NAVAL BASE SECONDARY SCHOOL
PRELIMINARY EXAMINATION, 2021

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Name ______________________________ **( )** **Class** ______________________

ADDITIONAL MATHEMATICS
Paper 1

4049/01

26 August 2021

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Item	***For examiner's use***
Presentation	
Accuracy	
Units	
Total	
Parent's Signature	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

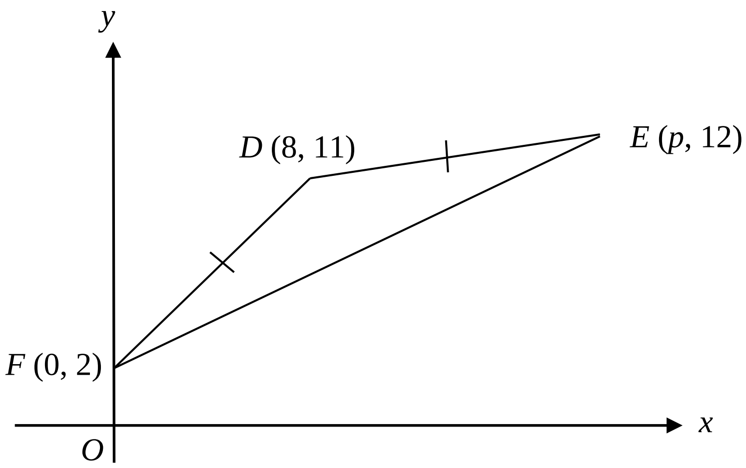
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

Answer **all** the questions.

1 The diagram shows an isosceles triangle DEF with vertices at D (8, 11), E (p, 12) and F (0, 2).



(i) Find the value of p. [3]

(ii) Find the equation of the perpendicular bisector of EF. [2]

2 **(i)** On the same axes, sketch the graphs of $y = 3\cos 2x$ and $y = 1 - \sin\frac{1}{2}x$ for $0 \le x \le \pi$. [4]

(ii) Hence, state the number of solutions for the equation $3\cos 2x + \sin\frac{1}{2}x = 1$ for $\boldsymbol{-2\pi \le x \le 2\pi}$. [1]

3 The coefficient of x in the expansion of $(1-ax)^4$ is -12.

(i) Show thar $a = 3$. [3]

(ii) Hence, find the coefficient of x^2 in the expansion of $(2x+1)(1-ax)^4$. [3]

4 **(i)** The function f is defined by $\mathrm{f}(x) = \dfrac{x^2 - 3}{x^2 + 5}$, $x > 0$.

Explain, with working, whether f is an increasing or a decreasing function. [4]

(ii) A point K moves along the curve $y = \dfrac{x^2 - 3}{x^2 + 5}$ in such a way that the y-coordinate of K is increasing at a rate of of 0.2 units per second.

Find the rate of increase of the x-coordinate of K when $x = 1$. [2]

5 The population of Singapore increased from 3.75 million at the beginning of 2000 to 5.79 million at the beginning of 2020.

Suppose that the population, P million, is modelled by the formula, $P = 3.75\mathrm{e}^{kt}$, where k is a constant and t is the number of years starting from the beginning of 2000.

(i) Find the estimated population of Singapore at the beginning of 2030, giving your answer correct to 3 significant figures. [4]

(ii) Find the year in which the population of Singapore will exceed 10 million. [2]

6 At any point (x, y) on a curve, $\dfrac{d^2y}{dx^2} = \dfrac{18}{(x-2)^3}$.

The gradient of the curve at (5, 20) is 2.

(i) Find the equation of the curve. [4]

(ii) Find the equation of the normal to the curve at (5, 20). [2]

7 **(i)** Express $3x^2 - 6x + 5$ and $-x^2 - 4x - 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [4]

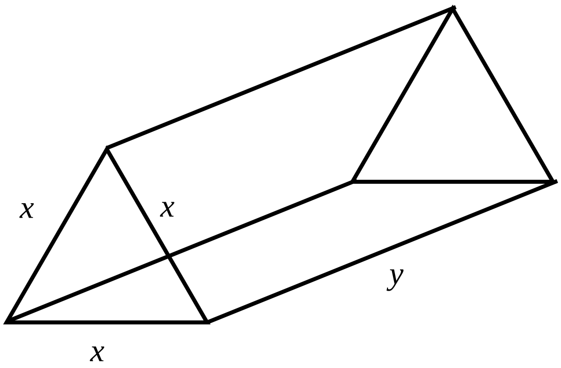
(ii) Using your answers from part **(i)**, explain why the two curves $y = 3x^2 - 6x + 5$ and $y = -x^2 - 4x - 3$ will not intersect. [3]

8 **Without using a calculator,**

(i) show that $\sin 75° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [2]

(ii) Hence, express $\operatorname{cosec}^2 75°$ in the form $a + b\sqrt{3}$, where a and b are integers. [5]

9 A piece of wire, 36 cm in length, is used to form the outline of a triangular prism. The cross-section of the prism is an equilateral triangle of side x cm and the length of the prism is y cm.



(i) Express y in terms of x. [2]

(ii) Show that the volume, V cm^3, of the prism is given by $V = \frac{\sqrt{3}}{2}\left(6x^2 - x^3\right)$. [2]

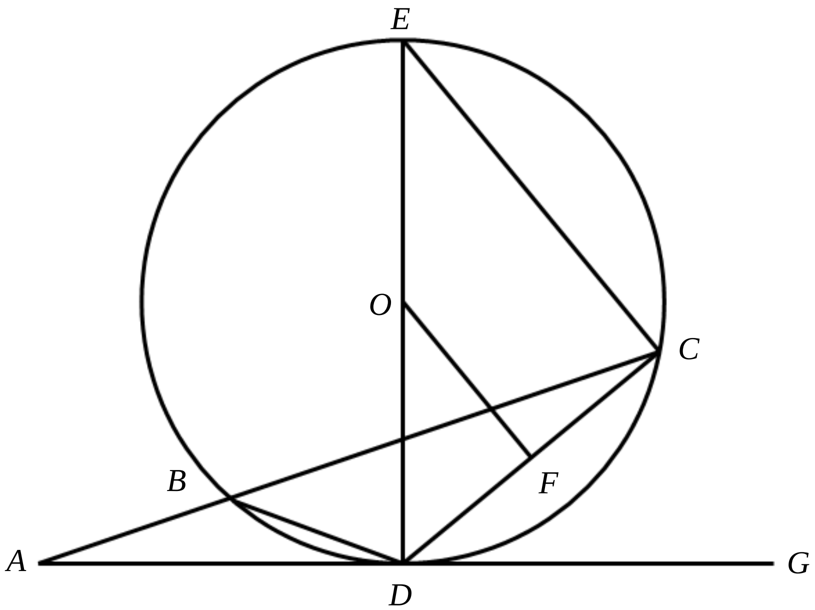
(iii) Find the stationary value of V and determine if it is a maximum or a minimum. [4]

10 The function $\mathrm{f}(x) = 2x^3 - 10x^2 + ax + b$, where a and b are constants, leaves a remainder of 3 and –3 when divided by $x+1$ and $x-2$ respectively.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in **(i)**, solve the equation $\mathbf{f(x) - 3 = 0}$, expressing non-integer roots in the form $c \pm \sqrt{d}$, where c and d are integers. [4]

11 In the diagram below, the line *ADG* is a tangent to the circle at *D*. *DE* is a diameter and *O* is the centre. *ABC* is a straight line intersecting the circle at points *B* and *C*.



(i) Prove that triangle ADB is similar to triangle ACD. [2]

(ii) If $AD = 2AB$, show that $AC = 4AB$. [3]

(iii) Given $DF = FC$, show that angle CDG = angle DOF. [3]

12 **(i)** Prove that $\sec x - \dfrac{\cos x}{1+\sin x} = \tan x$. [4]

(ii) Find, in radians, the **exact** value of the acute angle for which

$$\sec x - \frac{\cos x}{1+\sin x} = \frac{\cot x}{3}.$$ [2]

(iii) Explain why there is no solution between the curves $y = \sec x - \dfrac{\cos x}{1+\sin x}$ and $y = -\dfrac{\cot x}{3}$. [2]

13 A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by the expression $v = 2t^2 - 5t + 2$.

Find

(i) the time/s when the particle is instantaneously at rest, [2]

(ii) the range of time when the particle is decelerating, [3]

(iii) an expression for the displacement, s metres, in terms of t, [2]

(iv) the total distance travelled in the first 3 seconds. [3]

----------END OF PAPER----------