7 + 2(n-1)] \geq 1000$$
$$9\left(\frac{5}{3}\right)^{n-1} - 2n - 5 \geq 1000$$

n	$9\left(\frac{5}{3}\right)^{n-1} - 2n - 5$	
10	868.06	< 1000
11	1461.4	> 1000
12	2451.7	> 1000

Least n = 11

Q3 Solution	
(i)	**<u>Method ①:</u>** Since C has a vertical asymptote $x=2$, $2b-2=0$ $b=1$ **<u>Method ②:</u>** To find equation of vertical asymptote, equate the denominator to 0, $bx-2=0 \Rightarrow x=\frac{2}{b}$ $\frac{2}{b}=2 \Rightarrow b=1$ Substituting $(3,13)$ into $y=\frac{ax+1}{x-2}$, $13=\frac{3a+1}{3-2}$ $13=3a+1$ $a=4$
(ii)	By long division, $y=\frac{4x+1}{x-2}=4+\frac{9}{x-2}$ **<u>Method ①:</u>** $y=\frac{1}{x}$ ↓ [replace x with $x-2$] translate 2 units in the positive x-direction $y=\frac{1}{x-2}$ ↓ [replace y with $\frac{y}{9}$] scale parallel to y-axis by a scale factor of 9 $\frac{y}{9}=\frac{1}{x-2}$ $y=\frac{9}{x-2}$ ↓ [replace y with $y-4$] translate 4 units in the positive y-direction $y-4=\frac{9}{x-2}$ $y=4+\frac{9}{x-2}$

Method ②:

$y=\frac{1}{x}$

$\downarrow$ [replace x with $x-2$]

translate 2 units in the positive x-direction

$y=\frac{1}{x-2}$

$\downarrow$ [replace y with $y-\frac{4}{9}$]

translate $\frac{4}{9}$ units in the positive y-direction

$y=\frac{4}{9}+\frac{1}{x-2}$

$\downarrow$ [replace y with $\frac{y}{9}$]

scale parallel to y-axis by a scale factor of 9

$\frac{y}{9}=\frac{4}{9}+\frac{1}{x-2}$

$y=4+\frac{9}{x-2}$

Method ③:

$y=\frac{1}{x}$

$\downarrow$ [replace x with $\frac{x}{9}$]

scale parallel to x-axis by a scale factor of 9

$y=\frac{1}{\frac{x}{9}}$

$y=\frac{9}{x}$

$\downarrow$ [replace x with $x-2$]

translate 2 units in the positive x-direction

$y=\frac{9}{x-2}$

$\downarrow$ [replace y with $y-4$]

translate 4 units in the positive y-direction

$y-4=\frac{9}{x-2}$

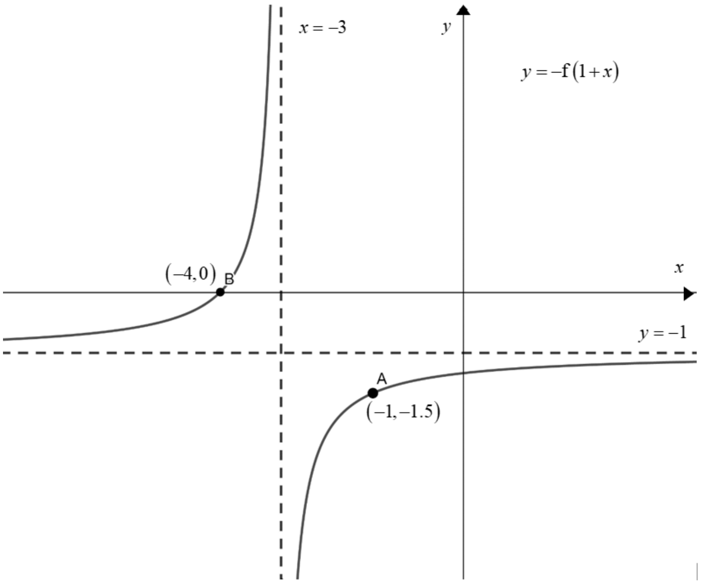
$y=4+\frac{9}{x-2}$

Q4 Solution	
(i)	$u_1 = 6: \quad \frac{1}{4}p + q + r = 6$ -----(1) $u_2 = 0: \quad \frac{1}{16}p + 2q + r = 0$ -----(2) $u_3 = -\frac{15}{4}: \quad \frac{1}{64}p + 3q + r = -\frac{15}{4}$ -----(3) Solving (1), (2) and (3), $p = 16, q = -3, r = 5$ $\therefore u_n = 16\left(\frac{1}{4^n}\right) - 3n + 5$
(ii)	**<u>Method ①:</u>** $\sum_{r=1}^{n}\left[16\left(\frac{1}{4^r}\right) - 3r + 5\right]$ $= 16\sum_{r=1}^{n}4^{-r} - 3\sum_{r=1}^{n}r + \sum_{r=1}^{n}5$ $= 16\left(\underbrace{4^{-1} + 4^{-1} + \ldots + 4^{-(n-1)} + 4^{-n}}_{\text{G.P.: } a=4^{-1}, r=4^{-1}, \text{number of terms}=n}\right) - 3\left(\underbrace{1 + 2 + \ldots + (n-1) + n}_{\text{A.P.: } a=1, d=1, \text{number of terms}=n}\right) + \left(\underbrace{5 + 5 + \ldots + 5 + 5}_{n \text{ times}}\right)$ $= 16 \times \frac{4^{-1}\left[1 - \left(4^{-1}\right)^n\right]}{1 - 4^{-1}} - 3 \times \frac{n}{2}(1+n) + 5n$ OR $16 \times \frac{4^{-1}\left[1 - \left(4^{-1}\right)^n\right]}{1 - 4^{-1}} - 3 \times \frac{n}{2}\left[2(1) + (n-1)(1)\right] + 5n$ $= \frac{16}{3}\left(1 - 4^{-n}\right) - \frac{3}{2}n(1+n) + 5n$ $= \frac{16}{3} - \frac{16}{3}\left(4^{-n}\right) + \frac{7}{2}n - \frac{3}{2}n^2$ where $A = \frac{16}{3}, B = -\frac{16}{3}, C = \frac{7}{2}$ and $D = -\frac{3}{2}$. **<u>Method ②:</u>** $\sum_{r=1}^{n}\left[16\left(\frac{1}{4^r}\right) - 3r + 5\right]$ $= 16\sum_{r=1}^{n}4^{-r} + \sum_{r=1}^{n}(-3r + 5)$ $= 16\left(\underbrace{4^{-1} + 4^{-2} + \ldots + 4^{-(n-1)} + 4^{-n}}_{\text{G.P.: } a=4^{-1}, r=4^{-1}, \text{number of terms}=n}\right) + \left(\underbrace{2 + (-1) + \ldots + \left[-3(n-1) + 5\right] + (-3n + 5)}_{\text{A.P.: } a=2, d=-3, \text{number of terms}=n}\right)$ $= 16 \times \frac{4^{-1}\left[1 - \left(4^{-1}\right)^n\right]}{1 - 4^{-1}} + \frac{n}{2}\left[2 + (-3n + 5)\right]$ OR $16 \times \frac{4^{-1}\left[1 - \left(4^{-1}\right)^n\right]}{1 - 4^{-1}} + \frac{n}{2}\left[2(2) + (n-1)(-3)\right]$ $= \frac{16}{3}\left(1 - 4^{-n}\right) - \frac{3}{2}n^2 + \frac{7}{2}n$ $= \frac{16}{3} - \frac{16}{3}\left(4^{-n}\right) + \frac{7}{2}n - \frac{3}{2}n^2$ where $A = \frac{16}{3}, B = -\frac{16}{3}, C = \frac{7}{2}$ and $D = -\frac{3}{2}$.

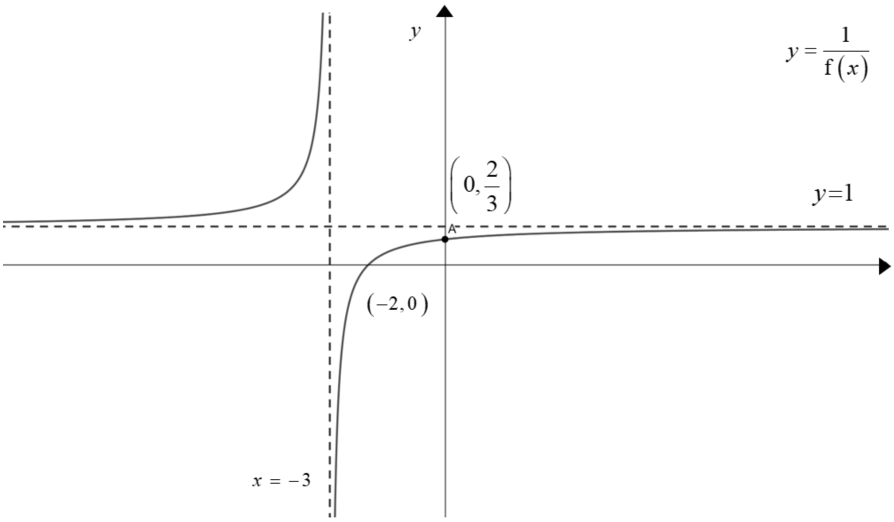
Q5 Solution	
(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{2+3x}\right)=\frac{\frac{1}{x}(2+3x)-3(\ln x)}{(2+3x)^2}=\frac{2+3x-3x\ln x}{x(2+3x)^2}$
(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}(x^3+2x)\right)=\frac{3x^2+2}{\sqrt{1-(x^3+2x)^2}}$
(b)	Differentiating $2xy-y^2=(1+y)^2$ implicitly with respect to *x*, $2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}=2(1+y)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ $(2+2y)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+2y\frac{\mathrm{d}y}{\mathrm{d}x}-2x\frac{\mathrm{d}y}{\mathrm{d}x}=2y$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2y}{4y-2x+2}=\frac{y}{2y-x+1}$ Where tangent is parallel to the *x*-axis, $\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow y=0$ When $y=0$, $\text{L.H.S.}=2x(0)-(0)^2=0$ $\text{R.H.S.}=(1+0)^2=1$ Hence there are no points on the curve where the tangent is parallel to the *x*-axis.

Q6 Solution

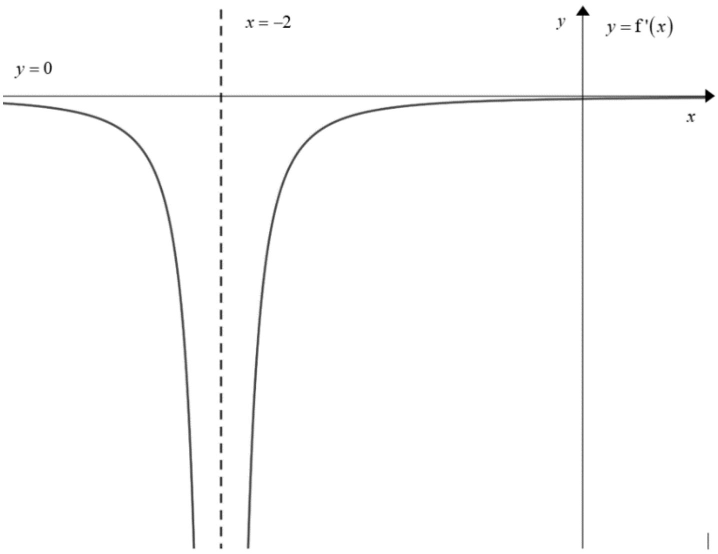
(i)

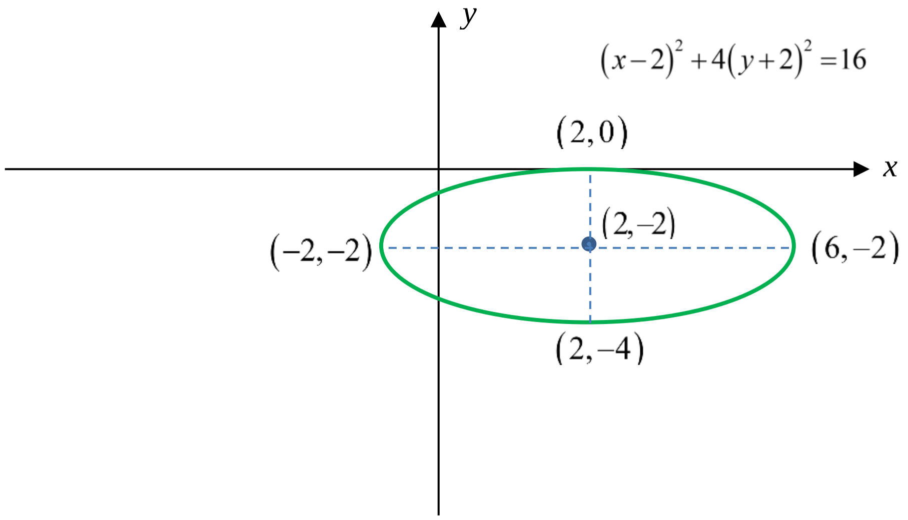


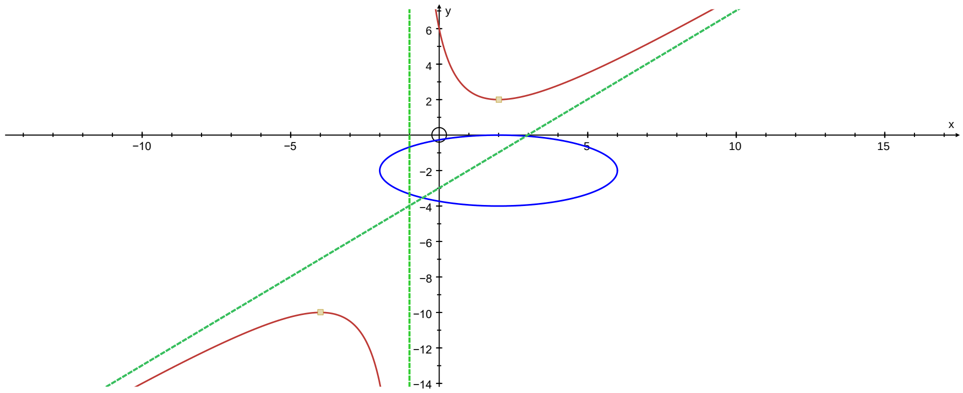
(ii)



(iii)

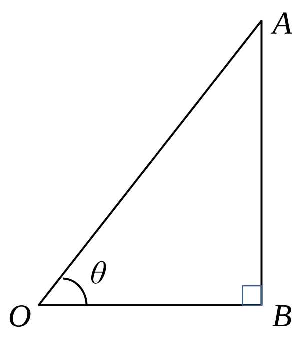
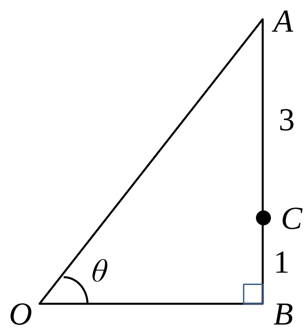


Q7 Solution	
(i)	$(x-2)^2+4(y+2)^2=16$ $\frac{(x-2)^2}{16}+\frac{4(y+2)^2}{16}=1$ $\frac{(x-2)^2}{4^2}+\frac{(y+2)^2}{2^2}=1$ Graph is an ellipse, centre at $(2,-2)$ <u>Axial Intercepts:</u> *x*-intercept: $(2,0)$ *y*-intercept: $(0,-0.268)$ and $(0,-3.73)$ <u>Turning Points:</u> Turning points: $(2,0)$ and $(2,-4)$ <u>Vertices:</u> 4 vertices: $(-2,-2)$, $(2,0)$,$(6,-2)$ and $(2,-4)$ 
(ii)	$y=\frac{x^2-2x+6}{x+1}$ $=(x-3)+\frac{9}{x+1}$ (By long division or other algebraic techniques) <u>Asymptotes:</u> Vertical asymptote: $x=-1$ Oblique asymptote: $y=(x-3)$ <u>Axial Intercepts:</u> *y*-intercept: $(0,6)$ <u>Turning Points:</u> Max point: $(-4,-10)$ Min point: $(2,2)$

	
(iii)	$(x-2)^2+4m^2\left(\frac{x^2-2x+6}{x+1}+2\right)^2=16$ Graphically, $(x-2)^2+4m^2\left(\frac{x^2-2x+6}{x+1}+2\right)^2=16$ represents the intersecting points between the 2 graphs $(x-2)^2+4m^2(y+2)^2=16$ and $y=\frac{x^2-2x+6}{x+1}$ $(x-2)^2+4m^2(y+2)^2=16$ $\frac{(x-2)^2}{4^2}+\frac{m^2(y+2)^2}{2^2}=1$ $\frac{(x-2)^2}{4^2}+\frac{(y+2)^2}{\left(2/m\right)^2}=1$ Since $m>0$, when $\frac{2}{m}=4$, the ellipse will touch the curve at the minimum point of $y=\frac{x^2-2x+6}{x+1}$ i.e. when $m=\frac{1}{2}$, there is one point of intersection i.e. one solution). Hence, for no solution (i.e. no points of intersection), $\frac{2}{m}<4$, i.e $m>\frac{1}{2}$.

Q8 Solution	
(i)	$\frac{1}{r!}-\frac{1}{(r+1)!}$ $=\frac{r+1-1}{(r+1)!}$ $=\frac{r}{(r+1)!}$
(ii)	$\sum_{r=1}^{N}\frac{r}{(r+1)!}$ $=\sum_{r=1}^{N}\left[\frac{1}{r!}-\frac{1}{(r+1)!}\right]$ $=\frac{1}{1!}-\frac{1}{2!}$ $+\frac{1}{2!}-\frac{1}{3!}$ $+\frac{1}{3!}-\frac{1}{4!}$ $+\ldots$ $+\frac{1}{(N-2)!}-\frac{1}{(N-1)!}$ $+\frac{1}{(N-1)!}-\frac{1}{N!}$ $+\frac{1}{N!}-\frac{1}{(N+1)!}$ $=1-\frac{1}{(N+1)!}$
(iii)	As $N\to\infty$, $\frac{1}{(N+1)!}\to 0$, $\sum_{r=1}^{N}\frac{r}{(r+1)!}\to 1$ Therefore the series is convergent. $\sum_{r=1}^{\infty}\frac{r}{(r+1)!}=1$

(iv)	$\sum_{r=2}^{r=N} \frac{r+2}{(r+3)!}$ $= \sum_{s-2=2}^{s-2=N} \frac{s}{(s+1)!}$ $\quad$ $(\text{Let } s = r+2 \text{ i.e. } r = s-2)$ $= \sum_{s=4}^{N+2} \frac{s}{(s+1)!}$ $= \sum_{s=1}^{N+2} \frac{s}{(s+1)!} - \sum_{s=1}^{3} \frac{s}{(s+1)!}$ $= \left(1 - \frac{1}{(N+3)!}\right) - \left(1 - \frac{1}{4!}\right)$ $= \frac{1}{24} - \frac{1}{(N+3)!}$

Q9 Solution	
(i)	**Method ①:** Let θ be the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Since $\underset{\sim}{b}$ and $\underset{\sim}{b}-\underset{\sim}{a}$ are perpendicular, $\underset{\sim}{b}\cdot\left(\underset{\sim}{b}-\underset{\sim}{a}\right)=0$ $\underset{\sim}{b}\cdot\underset{\sim}{b}-\underset{\sim}{b}\cdot\underset{\sim}{a}=0$ $\left\lvert\underset{\sim}{b}\right\rvert^2=\underset{\sim}{b}\cdot\underset{\sim}{a}=\underset{\sim}{a}\cdot\underset{\sim}{b}$ Since $\underset{\sim}{b}$ is a unit vector, $\underset{\sim}{a}\cdot\underset{\sim}{b}=1$ $\left\lvert\underset{\sim}{a}\right\rvert\left\lvert\underset{\sim}{b}\right\rvert\cos\theta=1$ $\left\lvert\underset{\sim}{a}\right\rvert\cos\theta=1$ $\left\lvert\underset{\sim}{a}\right\rvert=\frac{1}{\cos\theta}=\sec\theta$ **Method ②:** $\cos\theta=\frac{OB}{OA}$ $=\frac{\lvert\underset{\sim}{b}\rvert}{\lvert\underset{\sim}{a}\rvert}$ $=\frac{1}{\lvert\underset{\sim}{a}\rvert}$ since $\underset{\sim}{b}$ is a unit vector $\lvert\underset{\sim}{a}\rvert=\frac{1}{\cos\theta}=\sec\theta$ (shown) 
(ii)	By Ratio Theorem, $\underset{\sim}{c}=\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}$ $\left\lvert\underset{\sim}{c}\cdot\underset{\sim}{b}\right\rvert=\left\lvert\left(\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}\right)\cdot\underset{\sim}{b}\right\rvert$ $=\left\lvert\frac{1}{4}\underset{\sim}{a}\cdot\underset{\sim}{b}+\frac{3}{4}\underset{\sim}{b}\cdot\underset{\sim}{b}\right\rvert$ $=\left\lvert\frac{1}{4}\underset{\sim}{b}\cdot\underset{\sim}{b}+\frac{3}{4}\underset{\sim}{b}\cdot\underset{\sim}{b}\right\rvert$ since $\underset{\sim}{a}\cdot\underset{\sim}{b}=\underset{\sim}{b}\cdot\underset{\sim}{b}$ $=\left\lvert\underset{\sim}{b}\right\rvert^2$ $=1$ $\left\lvert\underset{\sim}{c}\cdot\underset{\sim}{b}\right\rvert$ is the length of projection of $\underset{\sim}{c}$ on $\underset{\sim}{b}$. 

(iii)

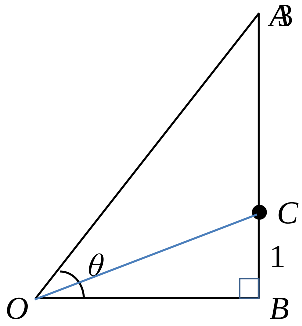
<u>Method ①:</u>

$$\frac{\left|\underset{\sim}{b}\times\underset{\sim}{c}\right|}{\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|}=\frac{\left|\underset{\sim}{b}\times\left(\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}\right)\right|}{\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|}$$

$$=\frac{\left|\frac{1}{4}\left(\underset{\sim}{b}\times\underset{\sim}{a}\right)\right|}{\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|},\ \text{where } \underset{\sim}{b}\times\underset{\sim}{b}=0$$

$$=\frac{1}{4}\frac{\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|}{\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|},\ \text{where } \left|\underset{\sim}{b}\times\underset{\sim}{a}\right|=\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|$$

$$=\frac{1}{4}$$



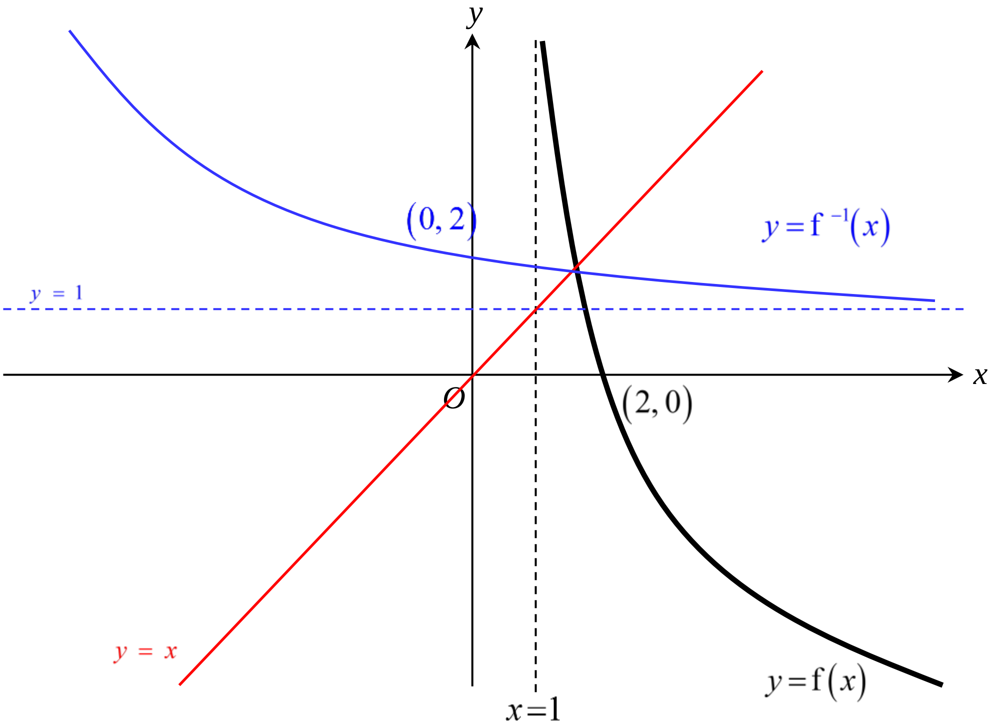
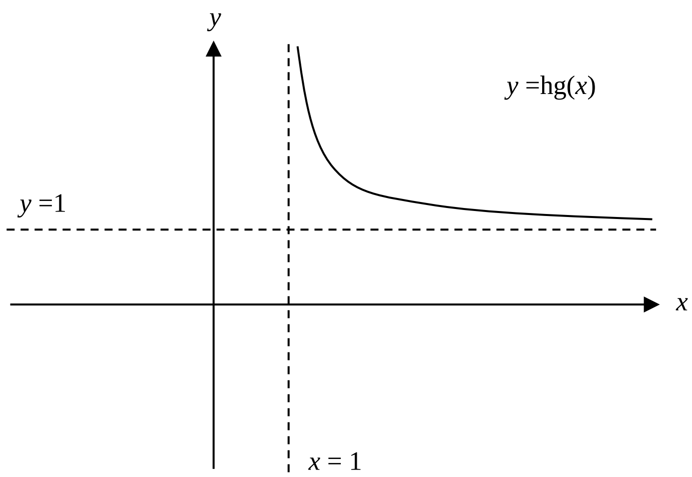
<u>Method ②:</u>

$$\frac{\left|\underset{\sim}{b}\times\underset{\sim}{c}\right|}{\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|}=\frac{\frac{1}{2}\left|\underset{\sim}{b}\times\underset{\sim}{c}\right|}{\frac{1}{2}\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|}$$

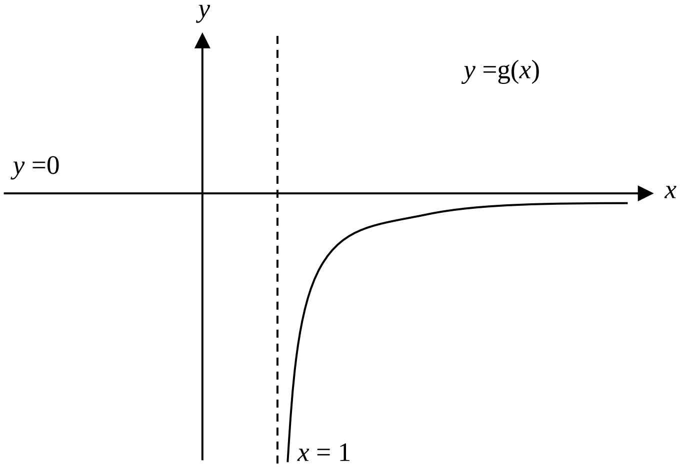
$$=\frac{\text{area of }\Delta OBC}{\text{area of }\Delta OAB}$$

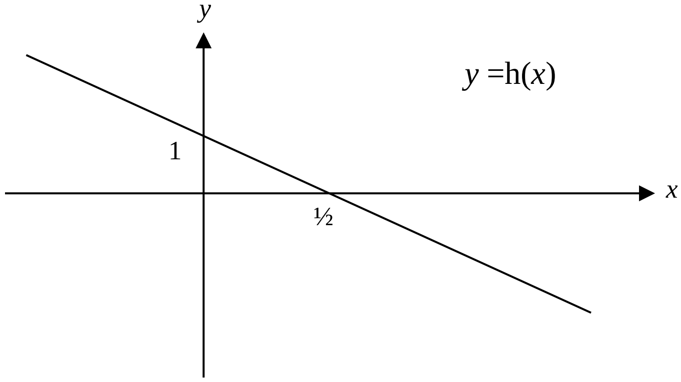
$$=\frac{\frac{1}{2}\times BC\times h}{\frac{1}{2}\times AB\times h}$$

$$=\frac{1}{4}$$

Q10 Solution	
(a)	
(b)(i)	For gh to exist, $R_h \subseteq D_g$. $R_h = (-\infty, \infty)$ $D_g = (1, \infty)$ Since $R_h \not\subseteq D_g$, gh does not exist.
(ii)	$hg(x) = h\left(\frac{1}{1-x^2}\right) = 1 - 2\left(\frac{1}{1-x^2}\right) = 1 - \frac{2}{1-x^2}$
(iii)	**<u>Method ①:</u> Composite Function** $D_{hg} = D_g = (1, \infty)$  $R_{hg} = (1, \infty)$

<u>Method ②:</u> Mapping Method





$$D_g = (1, \infty) \xrightarrow{g} R_g = (-\infty, 0) \xrightarrow{h} R_{hg} = (1, \infty)$$

(iv)	**Hence** Let $(\text{hg})^{-1}(4) = a$ $4 = \text{hg}(a)$ $4 = 1 - \frac{2}{1-a^2}$ $\frac{2}{1-a^2} = -3$ $2 = -3 + 3a^2$ $3a^2 = 5$ $a^2 = \frac{5}{3}$ $a = \sqrt{\frac{5}{3}}$ or $-\sqrt{\frac{5}{3}}$ Since $\text{D}_{\text{hg}} = \text{D}_{\text{g}} = (1, \infty)$, $a = \sqrt{\frac{5}{3}}$ **Otherwise** Let $y = 1 - \frac{2}{1-x^2}$ $\frac{2}{1-x^2} = 1 - y$ $1 - x^2 = \frac{2}{1-y}$ $x^2 = 1 - \frac{2}{1-y}$ $x = \sqrt{1 - \frac{2}{1-y}}$ or $-\sqrt{1 - \frac{2}{1-y}}$ Since $\text{D}_{(\text{hg})^{-1}} = \text{R}_{\text{hg}} = (1, \infty)$, $x = \sqrt{1 - \frac{2}{1-y}}$ $\therefore (\text{hg})^{-1}(x) = \sqrt{1 - \frac{2}{1-x}}$ $(\text{hg})^{-1}(4) = \sqrt{1 - \frac{2}{1-4}} = \sqrt{\frac{5}{3}}$

Q11 Solution	
(i)	$l_{AB}: r = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \lambda \in \mathbb{R}$ Since B lies on l_{AB}, $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, for some $\lambda \in \mathbb{R}$. $\begin{pmatrix} 2+\lambda \\ 2-\lambda \\ 3+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = -6$ Since B lies on p_1, $2+\lambda-2+\lambda+6+4\lambda = -6$ $\lambda = -2$ $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$ Therefore, coordinates of B is $(0,4,-1)$. Shortest distance $= \left\|\overrightarrow{AB}\right\|$ $= \left\| \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \right\|$ $= \left\| \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix} \right\|$ $= \sqrt{24}$ units
(ii)	Let the angle between the direction vector of path BC and the normal of p_1 be α. $\alpha = \cos^{-1} \dfrac{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{\sqrt{3}\sqrt{6}}$ $= \cos^{-1} \dfrac{2}{\sqrt{3}\sqrt{6}}$ $= 61.8744943°$ Angle that the path BC makes with $p_1 = 90° - 61.8744943°$ $= 28.1255057°$ $\approx 28.1°$ (to 1 d.p.)

(iii)	$l_{BC}: r=\begin{pmatrix}0\\4\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\1\\1\end{pmatrix}, \mu\in\mathbb{R}$ $\overrightarrow{OC}=\begin{pmatrix}0\\4\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\1\\1\end{pmatrix}$, for some $\mu\in\mathbb{R}$ $\overrightarrow{BC}=\mu\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\dfrac{\left\lvert\begin{pmatrix}\mu\\\mu\\\mu\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\2\end{pmatrix}\right\rvert}{\sqrt{6}}=2\sqrt{6}$ $\lvert 2\mu\rvert=12$ $\mu=6$ or $\mu=-6$ When $\mu=6$, $\overrightarrow{OC}=\begin{pmatrix}0\\4\\-1\end{pmatrix}+6\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}6\\10\\5\end{pmatrix}$. When $\mu=-6$, $\overrightarrow{OC}=\begin{pmatrix}0\\4\\-1\end{pmatrix}-6\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}-6\\-2\\-7\end{pmatrix}$. (Reject since coordinates are negative) $\therefore$ coordinates of C is $(6,10,5)$.
(iv)	**<u>Method ①:</u>** $\overrightarrow{BC}=\begin{pmatrix}6\\6\\6\end{pmatrix}$ $\overrightarrow{CD}=\begin{pmatrix}18\\22\\17\end{pmatrix}-\begin{pmatrix}6\\10\\5\end{pmatrix}=\begin{pmatrix}12\\12\\12\end{pmatrix}=2\overrightarrow{BC}$ Since $\overrightarrow{CD}=2\overrightarrow{BC}$, *B*, *C* and *D* are collinear. Hence, the destroyer do not need to change its path to reach the final target. **<u>Method ②:</u>** Assume *D* lies on line *BC*. $\begin{pmatrix}18\\22\\17\end{pmatrix}=\begin{pmatrix}0\\4\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\mu=18$ is consistent for all three equations. Hence *D* lies on line *BC*, and the destroyer do not need to change its path to reach the final target.

Q12 Solution

(a)(i)

By similar triangles,

$$\frac{20-y}{20}=\frac{x}{R}$$

$$20-y=\frac{20x}{R}$$

$$y=20-\frac{20x}{R}$$

$$V=\frac{1}{3}\pi x^2 y=\frac{1}{3}\pi x^2\left(20-\frac{20x}{R}\right)=\frac{20\pi}{3}x^2-\frac{20\pi}{3R}x^3$$

(ii)

$$\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{40\pi}{3}x-\frac{20\pi}{R}x^2$$

At stationary point, $\frac{\mathrm{d}V}{\mathrm{d}x}=0$

$$\frac{40\pi}{3}x-\frac{20\pi}{R}x^2=0$$

$$20\pi x\left(\frac{2}{3}-\frac{x}{R}\right)=0$$

$x=0$ (reject) or $x=\frac{2}{3}R$

x	$\frac{2}{3}R^-$	$\frac{2}{3}R$	$\frac{2}{3}R^+$
$\frac{\mathrm{d}V}{\mathrm{d}x}$	+	0	−
shape	/	—	\

Hence V is maximum when $x=\frac{2}{3}R$

OR

$$\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=\frac{40\pi}{3}-\frac{40\pi}{R}x$$

$$=\frac{40\pi}{3}-\frac{40\pi}{R}\left(\frac{2R}{3}\right)\quad \text{when } x=\frac{2}{3}R$$

$$=\frac{40\pi}{3}-\frac{80\pi}{3}$$

$$=-\frac{40\pi}{3}<0$$

Hence V is maximum when $x=\frac{2}{3}R$

	$\therefore$ largest possible volume $=\frac{20\pi}{3}\left(\frac{2}{3}R\right)^2-\frac{20\pi}{3R}\left(\frac{2}{3}R\right)^3$ $=\frac{80\pi R^2}{27}-\frac{160\pi R^2}{81}$ $=\frac{80\pi R^2}{81}$ unit^3 or $3.10R^2$
(b)(i)	Since $\tan 30^0=\frac{r}{h} \Rightarrow r=\frac{h}{\sqrt{3}}$ OR Since $\tan 60^0=\frac{h}{r} \Rightarrow r=\frac{h}{\sqrt{3}}$ OR By sine rule, $\frac{r}{\sin 30^0}=\frac{h}{\sin 60^0} \Rightarrow r=\frac{h}{\sqrt{3}}$ OR By Pythagoras Theorem, $\sqrt{r^2+h^2}=2r \Rightarrow r=\frac{h}{\sqrt{3}}$
(ii)	Let E be the volume the charged energy occupies $E=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2h=\frac{1}{9}\pi h^3$ $\frac{\mathrm{d}E}{\mathrm{d}h}=\frac{1}{3}\pi h^2$ $\frac{\mathrm{d}E}{\mathrm{d}t}=\frac{\mathrm{d}E}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $8=\frac{1}{3}\pi(5)^2\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $\frac{\mathrm{d}h}{\mathrm{d}t}=0.306$ unit/s or $\frac{24}{25\pi}$ unit/s