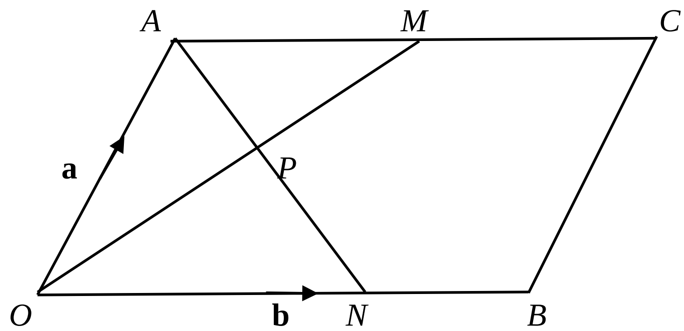
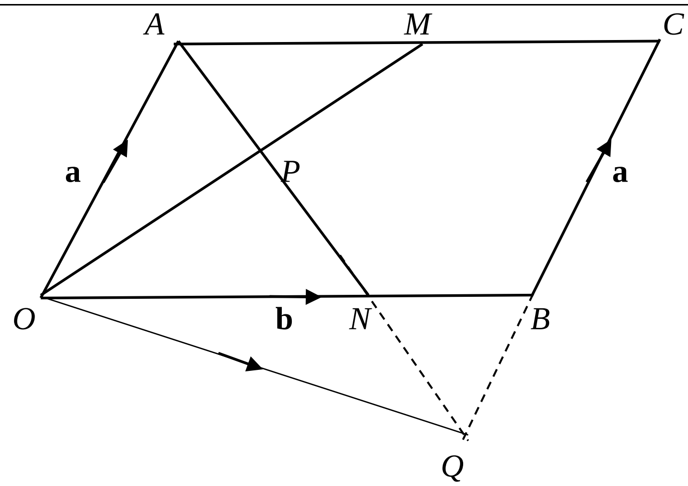
2021 JC 2 H2 Maths Common Test Markers Report

1	Solution [6] Abstract Vector	
(a)	 Let $\overrightarrow{OP}=\lambda\overrightarrow{OM}$, then $\overrightarrow{OP}=\lambda\left(\mathbf{a}+\frac{1}{2}\mathbf{b}\right)$. Let $\overrightarrow{AP}=\mu\overrightarrow{AN}$, then $\overrightarrow{AP}=\mu\left(\overrightarrow{OP}-\overrightarrow{OA}\right)=\mu\left(-\mathbf{a}+\frac{2}{3}\mathbf{b}\right)$. Next, we have $\overrightarrow{OA}+\overrightarrow{AP}=\overrightarrow{OP}$ $\Rightarrow \mathbf{a}+\mu\left(-\mathbf{a}+\frac{2}{3}\mathbf{b}\right)=\lambda\left(\mathbf{a}+\frac{1}{2}\mathbf{b}\right)$ Since **a** and **b** are non-zero and non-parallel vectors, upon comparing **a** and **b** on both sides of equation, we have: $1-\mu=\lambda \text{ -----}(1)$ $\frac{2}{3}\mu=\frac{1}{2}\lambda \text{ -----}(2)$ From (2) we have $\lambda=\frac{4}{3}\mu$ and $1-\mu=\frac{4}{3}\mu\Rightarrow\frac{7}{3}\mu=1\Rightarrow\mu=\frac{3}{7}$ and so $\lambda=\frac{4}{7}$ Therefore, $\overrightarrow{OP}=\frac{2}{7}(2\mathbf{a}+\mathbf{b})$	Many students struggled to form a vector equation in terms of λ and μ. Of those who could, a number did not correctly use λ and μ as defined in the question (i.e. they defined their own λ and μ).

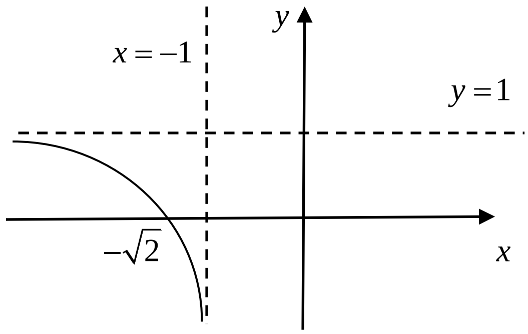
<table>
<tr>
<td>(b)</td>
<td>


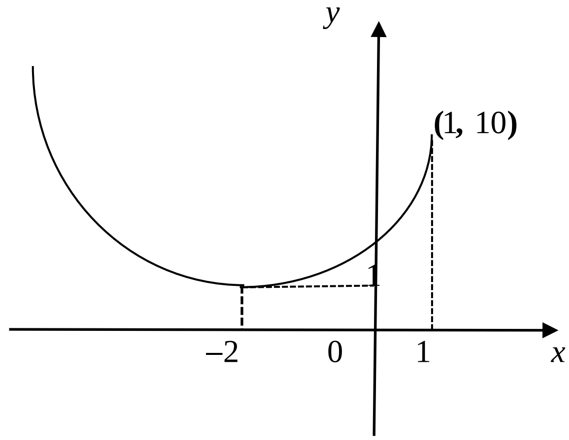
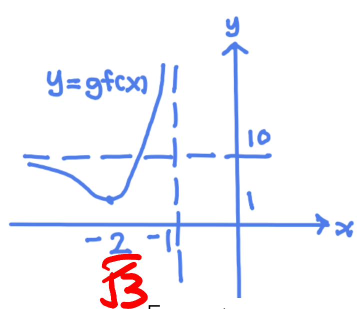
Q lies on AN produced such that $AN : NQ = 2 : 1$.
Applying Ratio Theorem,
$\overrightarrow{ON} = \frac{2}{3}\overrightarrow{OQ} + \frac{1}{3}\overrightarrow{OA}$
$\overrightarrow{OQ} = \frac{3}{2}\overrightarrow{ON} - \frac{1}{2}\overrightarrow{OA}$
$= \frac{3}{2} \times \frac{2}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$
$= \mathbf{b} - \frac{1}{2}\mathbf{a}$
Then we have
$\overrightarrow{QB} = \overrightarrow{OB} - \overrightarrow{OQ} = \mathbf{b} - \left(\mathbf{b} - \frac{1}{2}\mathbf{a}\right) = \frac{1}{2}\mathbf{a}$.
Since $\overrightarrow{BC}$ parallel $\overrightarrow{QB}$ and B is a common point, the points Q, B and C are colliear.
</td>
<td>Generally students had the right idea on how to attempt this question, but some struggled with algebraic errors.</td>
</tr>
</table>

2	Solution [6] Complex Numbers	
	$z+w^{*}=4$ $z=4-w*$ $z*=4-w \quad (1)$ Sub (1) into $2\mathrm{i}z^{*}-w=1$ $\Rightarrow 2\mathrm{i}(4-w)-w=1$ $8\mathrm{i}-2\mathrm{i}w-w=1$ $w+2\mathrm{i}w=-1+8\mathrm{i}$ $w(1+2\mathrm{i})=-1+8\mathrm{i}$ $w=\frac{-1+8\mathrm{i}}{1+2\mathrm{i}}\times\frac{1-2\mathrm{i}}{1-2\mathrm{i}}$ $=\frac{-1+16+8\mathrm{i}+2\mathrm{i}}{5}$ $=3+2\mathrm{i}$ Therefore $z=4-(3+2\mathrm{i})*$ $=1+2\mathrm{i}$	Students struggled to correctly manipulate the conjugate. Students generally preferred to allow $w = x + \mathrm{i}y$ rather than solving for w directly. Almost all forgot to indicate that x, y needed to be real numbers. Students should be encouraged to make greater use of the GC in their working or to check their answers.
	Alternative Method Let $z=a+b\mathrm{i}$ and $w=c+d\mathrm{i}$ where $a,b,c,d\in\mathbb{R}$. From $2\mathrm{i}z^{*}-w=1$, $2\mathrm{i}(a-b\mathrm{i})-(c+d\mathrm{i})=1$ $2a\mathrm{i}+2b-c-d\mathrm{i}=1$ $(2b-c)+(2a-d)\mathrm{i}=1$ Comparing real and imaginary parts, $2b-c=1 \quad (1)$ $2a-d=0 \quad (2)$ From $z+w^{*}=4$, $a+b\mathrm{i}+c-d\mathrm{i}=4$ $(a+c)+(b-d)\mathrm{i}=4$ Comparing real and imaginary parts, $a+c=4 \quad (3)$ $b-d=0 \quad (4)$ Solving all 4 equations using GC, we get $a=1,\ b=2,\ c=3,\ d=2$ Therefore, $z=1+2\mathrm{i},\ \mathrm{w}=3+2\mathrm{i}$.	This was a preferred method by students, and there were some good solutions. However, problems similar to the previous method were present: students forgot to indicate the real and imaginary parts needed to be real. Students also made algebraic errors after establishing the 4 equations – they should be encouraged to use GC to solve simultaneous equations or minimally check their answers using GC.

3	Solution [5] Complex numbers	
(i)	$\lvert z\rvert = 2 \Rightarrow \lvert w\rvert = \sqrt{2}$ $\therefore w = \sqrt{2}\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)$ $= 1 + \mathrm{i}$	This was generally well done.
(ii)	$\arg\left((zw)^n\right) = n\left(\arg(z) + \arg(w)\right)$ $= n\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{7\pi}{12}n$ For $(zw)^n$ to be purely imaginary, $\arg\left((zw)^n\right) = \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$. $\Rightarrow \frac{7\pi}{12}n = \frac{\pi}{2} + k\pi$ $n = \frac{2k+1}{2} \times \frac{12}{7}$ $= \frac{6(2k+1)}{7}$ Therefore, the least integer value of n = 6, (when k = 3)	Students generally had the right idea, but did not always appreciate that n needed to be an integer. $n = \frac{6}{7}$ was a common incorrect answer. Some students were not able to get that there were infinite values for n and k, and instead incorrectly gave $\frac{7\pi}{12}n = \frac{\pi}{2}$ or $-\frac{\pi}{2}$

4	Solution [6] Summation	Comments
	$\mathrm{f}(r)-\mathrm{f}(r-1)=\left(1-\frac{3}{(r+2)!}\right)-\left(1-\frac{3}{(r+1)!}\right)$ $=\frac{3}{(r+1)!}-\frac{3}{(r+2)!}=\frac{3(r+2)-3}{(r+2)!}$ $=\frac{3(r+1)}{(r+2)!}$. Hence, $a=3$	Most students are able to get the value of a.
(i)	From (i), $\frac{r+1}{(r+2)!}=\frac{1}{3}\left(\mathrm{f}(r)-\mathrm{f}(r-1)\right)$ Hence, $\sum_{r=1}^{n}\frac{r+1}{(r+2)!}=\frac{1}{3}\sum_{r=1}^{n}\left(\mathrm{f}(r)-\mathrm{f}(r-1)\right)$ $=\frac{1}{3}\big(f(1)-f(0)$ $+\mathrm{f}(2)-\mathrm{f}(1)$ $+\mathrm{f}(3)-f(2)$ + ... + ... $+\mathrm{f}(n-1)-\mathrm{f}(n-2)$ $+\mathrm{f}(n)-\mathrm{f}(n-1)\big)$ $=\frac{1}{3}\left(f(n)-f(0)\right)=\frac{1}{3}\left(1-\frac{3}{(n+2)!}-\left(1-\frac{3}{2!}\right)\right)$ $=\frac{1}{3}\left(\frac{3}{2}-\frac{3}{(n+2)!}\right)$ $=\frac{1}{2}-\frac{1}{(n+2)!}$	Most of students are also able to demonstrate method of difference.
(ii)	For the sum $\sum_{r=3}^{n}\frac{r-1}{r!}$, we replace '$r$' by '$k$+2'. Then we have: $\sum_{r=3}^{n}\frac{r-1}{r!}=\sum_{k+2=3}^{k+2=n}\frac{k+2-1}{(k+2)!}$ $=\sum_{k=1}^{n-2}\frac{k+1}{(k+2)!}$ $=\frac{1}{2}-\frac{1}{(n-2+2)!}$ (using result in part (i)) $=\frac{1}{2}-\frac{1}{n!}$	This part proves to be challenging to some students as they are not able to replace according. For some they may recognize what to replace but forget to change the starting and ending values of r.

5	Solution [7] Function	
(i)	Sketch of $\mathrm{f}(x)=1-\dfrac{1}{x^2-1}$ for $x\in\mathbb{R},\ x<-1$:  To find $\mathrm{f}^{-1}(x)$: Let $y=1-\dfrac{1}{x^2-1}$, then $\dfrac{1}{x^2-1}=1-y$ $x^2-1=\dfrac{1}{1-y}$ $x^2=1+\dfrac{1}{1-y}$ $\Rightarrow x=\pm\sqrt{1+\dfrac{1}{1-y}}$ Since $x<-1$, $x=-\sqrt{1+\dfrac{1}{1-y}}$ Thus, $\mathrm{f}^{-1}(x)=-\sqrt{1+\dfrac{1}{1-x}}$, $D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}=(-\infty,1)$ (from sketch of f(x))	The most common mistake is fail to justify the choice of x.

(ii)	Sketch of $\mathrm{g}(x)=x^2+4x+5$ for $x\in\mathbb{R},\ x\le 1$:  $R_{\mathrm{g}}=[1,\ \infty)$ For fg, $R_{\mathrm{g}}=[1,\ \infty)\not\subset D_{\mathrm{f}}=(-\infty,\ -1)$, thus fg does not exist For gf, $R_{\mathrm{f}}=(-\infty,\ 1)\subseteq(-\infty,\ 1]=D_{\mathrm{g}}$, thus gf exists. To find range of gf, we use the mapping method: $(-\infty,\ -1)\xrightarrow{\mathrm{f}}(-\infty,\ 1)\xrightarrow{\mathrm{g}}[1,\ \infty)$ Thus, $R_{\mathrm{gf}}=[1,\ \infty)$ Alternatively. Graph of $y=\mathrm{gf}(x)$  Thus, $R_{\mathrm{gf}}=[1,\ \infty)$	This part is not very well done especially finding the range of f & g. Some students are not sure how to check whether composite functions exist. Another common statement made by students is that $R_{gf}=R_g$ which is not necessary true. Students also state the range of gf without justification.

<table>
<tr><td>6</td><td>Solution [7] Integration & its application</td><td></td></tr>
<tr><td>(i)</td><td>

$\int \sin t\left(e^t\right)\,\mathrm{d}t$

$=\left[e^t \sin t\right] - \int e^t \cos t\ \mathrm{d}t$

$= e^t \sin t - \left\{\left[e^t \cos t\right] + \int e^t \sin t\ \mathrm{d}t\right\}$

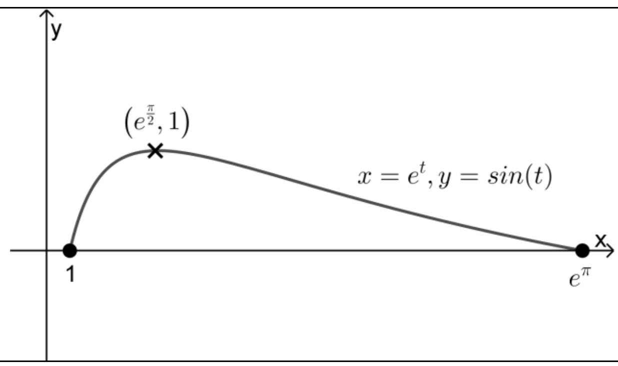
$= e^t \sin t - e^t \cos t - \int e^t \sin t\ \mathrm{d}t$

Since $\int e^t \sin t\ \mathrm{d}t = e^t \sin t - e^t \cos t - \int e^t \sin t\ \mathrm{d}t$

$\Rightarrow 2\int e^t \sin t\ \mathrm{d}t = e^t \sin t - e^t \cos t + c$

$\Rightarrow \int e^t \sin t\ \mathrm{d}t = \frac{1}{2}\left(e^t \sin t - e^t \cos t\right) + c$

</td><td></td></tr>
<tr><td>(ii)</td><td>



</td><td>Many students failed to sketch the graph according to the specified domain.

Many students failed to indicate the correct x-intercept.</td></tr>
<tr><td>(iii)</td><td>

$\int_1^{e^a} y\ \mathrm{d}x$

$= \int_1^{e^a} \sin t\ \mathrm{d}x$

$= \int_0^a \sin t\left(e^t\right)\mathrm{d}t$

$= \frac{1}{2}\left[e^t \sin t - e^t \cos t\right]_0^a$

$= \frac{1}{2}\left(e^a \sin a - e^a \cos a + 1\right)$

</td><td>

Many failed to make the connection that

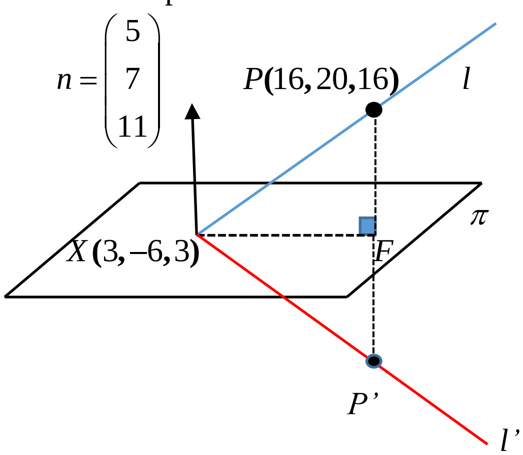
$\int_1^{e^a} y\ \mathrm{d}x$

$= \int_1^{e^a} \sin t\ \mathrm{d}x$

$= \int_0^a \sin t\left(e^t\right)\mathrm{d}t$

</td></tr>
</table>

7	Solution [11] 3-D Vectors	
(i)	Let $A=(-1,0,1)$, $B=(2,1,-1)$ and $C=(1,-3,2)$. Then $\overrightarrow{OA}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}2\\1\\-1\end{pmatrix}$ and $\overrightarrow{OC}=\begin{pmatrix}1\\-3\\2\end{pmatrix}$. Next, we have $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}2\\1\\-1\end{pmatrix}-\begin{pmatrix}-1\\0\\1\end{pmatrix}=\begin{pmatrix}3\\1\\-2\end{pmatrix}$ $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}1\\-3\\2\end{pmatrix}-\begin{pmatrix}-1\\0\\1\end{pmatrix}=\begin{pmatrix}2\\-3\\1\end{pmatrix}$ Then $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}3\\1\\-2\end{pmatrix}\times\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}-5\\-7\\-11\end{pmatrix}$ Thus, we have $\mathbf{r}\bullet\begin{pmatrix}5\\7\\11\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}\bullet\begin{pmatrix}5\\7\\11\end{pmatrix}=-5+0+11=6$ $\therefore$ The Cartesian equation of the plane π is $5x+7y+11z=6$	There are a number of students who made arithmetic mistakes, leading to incorrect answer.
(ii)	The line l has equation of the form: $l:\mathbf{r}=\begin{pmatrix}16\\20\\16\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\1\end{pmatrix},\ \lambda\in\mathbf{R}$ Then to find the intersection point of the line and plane, we solve: $5(16+\lambda)+7(20+2\lambda)+11(16+\lambda)=6$ $\Rightarrow 80+5\lambda+140+14\lambda+176+11\lambda=6$ $\Rightarrow 396+30\lambda=6$ $\Rightarrow 30\lambda=-390$ $\Rightarrow \lambda=-13$ So, $\mathbf{r}=\begin{pmatrix}16-13\\20-26\\16-13\end{pmatrix}=\begin{pmatrix}3\\-6\\3\end{pmatrix}$ Thus, the intersection point of the line l and plane π is	Generally, well done.

	$(3,-6,3)$	
(iii)	Let θ be the acute angle between the line l and plane π. Then $\sin\theta = \dfrac{\left\lvert \begin{pmatrix}1\\2\\1\end{pmatrix} \bullet \begin{pmatrix}5\\7\\11\end{pmatrix} \right\rvert}{\sqrt{1^2+2^2+1^2}\sqrt{5^2+7^2+11^2}}$ $= \dfrac{30}{\sqrt{6}\sqrt{195}}$ $\Rightarrow \theta = 61.3°$ (1 dec place)	$\cos\theta = \lvert \hat{\mathbf{m}} \times \hat{\mathbf{n}} \rvert$ leads to an acute angle θ that gives the angle between the normal of the plane and the line.
(iv)	To find the reflected line l' of l in the plane π, we first consider finding the foot of the perpendicular F from the point P to the plane π.  Let F be the foot of perpendicular from point P to plane π. Then F is the point of intersection between line PF and π.. $l_{PF}: \mathbf{r} = \begin{pmatrix}16\\20\\16\end{pmatrix} + k\begin{pmatrix}5\\7\\11\end{pmatrix}, \ k \in \mathbf{R}$ Since F is the point of intersection, $\begin{pmatrix}16+5k\\20+7k\\16+11k\end{pmatrix} \bullet \begin{pmatrix}5\\7\\11\end{pmatrix} = 6$ $80+25k+140+49k+176+121k=6$ $\Rightarrow 195k = -390$ $\Rightarrow k = -2$	Many students failed to realize that they have to first find the foot of perpendicular of point P on the plane. For those who attempt to find the foot of perpendicular of F and the reflected point P', they committed many arithmetic mistakes.

	$\overrightarrow{OF}=\begin{pmatrix}16\\20\\16\end{pmatrix}+(-2)\begin{pmatrix}5\\7\\11\end{pmatrix}=\begin{pmatrix}6\\6\\-6\end{pmatrix}$ $\overrightarrow{PF}$ $=\overrightarrow{OF}-\overrightarrow{OP}$ $=\begin{pmatrix}16\\20\\16\end{pmatrix}+(-2)\begin{pmatrix}5\\7\\11\end{pmatrix}-\begin{pmatrix}16\\20\\16\end{pmatrix}$ $=\begin{pmatrix}-10\\-14\\-22\end{pmatrix}$ Next we have $\overrightarrow{XP'}$ $=\overrightarrow{XP}+\overrightarrow{PP'}$ $=\overrightarrow{XP}+2\overrightarrow{PF}$ $=\left(\begin{pmatrix}16\\20\\16\end{pmatrix}-\begin{pmatrix}3\\-6\\3\end{pmatrix}\right)+2\begin{pmatrix}-10\\-14\\-22\end{pmatrix}$ $=\begin{pmatrix}-7\\-2\\-31\end{pmatrix}$ Thus the equation of the reflected line *l'* is $\mathbf{r}=\begin{pmatrix}3\\-6\\3\end{pmatrix}+\mu\begin{pmatrix}7\\2\\31\end{pmatrix}, \mu\in\mathbf{R}$	

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<table>
<tr><td>8</td><td>Solution [12] AP GP</td><td></td></tr>
<tr><td>(a)
(i)</td><td>$R_0 = \frac{u_{n+2}}{u_{n+1}} = \frac{2401}{686} = \frac{7}{2}$</td><td rowspan="2">Majority used the correct technique of comparing consecutive terms to obtain the common ration R_0 in (a)(i). However, major challenge here is the interpretation and understanding of the question. Many failed to do this correctly. For example, Q8 said that "… by the nth day, a TOTAL of 268 people were infected …" This refers to $S_n = 268$ and NOT u_n. But majority interpreted this as u_n.

Thus to solve (i) and to make use of the given formula $u_n = R_0^{n-1}$, we need to identify the TERMS which are $u_{n+1}=686$ and $u_{n+2}=2401$.

Likewise to use 268 in (ii), we have to equate it to Sn of a GP, not the u_n.</td></tr>
<tr><td>(a)
(ii)</td><td>Let a = 16
$S_n = \frac{a\left(R_0{}^n - 1\right)}{\left(R_0 - 1\right)}$
$268 = \frac{16\left(\left(\frac{7}{2}\right)^n - 1\right)}{\left(\frac{7}{2} - 1\right)}$
$\frac{670}{16} = \left(\left(\frac{7}{2}\right)^n - 1\right)$
$\frac{343}{8} = \left(\frac{7}{2}\right)^n$
$n = 3$
The virus had been spreading for 3 days before it was identified.

Alternative solutions:
$u_{n+1} = 16\left(\frac{7}{2}\right)^n \Rightarrow 686 = 16\left(\frac{7}{2}\right)^n \Rightarrow n = 3 \quad or$
$u_{n+2} = 16\left(\frac{7}{2}\right)^{n+1} \Rightarrow 2401 = 16\left(\frac{7}{2}\right)^{n+1} \Rightarrow n = 3$</td></tr>
<tr><td>(a)
(iii)</td><td>As $\left|R_0\right| < 1$ the infinity sum of the GP will converge. Hence, eventually there will be no new infections.
OR
As $\left|R_0\right| < 1,\ R_0^{n-1} \to 0 \;\&\; u_n \to 0$ as $n \to \infty$. Thus there will be no new infection in the long run.</td><td>Majority could get the final answer of zero new infection but was not able to get the full credit due to lack of explanation.

Some students explained that $R_0 \to 0$. This is not true, R_0 is less than 1, but it need not tend to 0. Instead $R_0^{n-1} \to 0$ as $n \to \infty$.</td></tr>
<tr><td>(b)</td><td><table><tr><th>Day</th><th>Morning</th><th>Evening</th></tr><tr><td>1</td><td>200</td><td>$(200 - 10)$</td></tr></table></td><td>Again, main challenge in (b) is the interpretation of the question. Majority used</td></tr>
</table>

2	$(200-10)\times 1.2$	$((200-10)\times 1.2-10)$
3	$((200-10)\times 1.2-10)\times 1.2$ $=200\times 1.2^2-10(1.2^2+1.2)$	$\left(((200-10)\times 1.2-10)\times 1.2-10\right)$ $=200\times 1.2^2-10(1.2^2+1.2+1)$
…		
n	$200\times 1.2^{n-1}-10\left(1.2^{n-1}+1.2^{n-2}+...+1.2\right)$	$200\times 1.2^{n-1}-10\left(1.2^{n-1}+1.2^{n-2}+...+1.2+1\right)$

v_n denotes the number of people in hospital on the morning of the n^{th} day.

$$v_n=200\times 1.2^{n-1}-10\left(1.2^{n-1}+1.2^{n-2}+...+1.2\right)$$
$$=200\times 1.2^{n-1}-10\left(1.2\frac{1.2^{n-1}-1}{1.2-1}\right)$$
$$=200\times 1.2^{n-1}-60\left(1.2^{n-1}-1\right)$$
$$=140\times 1.2^{n-1}+60$$

Therefore $k = 60$

As capacity is 500 beds and when the hospital exceeds capacity

$$v_n>500$$
$$140\times 1.2^{n-1}+60>500$$
$$1.2^{n-1}>\frac{22}{7}$$
$$(n-1)\ln 1.2>\ln\left(\frac{22}{7}\right)$$
$$n>\frac{\ln\left(\frac{22}{7}\right)}{\ln(1.2)}+1=7.28$$

Therefore least value of n is 8.

a good skill of listing down the "before" and "after" to observe pattern before writing down the general form. There was also appropriate application of GP sum.

Many could start with 200 on the first day morning, but failed to recognize that "during the course of the day, … discharge 10 patients …". Thus "–10" should appear at the END of each day BEFORE evening starts. Thus $v_2=200\times 1.2-10$ is NOT correct.

Some students tried to write down the general form directly without listing down the first few cases. Most of failed to get the correct general term.

Some attempted to use the given v_n and substituted $v_2=228$ to obtain k. No credit awarded as this is a "Show" question. Students are to derived the expression for v_n.

To find the "least value of n", it translated to solve for n such that $v_n > 500$. Some used "=" instead. If "=" were used, students were expected to justify why they rounded up (or down).

9	Solution [6] P&C	
(a)	Total ways $=\binom{10}{5}\times\frac{5!}{5}=6048$	Many students failed to recognize that the 5 flavours were not fixed. Hence there is a need for $\binom{10}{5}$.
(b)	Case 1: 3 scoops of same flavor, 2 other scoops different flavors. i.e. AAA B C Number of ways $=\binom{3}{1}\frac{5!}{3!}=60$ Explanation: • $\binom{3}{1}$ ways to determine which flavor is the one with 3 scoops • $\frac{5!}{3!}$ ways to arrange all 5 scoops in a row with 3 being identical. Case 2: 2 scoops each for 2 flavors and 1 scoop for the last flavor. i.e. AA BB C Number of ways $=\binom{3}{2}\times\frac{5!}{2!2!}=90$ Total number of ways $=60+90=150$	Important concept here is the arrangement of identical objects. Thus, it is crucial to identify how many identical flavours are there in the selection. So, the first step to this questions is to consider the different cases possible for all 3 flavours to be included with varying repetition of some flavours.

10	Solution [6] Hypothesis Testing	
(i)	The sample Mr Tan has chosen is indeed a random one as every JC student in his school has an equal chance of being selected for the survey. The event that a student is chosen is independent of the event that any other student is chosen.	A small group of students mentioned unbiasedness instead of equal chance of being selected. Majority did not mention about the independent property.
(ii)	The unbiased estimates are: $\bar{x} = \frac{\sum x}{n} = \frac{1117}{50} = 22.34$ $s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right) = \frac{1}{49}\left(25061 - \frac{1117^2}{50}\right)$ $= \frac{5361}{2450}$ **(**2.188163265**)**	Generally ok. Some students used the wrong formula $s^2 = \frac{n}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$
(iii)	Let μ be population mean time for time duration spent in canteen by JC student for lunch. Then we have: $H_0: \mu = 22$ $H_1: \mu \neq 22$ We perform a 2-tail test at 5% level of significance, $\alpha = 0.05$ Under H_0, $\bar{X} \sim N\left(22, \frac{s^2}{50}\right)$ approximately Or $Z = \frac{\bar{X} - 22}{\sqrt{\frac{5361}{2450}}/\sqrt{50}} \sim N(0, 1)$ Using GC, the *p* value is $p = 0.104 > 0.05$. Since $p > \alpha$, we do not reject H_0. Thus, at the 5% level of significance, there is insufficient evidence to claim that the time duration JC students in his school spent for their lunch in the school canteen, differs from 22 minutes. Thus his belief is valid.	Many students did not define μ. Wrong hypothesis statemetns include: $H_0 = 22$ vs $H_1 \neq 22$ $H_0: \bar{x} = 22$ vs $H_1: \bar{x} \neq 22$ $H_0: \mu = 22$ vs $H_1: \mu > 22$ Common errors: $\bar{X} \sim N\left(22.34, \frac{s^2}{50}\right)$ approx; $\bar{X} \sim N\left(22, s^2\right)$ approx ; wrong use of CLT ; $Z = \frac{22.34 - 22}{\sqrt{\frac{5361}{2450}}/\sqrt{50}} \sim N(0, 1)$ $p = 0.104 < 0.05$ leading to wrong conclusion; Since $p > \alpha$, we do not reject H_0 and there is insufficient evidence to claim that the time spent by JC students for lunch *is 22 minutes* (instead of differs from 22 minutes)

<table>
<tr><td>11</td><td>Solution [6] DRV</td><td></td></tr>
<tr><td>(i)</td><td>

x	0	1	2	3	4
P(X = x)	$\frac{15}{64}$	$\frac{26}{64}$	$\frac{16}{64}$	$\frac{6}{64}$	$\frac{1}{64}$

$$P(X=0)=\frac{1}{4}\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^2+\frac{1}{4}\left(\frac{1}{2}\right)^3+\frac{1}{4}\left(\frac{1}{2}\right)^4=\frac{15}{64}$$

$$P(X=1)=\frac{1}{4}\left(\frac{1}{2}\right)+\frac{1}{4}\left(\binom{2}{1}\left(\frac{1}{2}\right)^2\right)+\frac{1}{4}\left(\binom{3}{1}\left(\frac{1}{2}\right)^3\right)$$
$$+\frac{1}{4}\left(\binom{4}{1}\left(\frac{1}{2}\right)^4\right)=\frac{26}{64}$$

$$P(X=2)=\frac{1}{4}\left(\frac{1}{2}\right)^2+\frac{1}{4}\left(\binom{3}{2}\left(\frac{1}{2}\right)^3\right)+\frac{1}{4}\left(\binom{4}{2}\left(\frac{1}{2}\right)^4\right)=\frac{16}{64}$$

$$P(X=3)=\frac{1}{4}\left(\frac{1}{2}\right)^3+\frac{1}{4}\left(\binom{4}{3}\left(\frac{1}{2}\right)^4\right)=\frac{6}{64}$$

$$P(X=4)=\frac{1}{4}\left(\frac{1}{2}\right)^4=\frac{1}{64}$$

</td><td>Very badly done. Many students missed out the case when x = 0 or miscalculated the relevant values.</td></tr>
<tr><td>(ii)</td><td>

$$E(X)=(0)\left(\frac{15}{64}\right)+(1)\left(\frac{26}{64}\right)+(2)\left(\frac{16}{64}\right)+(3)\left(\frac{6}{64}\right)$$
$$+(4)\left(\frac{1}{64}\right)=\frac{80}{64}=1.25$$

$$E(X^2)=(0^2)\left(\frac{15}{64}\right)+(1^2)\left(\frac{26}{64}\right)+(2^2)\left(\frac{16}{64}\right)+(3^2)\left(\frac{6}{64}\right)$$
$$+(4^2)\left(\frac{1}{64}\right)=\frac{160}{64}=2.5$$

$$\text{Var}(X)=E(X^2)-E(X)^2=2.5-1.25^2=0.9375$$

</td><td>Students generally are able to apply what they have found correctly but due to the incorrect values found in part (i), marks were not awarded.</td></tr>
</table>

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12	Solution [6] Probability	
(a)	Let A be the event that the person has the disease, and let B be the event that the person tests positive. $\mathrm{P}(A\mid B)=\dfrac{\mathrm{P}(A\cap B)}{P(B)}$ $=\dfrac{\left(\frac{1}{10000}\right)(0.999)}{\left(\frac{1}{10000}\right)(0.999)+\left(\frac{9999}{10000}\right)(0.001)}$ $=0.0908$ (3 s.f.)	Some students did not realise that this part is testing for conditional probability. Many had difficulties working for the correct answer for the denominator of the conditional prob due to errors in decimal point placing or simply quoting it as 0.999.
(b) (i)	Let M be the event a person wears a mask, and let C be the event the person caught the disease. | | ***C*** | ***C'*** | | | ***M*** | 7 | 24 | **31** | | ***M'*** | 6 | 13 | **19** | | | **13** | **37** | **50** | Since $\mathrm{P}(M\cap C)=\dfrac{7}{50}\neq\dfrac{403}{2500}=\left(\dfrac{13}{50}\right)\left(\dfrac{31}{50}\right)=\mathrm{P}(M)\mathrm{P}(C)$ The events M and C are **not** independent	Generally ok. Some students wrongly stated $\mathrm{P}(M\cap C)=\dfrac{7}{31}$ Some students merely stated that event M and C are independent with explanation statements that lack mathematics calculations backing.
	<u>Alternative</u> $\mathrm{P}(M\mid C)=\dfrac{7}{13}$, $\mathrm{P}(M)=\dfrac{31}{50}$ Since $\mathrm{P}(M\mid C)=\dfrac{7}{13}\neq\dfrac{31}{50}=\mathrm{P}(M)$ the events M and C are **not** independent.	Fewer students used this approach although it is also a good method of explanation.
(b) (ii)	Method 1: $\mathrm{P}(C\mid M)=\dfrac{7}{31}=0.226$ $\mathrm{P}(C)=\dfrac{13}{50}=0.26$ Since $\mathrm{P}(C\mid M)<\mathrm{P}(C)$, wearing a mask **decreases** the probability of catching the disease. Hence, we should **disagree** with the comment. Method 2:	Very few students argue using Method 1. Majority of the students use Method 2 in their argument. However quite a substantial number of students mix up use of $\mathrm{P}(C\cap M)$ and $\mathrm{P}(C\mid M)$. We should use conditional probabilities in the given argument as in the given

	$\mathrm{P}\left(C \mid M\right) = \frac{\frac{7}{50}}{\frac{31}{50}} = \frac{7}{31} = 0.2258$ $\mathrm{P}\left(C \mid M'\right) = \frac{\frac{6}{50}}{\frac{19}{50}} = \frac{6}{19} = 0.3158$ Compare the above probabilities & Disagree	sample, the number of people wearing masks is more than those not wearing masks. An example of this **wrong** use of probabilities is in the argument that $\mathrm{P}\left(C \cap M\right) = \frac{7}{50}$ $> \frac{6}{50} = \mathrm{P}\left(C \cap M'\right)$ and hence wearing mask is pointless.

13	Solution [8] Binomial Distribution	
(i)	The assumption needed is the probability that a randomly chosen scientific calculator is faulty is a constant.	There were many well expressed correct answers to this part, perhaps best expressed as 'the probability of a scientific calculator being faulty is constant'. Incorrect assumptions were seen such as 'a calculator is either faulty or not' or 'The event of a calculator being faulty is independent of another being faulty'. These conditions are required for a binomial distribution, but they are covered in the description given in the question, so they are not assumptions that need to be made.
(ii)	For the case $n = 30$, $X \sim B\left(30, \frac{1}{40}\right)$ mean of distribution = $\mathrm{E}(X) = np = 30 \times \frac{1}{40} = 0.75$ variance = $\mathrm{Var}(X) = npq = 30 \times \frac{1}{40} \times \frac{39}{40} = 0.73125$. Then, we have $\mathrm{P}(\lvert X-\mu \rvert < 2\sigma)$ $= \mathrm{P}\left(\lvert X - 0.75 \rvert < 2\sqrt{0.73125}\right)$ $= \mathrm{P}(\lvert X - 0.75 \rvert < 1.7103)$ $= \mathrm{P}(0.75 - 1.7103 < X < 0.75 + 1.7103)$ $= \mathrm{P}(0 \le X < 2.4603)$ $= \mathrm{P}(X \le 2)$ $= 0.962$ (3 s.f.)	Many responses rounded off the value of the variance to 3 sf which should not be done as the value 0.73125 is exact, no rounding is needed. Many responses seem to have the mistaken impression to need to use Normal distribution or CLT for this part. Uncommon mistakes include failing to square root the variance for the standard deviation. Common errors will include failure to consider the modulus or failure to recognize that the case X = 0 should also be included. Note that a lot of solutions have the correct answer although they have committed one of the common errors for which no credit can be awarded for the answer.

<table>
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<td>(iii)</td>
<td>
Let $X \sim \mathrm{B}\left(n, \frac{1}{40}\right)$. Then:
$P(X > 2) \le 0.05$
$\Rightarrow 1 - P(X \le 2) \le 0.05$
$\Rightarrow P(X \le 2) \ge 0.95$

Using GC to solve for the inequality:
<table>
<tr><th>n</th><th>$P(X \le 2)$</th></tr>
<tr><td>32</td><td>0.954776</td></tr>
<tr><td>33</td><td>0.951150</td></tr>
<tr><td>34</td><td>0.947386</td></tr>
</table>

Therefore, the largest value of n is 33.
The educational store manager needs to order at most 33 scientific calculators from the company.

The GC command for calculating $P(X \le 2)$ above is binomcdf$\left(n, \frac{1}{40}, 2\right)$.
</td>
<td>
Many are able to give the first line although some responses gave a strict inequality for the interpretation of ‘at most’ which is incorrect.

Many responses are unable to compare the decimal values properly leading to the wrong conclusion.

Most responses have shown ability to use the correct command for GC.
</td>
</tr>
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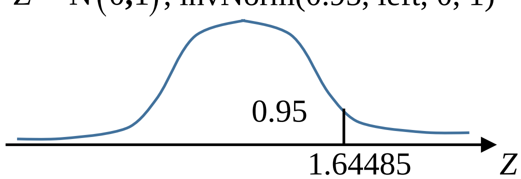
14	Solution [10] Normal Distribution	
(a) (i)	Let G: mass of Garoupa fish in kg. Then $G \sim \mathrm{N}\left(0.5, 0.025^2\right)$ Let S: mass of Snapper fish in kg. Then $S \sim \mathrm{N}\left(0.45, 0.02^2\right)$ Let T be the total price for 2 Garoupa and 3 Snapper Then $T = (G_1 + G_2) + (S_1 + S_2 + S_3)$ We then have: $\mathrm{E}(T) = 2 \times 0.5 + 3 \times 0.45$ $= 2.35$ $\mathrm{Var}(T) = 2 \times 0.025^2 + 3 \times 0.02^2$ $= 0.00245$ Thus $T \sim \mathrm{N}(2.35, 0.00245)$ Hence using GC, we have $\mathrm{P}(T > 2.25) = 0.978$ (3 s.f.)	Not well done at all. Many students calculated the wrong value of variance and did not define the relevant variables.
(a) (ii)	Suppose Mrs Tan decided to buy k Snapper fishes. Then $L = 18.5(S_1 + S_2 + S_3 + \ldots\ldots + S_k)$ and so, $\mathrm{E}(L) = 18.5 \times k \times 0.45 = 8.325k$ $\mathrm{Var}(L) = 18.5^2 \times k \times 0.02^2 = 0.1369k$ $L \sim N(8.325k, 0.1369k)$ We then need to have $P(L \le 50) \ge 0.95$. Standardizing the variable L : $P\left(Z \le \frac{50 - 8.325k}{\sqrt{0.1369k}}\right) \ge 0.95$ $\Rightarrow \frac{50 - 8.325k}{\sqrt{0.1369k}} \ge 1.64485$ For $Z \sim \mathrm{N}(0,1)$, invNorm(0.95, left, 0, 1) =1.64485 	Very badly done. Many students were not able to obtain the correct values of E(L) and Var(L) and many were not able to standardize the relevant variable. Students who used GC method did not write down a table of values but instead conclude the answer as 5.

<table>
<tr>
<td></td>
<td>Using GC to solve for the above inequality, we have

<table>
<tr><th>k</th><th>$\frac{50-8.325k}{\sqrt{0.1369k}}$</th></tr>
<tr><td>4</td><td>22.568</td></tr>
<tr><td>5</td><td>10.123</td></tr>
<tr><td>6</td><td>0.05517</td></tr>
</table>

Thus, the maximum number of Snapper that can be bought is 5.</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Given $G \sim \mathrm{N}\left(0.5, 0.025^2\right)$, $P\left(G > 0.48\right) = 0.7881446663$
We define the random variable Y : number of Garoupa fish out of 6 purchased with mass greater then 0.48 kg.
Then $Y \sim B\left(6, 0.7881446663\right)$.
So $\mathrm{P}\left(Y > 3\right) = 1 - \mathrm{P}\left(X \le 3\right)$
= 0.886 (3 s.f.)</td>
<td>Badly done. Many students were very careless by writing
$\mathrm{P}\left(Y > 3\right) = 1 - \mathrm{P}\left(X \le 2\right)$</td>
</tr>
<tr>
<td>(c)</td>
<td>P (lightest Groupa has mass exceeding 0.48 kg)
$= \left[\mathrm{P}\left(X > 0.48\right)\right]^6$
$= \left[0.7881446663\right]^6$
$= 0.240$</td>
<td>Generally quite well done, except a few who could not round this to 3sf.</td>
</tr>
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