YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME

CG

MATHEMATICS 9758/02

Paper 2

13 SEPTEMBER 2023

Candidates answer on the Question Paper. **3 hours**
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

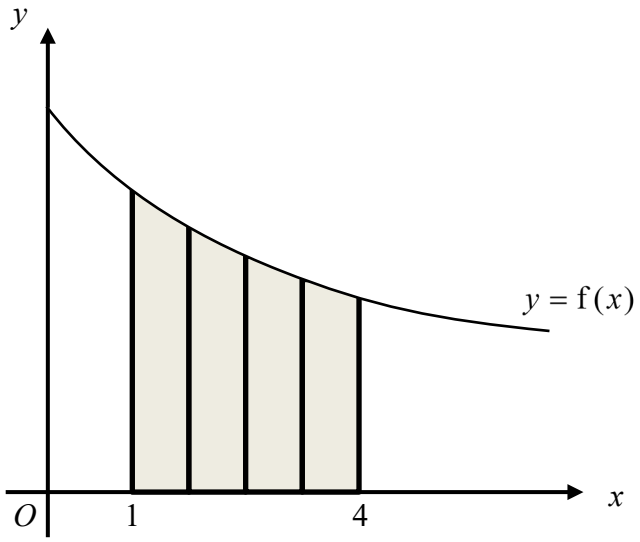
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Presentation	
Total	/ 100

Section A: Pure Mathematics [40 marks]

1 The diagram shows a sketch of the curve $y = \mathrm{f}(x)$. The shaded region under the curve between $x = 1$ and $x = 4$, shown in the diagram, is A. This region is split into 4 vertical strips of equal width, h units.



(a) State the value of h and show, by drawing on the above diagram, that $\sum_{n=1}^{4}[\mathrm{f}(1+nh)]h$ is less than the area of A. [2]

(b) Find a similar expression that is greater than the area of A. [1]

You are now given that $\mathrm{f}(x) = -2\ln(x+1) + 7$.

(c) Use the expression in part **(a)** and your expression in part **(b)** to find lower and upper bounds for the area of A. [2]

(d) The curve with equation $y = \mathrm{g}(x)$ is obtained when the curve with equation $y = \mathrm{f}(x)$ is translated by b units in the positive y-direction, where b is a positive constant.

Given that $\sum_{n=1}^{4}[\mathrm{g}(1+nh)]h = \sum_{n=1}^{4}[\mathrm{f}(1+nh)]h + c$, where c is a positive constant, express c in terms of b. [1]

2 The complex number z is given by $z = \dfrac{3\mathrm{i} - \sqrt{3}}{k - \mathrm{i}}$, where k is a real constant.

(a) Given that $k = 1$, find the exact values of $|z|$ and $\arg(z)$. [4]

(b) Given instead that $z = z^*$, find the value of k. [3]

3 **(a)** Show that $\ln\left(1-\frac{1}{r^2}\right)$ can be expressed as $\ln(r-1)-2\ln r+\ln(r+1)$, where $r>1$. [1]

The sum $\sum_{r=2}^{n}\ln\left(1-\frac{1}{r^2}\right)$ is denoted by S_n.

(b) Find an expression for S_n in terms of n. [4]

(c) Find the smallest value of n for which S_n is within 0.05 of the sum to infinity. [3]

4 **(a)** Vectors **u** and **v** are such that $\mathbf{u}\neq\mathbf{0}$ and $\mathbf{v}\times\mathbf{u}=\mathbf{0}$.

(i) Find a linear relationship between **u** and **v**. [2]

(ii) Find a unit vector **n** such that $(\mathbf{i}-2\mathbf{j}-2\mathbf{k})\times\mathbf{n}=\mathbf{0}$. [2]

(b) Referred to the origin O, the points A, B and C have position vectors **a**, **b** and $4\mathbf{a}-\frac{2}{3}\mathbf{b}$ respectively.

(i) Given that the area of triangle ABC is 14, show that $|\mathbf{a}\times\mathbf{b}|=12$. [3]

(ii) Given further that $|\mathbf{a}|=5$, $|\mathbf{b}|=3$ and $\angle AOB$ is obtuse, deduce the exact value of $\cos\angle AOB$. [2]

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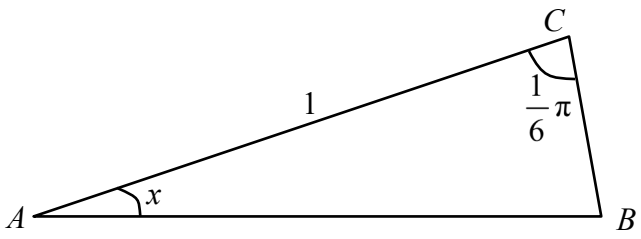


In the triangle ABC, $AC=1$, angle $BAC=x$ radians and angle $ACB=\frac{1}{6}\pi$ radians (see diagram).

(a) Show that $AB=\dfrac{1}{\cos x+\sqrt{3}\sin x}$. [3]

(b) Given that x is a sufficiently small angle, show that

$$AB\approx 1+ax+bx^2,$$

for constants a and b to be determined exactly. [3]

(c) The first two terms in the Maclaurin series for $\ln(px+q)$ are equal to the first two terms in the series expansion in part **(b)**. Using standard series from the List of Formulae (MF26), find the exact values of p and q. [4]

Section B: Probability and Statistics [60 marks]

6 For events A and B, it is given that $\mathrm{P}(A'\cup B')=0.82$ and $\mathrm{P}(A\,|B)=0.4$.

(a) Find $\mathrm{P}(B)$. [2]

(b) Hence find an inequality for $\mathrm{P}(A\cap B')$. [2]

A third event C is such that $\mathrm{P}(C)=0.1$. A and C are mutually exclusive. B and C are independent.

(c) Find $\mathrm{P}(A'\cap B\cap C')$. [3]

7 There are 2 red discs, 6 green discs and 1 blue disc on a table. The discs are identical in all aspects except colour.

(a) The 9 discs are arranged in a line. Find the number of different possible arrangements such that the 2 red discs and the blue disc are next to one another. [2]

(b) All the 9 discs are put into a bag. Jack takes 3 discs from the bag at random. The random variable R is the number of red discs taken.

(i) Tabulate the probability distribution of R. [3]

(ii) For each red disc taken, Jack gains nine points. Otherwise, he loses three points for each disc of any other colour taken. Find the expected change in points Jack has after taking the 3 discs. [3]

8 A company produces packets of scallops with each packet stated to weigh 500 grams. In a quality control inspection, the production manager takes a random sample of 35 packets packed on a particular day at the factory.

The masses, x grams, are summarised as follows.

$$\sum(x-500)=198.5 \qquad \sum(x-500)^2=7188$$

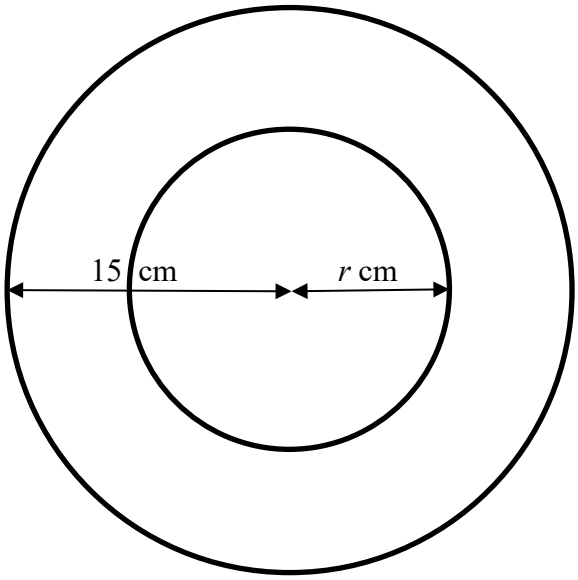
(a) Find unbiased estimates of the population mean and variance. [2]

(b) Test, at the 2% level of significance, whether the mean mass of packets of scallops is 500 grams. [4]

To increase customer satisfaction, the company changes the machine setting and claims that the mean mass of packets of scallops is now at least μ_0 grams. To investigate the claim, the production manager takes a random sample of 40 packets of scallops after the change. The mean and standard deviation of this sample are found to be 510 grams and 11.7 grams respectively.

(c) Given that there is no reason to reject the company's claim at the 5% level of significance, find the range of possible values of μ_0, giving your answer correct to 2 decimal places. [4]

9 A dart board is designed with an inner radius of r cm and an outer radius of 15 cm, as shown in the diagram below.



A dart throw is considered a 'hit' if it lands within the inner circle. It is assumed that any dart throw lands randomly on the dart board.

(a) Show that the probability of a 'hit' is $\frac{r^2}{225}$. [1]

(b) Wei plays a round of 8 dart throws.

(i) The probability that Wei obtains more than six 'hits' is at most 0.08. Write down an inequality satisfied by r. Hence find the range of possible values of r. [2]

It is now given that $r = 6$.

(ii) Find the probability that Wei's fifth throw is the second time that he obtains a 'hit'. [2]

(iii) Find the least number of throws that Wei needs to attempt such that the probability of obtaining at least 1 'hit' from the attempts is at least 0.7. [3]

(iv) Chan also plays a round of 8 dart throws. The number of 'hits' Chan obtained is more than the expected number of 'hits' Wei obtained. Find the probability that the number of 'hits' Chan obtained is fewer than twice the expected number of 'hits' Wei obtained. [3]

10 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.

The masses, in kg, of pumpkins and cabbages sold in a supermarket are modelled as having independent normal distributions with means and standard deviations as shown in the following table.

	Mean Mass	Standard Deviation
Pumpkins	3.7	0.4
Cabbages	0.8	0.12

(a) The probability that the mass of a randomly chosen pumpkin is between 3.2 kg and m kg is 0.6. Show that $m = 3.916$ correct to 3 decimal places. [2]

(b) 20 pumpkins are randomly selected. Find the probability that more than 5 pumpkins each have a mass greater than m kg. [3]

(c) Find the least value of n such that the probability that the mean mass of n pumpkins is at most 3.8 kg is more than 0.95. [3]

Pumpkins are sold at \$5 per kg and cabbages at \$3 per kg.

(d) Find the probability that the total selling price of three randomly chosen pumpkins and four randomly chosen cabbages is less than \$60. [4]

11 Jai is investigating how the mass of 30-year-old males (m kg) is related to the average mass of fried chicken consumed per week (x kg). He found that the relationship between x and m can be modelled by the equation $m = a + bx$, where a and b are constants.

(a) Given that $a = 62.3$ and $b = 0.765$, explain the meaning of a and b in the context of the question. [2]

Jai is now investigating how the percentage body fat of 30-year-old males is related to the average mass of fried chicken consumed per week. The average mass of fried chicken consumed per week (x kg) and the percentage body fat (y %) of a sample of eight 30-year-old males are given in the table.

x	0.5	0.9	1.3	1.7	2.1	2.5	2.9	3.4
y	16.5	24.6	29.3	32.7	34.3	35.2	35.9	36.5

(b) Sketch a scatter diagram of these data. [1]

(c) By calculating the relevant product moment correlation coefficients, determine whether the relationship between x and y is modelled better by $y = p + qx$ or by $y = p + qe^{-x}$, where p and q are constants. Explain how you decide which model is better and state the equation in this case. [4]

(d) Use your equation in part **(c)** to estimate the percentage body fat of a 30-year-old male who consumes an average of 2.3 kg of fried chicken per week. Explain whether your estimate is reliable. [2]

(e) It is found that all the values of the percentage body fat were recorded wrongly. The actual values are 5% lower than the recorded values.

(i) Explain whether the product moment correlation coefficients calculated in part **(c)** would differ. [1]

(ii) Re-write the equation from part **(c)** in terms of x and z, where z is the correct percentage body fat. [2]