JURONG PIONEER JUNIOR COLLEGE
9749 H2 PHYSICS

DYNAMICS

Content

(1) Newton's laws of motion
(2) Linear momentum and its conservation

Learning Outcomes

Students should be able to:

(a) state and apply each of Newton’s laws of motion

(b) show an understanding that mass is the property of a body which resists change in motion (inertia)

(c) describe and use the concept of weight as the force experienced by a mass in a gravitational field

(d) define and use linear momentum as the product of mass and velocity

(e) define and use impulse as the product of force and time of impact

(f) relate resultant force to the rate of change of momentum

(g) recall and solve problems using the relationship $F = ma$, appreciating that resultant force and acceleration are always in the same direction

(h) state the principle of conservation of momentum

(i) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required)

(j) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation

(k) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Introduction

If you see the velocity of an object change in either magnitude or direction, you know that something must have caused that change (or acceleration). An interaction that can cause an acceleration of a body is called a force, which is loosely speaking, a *push* or a *pull* on the body.

The relationship between a force and the acceleration it causes was first understood by Isaac Newton (1642 – 1727). The study of the relationship is called Newtonian mechanics. We shall focus on its 3 primary laws of motion.

Sir Isaac Newton

Newtonian mechanics does <u>not</u> apply in the following situations:

- When the speeds of the interacting bodies are very large (i.e. approaching the speed of light). In this case, Einstein's special theory of relativity replaces Newtonian mechanics.
- When the interacting bodies are on the atomic scale. In this case, Quantum mechanics replaces Newtonian mechanics.

Physicists now view Newtonian mechanics as a special case of these two more comprehensive theories.

Linking Kinematics and Dynamics

In Kinematics, when we look at ***describing*** the motion of objects, we are concerned only with how the **displacement, velocity** and **acceleration** of the objects change with **time**.

We then try to answer questions like:

a) How far has the object moved from the starting/reference point?
b) How long does the object take to move from point A to point B?
c) How fast must the object move to travel from point A to point B within a certain time interval?

In Dynamics, we will discuss the ***cause*** of motion using two quantities: **force** and **mass**, and study their relationships to the motion of objects. We then try to answer questions like:

a) **Why** does something move?
b) **Why** does something CONTINUE to move?
c) **What causes** something to STOP once it is moving?

1 Newton's laws of motion

(a) State and apply each of Newton's laws of motion.
(b) Show an understanding that mass is the property of a body which resists change in motion (inertia).

1.1 Newton's first law of motion

- Suppose you send a book sliding across a carpet by applying a horizontal force to it with your hand. After you stop pushing, the book slows down and comes to rest soon after. If you want it to continue sliding, you will have to keep pushing it across the carpet. How about if you now give a push to the book on a frozen lake of ice? In this case, the book would probably slide much further on its own although eventually it will still come to rest.

- What is it that causes the book to come to rest in these two instances? It is the friction between the book and the surface; the friction between the book and the carpet is much higher than that between the book and the frozen lake.

- If we can eliminate friction completely, **the book will never slow down, and we would need no force at all for the book to keep moving with constant velocity**.

- Most moving bodies on Earth visible to us tend to come to rest in the absence of an applied force. This is because moving bodies on Earth are continuously subjected to effects of resistive forces, be it from the ground, air or even between mechanical parts.

- Therefore it is a common <u>misconception</u> that a <u>force must always be applied</u> to keep a body moving at <u>constant</u> velocity.

- **Newton's first law of motion** states that:

A body will continue in its state of rest or uniform motion in a straight line unless a net external force acts on it.

- Motion that is uniform in a straight line implies that <u>**velocity is constant**</u>. In other words, there is <u>**no acceleration**</u>.

- Newton's first law of motion tells the effects of what a force does. A force when applied on an object causes it to <u>accelerate (change in velocity).</u>

- We may have more than one force acting on the object. As such, we consider the effect of the <u>**resultant force**</u> acting on the object.

- Hence, if an object is <u>at rest</u> or <u>moving with constant velocity</u>,
  EITHER no force acts on it, **OR** the resultant force acting on it is zero.

- Newton's first law of motion is often called the law of inertia.
  The inertia of a body is the reluctance of a body to change its state of rest or motion. The <u>mass</u> of a body is a <u>measure of its inertia</u>. The larger the mass, the greater the inertia. It is more difficult to kick a large rock and expect it to move compared to doing the same to a small pebble.

- Mass is the property of a body which resists change in motion (inertia).

- To change the <u>state of motion</u> (i.e. velocity) of an object, a force (push or pull) must be applied to the object. However, the state of motion may remain unchanged even when a force is applied to the object.

- Inertia can be used to explain why a force is needed to:

  - move a stationary object,
  - stop a moving object,
  - change the direction of an object moving in a straight line.

(c) describe and use the concept of weight as the force experienced by a mass in a gravitational field

- When an object is brought from place to place, its mass remains the same. However, its weight may vary considerably from place to place.

  While it is true that you will weigh less if you have less mass, this cannot account fully for the difference. For example, you will weigh six times lighter on the Moon than on the Earth. This is not because you have lost mass. You still have the same mass. The difference in weight arises because of the difference in the **gravitational field strength** of the Earth and that of the Moon.

The **weight** of a body is the gravitational force exerted on it by a gravitational field.

weight = mass x acceleration of free fall or $w = mg$

- SI Unit : newton (N)
- Mass is a scalar quantity, while weight is a vector quantity.
- The direction of weight is always in the direction of the gravitational field strength. In the context of Earth, weight will always point towards the centre of the Earth.

1.2 Newton's second law of motion

(d) Define and use linear momentum as the product of mass and velocity.
(e) Define and use impulse as the product of force and time of impact.
(f) Relate resultant force to the rate of change of momentum.
(g) Recall and solve problems using the relationship F = ma, appreciating that resultant force and acceleration are always in the same direction.

1.2.1 Linear momentum

- It gives a measure of how an object will respond to an externally applied force, e.g. how much force is required to get the object to stop.

The linear momentum p of a body is defined as the product of its mass m and its velocity v.

Mathematically:

$$\boxed{p = m\,v}$$

Linear momentum is a vector quantity and its unit is kg m s^{-1} or N s.
Direction of momentum of body is in the direction of its velocity.

1.2.2 Defining Newton’s second law of motion

- **Newton's second law of motion** states that:

> **The rate of change of momentum of a body is proportional to the resultant force that acts on it and the momentum change takes place in the direction of the force.**

- Mathematically:

$$\boxed{\frac{\mathrm{d}p}{\mathrm{d}t} \propto F}$$

$$\Rightarrow F = k\frac{\mathrm{d}p}{\mathrm{d}t}$$

where k is the constant of proportionality. k is taken to be 1 when the quantities used in computation are expressed in SI base units.

Hence:

$$\boxed{F = \frac{\mathrm{d}p}{\mathrm{d}t}}$$

where

F is the resultant force (or net force) acting on the body; and
$\frac{\mathrm{d}p}{\mathrm{d}t}$ is the rate of change of momentum of the body.

- If the effect of forces acting on a body is analysed over a certain time interval Δt, the following equation can be used:

$$\boxed{\langle F \rangle = \frac{\Delta p}{\Delta t}}$$

where

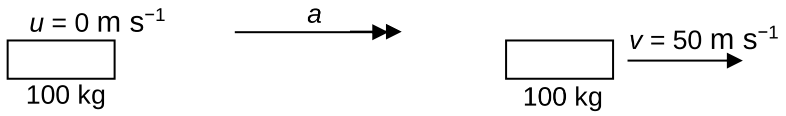
$\langle F \rangle$ is the *average* resultant force (or *average* net force) acting on the body; and
Δp is the change in momentum of the body over the time interval Δt.

Example 1

An object of mass 100 kg accelerates uniformly towards the right from rest to 50 m s^{-1} over a period of 2.0 s.

Calculate
(i) the change in the object's momentum, and
(ii) the net force causing the acceleration.

Solution



Take right as positive:
(i) Change in momentum $\Delta p = mv - mu$

$= 100\,(50) - 100\,(0)$
$= 5000$ kg m s^{-1} (directed towards the right)

(ii) Net force causing the acceleration $= \dfrac{\Delta p}{\Delta t}$

$= \dfrac{5000}{2.0}$

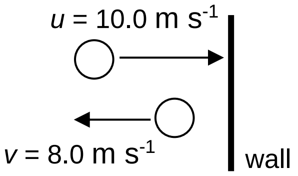
$= 2500$ N (directed towards the right)

Example 2

A ball of mass 0.10 kg hits a smooth vertical wall normally with a speed of 10.0 m s^{-1} and bounces off the wall normally with a speed of 8.0 m s^{-1} after 0.20 s.

Calculate the net force exerted on the ball during the bounce.

Solution



Take left (away from wall) as positive:
Change in momentum of the ball $= mv - mu$

$= 0.10(8.0) - 0.10(-10.0)$

$= 1.8 \text{ kg m s}^{-1}$

From Newton's second law,

net force exerted on the ball during the bounce $= \dfrac{\Delta p}{\Delta t} = \dfrac{1.8}{0.20} = 9.0\text{ N}$

Direction: (away from wall)

Note: The net force acting on the ball is the impact force that the wall exerts on the ball.

1.2.3 Applying Newtons' second law on a body of constant mass

- From Newton's second law of motion,

  resultant force, $F = \dfrac{\text{momentum change}}{\text{time}}$

  $= \text{mass} \times \dfrac{\text{velocity change}}{\text{time}}$

  $= \text{mass} \times \text{acceleration}$

  Therefore $\boxed{F = m\,a}$

  where
  m is the mass of the body; and
  a is the acceleration of the body.

- The magnitude of the resultant force is obtained from $F = m\,a$.

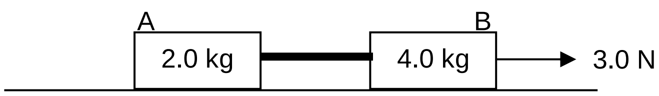
- The direction of the resultant force is the same as that of the acceleration (or momentum change)

- **Steps for determining the acceleration of a constant mass:**

  - Draw a force diagram that shows all the forces acting on the body.
  - Determine the resultant force acting on the body.
  - Use $a = \dfrac{F}{m}$ to determine the acceleration of the body.

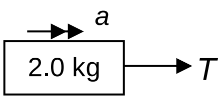
Example 3

Block A of mass 2.0 kg and B of mass 4.0 kg connected by a rope of negligible mass are at rest on a frictionless surface as shown below. If a force of 3.0 N is applied on B, determine the acceleration of A and B.

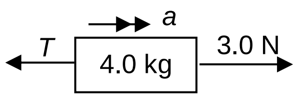


Solution

Force diagram for A,



Force diagram for B,



Take rightwards to be positive:

using $F = ma$

$T = 2.0\,a$ --------- (1)

using $F = ma$

$3.0 - T = 4.0\,a$ --------- (2)

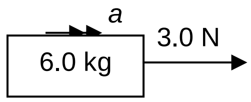
where T is the tension in the rope.

Solving (1) and (2), $T = 1.0$ N

$a = 0.50$ m s^{-2}

Alternatively,

Treating A and B as one body

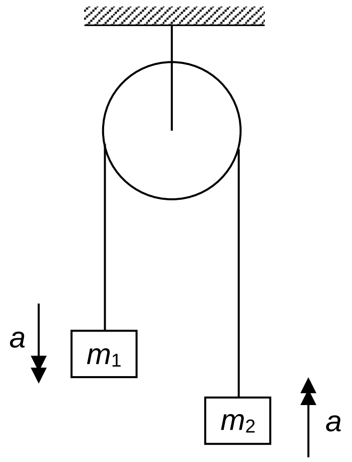


using $F = ma$

$$3.0 = (4.0 + 2.0)\,a$$
$$a = 0.50 \text{ m s}^{-2}$$

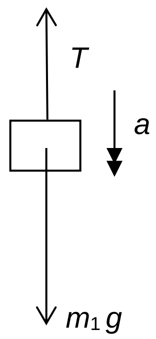
Example 4

For the pulley system shown, determine the acceleration a of the masses m_1 and m_2 ($m_1 > m_2$) and the tension T in the cable in terms of m_1, m_2 and the acceleration of free fall g.

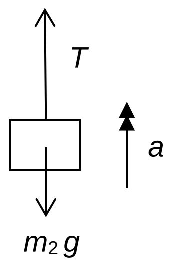


Solution

Force diagram for mass m_1:



Force diagram for mass m_2:



$m_1 g - T = m_1 a$ ---------- (1)

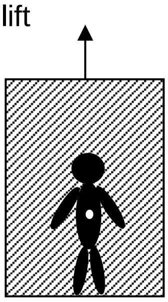
$T - m_2 g = m_2 a$ ---------- (2)

(1) + (2), $$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

Therefore, $$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$

Example 5

Determine the force that the floor of a lift exerts on an 80.0 kg man when the lift

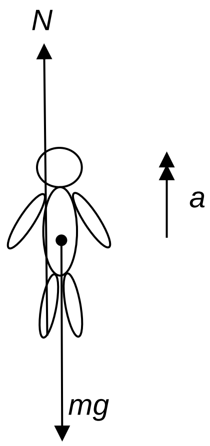


(a) is at rest;
(b) rises with a constant velocity of 2.00 m s^{-1};
(c) rises with a constant acceleration of 2.00 m s^{-2}; and
(d) descends with a constant acceleration of 2.00 m s^{-2}.

(Take g = 9.80 m s^{-2})

Solution

Let N be the force exerted by the lift on the man.
N is also known as the **apparent weight** of the man.



Let a be the acceleration of the system (lift and man).

Taking upwards as positive and by Newton's second law,

$$N - mg = ma$$
$$N = m(a + g)$$

(a) At rest, $a = 0$

$$N = m(0 + g) = mg = (80.0)(9.80) = 784 \text{ N}$$

(b) At constant velocity upwards, $a = 0$

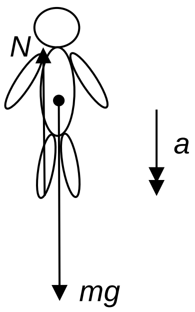
$$\therefore N = mg = 784 \text{ N}$$

(c) At constant acceleration upwards, a = 2.00 m s^{-2}

$$\therefore N = m(a + g) = (80.0)(2.00 + 9.80) = 944 \text{ N}$$

Note:
- Apparent weight N is larger than the true weight mg.
- The man feels 'heavier' as the lift is pushing up on him with a larger force than his true weight.

(d) Taking downwards as positive:



$$mg - N = ma$$
$$N = m(g - a)$$

At constant acceleration downwards, a = 2.00 m s^{-2}

$$\therefore N = m(g - a) = (80.0)(9.80 - 2.00) = 624 \text{ N}$$

Note:
- Apparent weight N is smaller than the true weight mg.
- The man feels 'lighter' as the lift is pushing up on him with a smaller force than his true weight.

1.2.4 Applying Newtons' second law on a <u>system of changing mass</u>

- There are situations where forces are acting on a system that has a <u>changing mass with respect to time</u> and a <u>velocity change by the system due to the application of these forces</u>.

- Examples of **changing mass systems**: fluids; system of particles.

- From Newton's second law of motion,

  resultant force, $F = \dfrac{\text{momentum change}}{\text{time}}$

  $$= \frac{\text{mass}}{\text{time}} \times \text{velocity change}$$

  $$\boxed{F = \Delta v \frac{dm}{dt}}$$

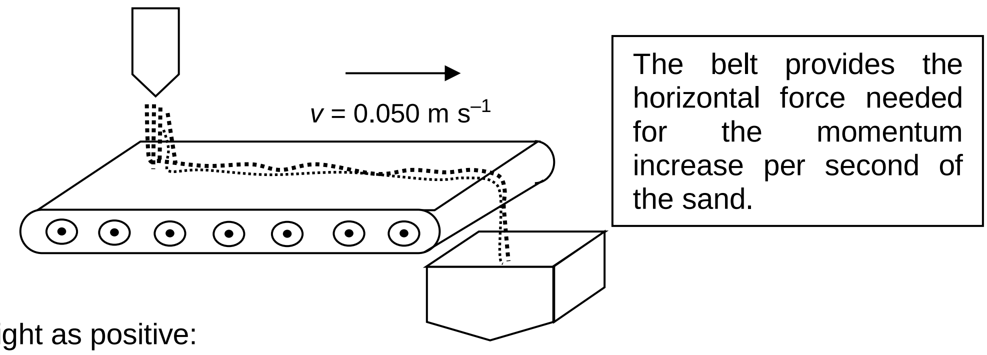
  where

  Δv is the velocity change; and

  $\dfrac{dm}{dt}$ is the rate of change of mass of the system (unit: kg s^{-1}).

- Suppose sand is allowed to fall vertically at a steady rate of 0.10 kg s^{-1} onto a horizontal conveyor belt moving at a constant velocity of 0.050 m s^{-1} as shown below.

  The initial horizontal velocity of the sand is zero.
  The final horizontal velocity of the sand is 0.050 m s^{-1}.



  Take right as positive:
  For every 1 second,
  - mass of sand that has fallen onto the conveyor belt = 0.10 kg
  - velocity change Δv of the sand = 0.050 m s^{-1} (horizontally, rightwards)
  - momentum change Δp of the sand = $m\Delta v = 0.10 \times 0.050 = 0.0050 \text{ kg m s}^{-1}$ (horizontally rightwards)
  - resultant force acting on the sand $= \dfrac{\Delta p}{\Delta t} = \dfrac{0.0050 \text{ kg m s}^{-1}}{1 \text{ s}}$

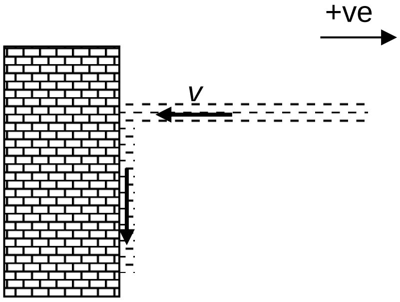
    $= 0.0050 \text{ kg m s}^{-2}$

    $= 0.0050 \text{ N}$ (horizontally rightwards)
  - The belt provides this extra horizontal force needed for the momentum increase per second of the sand.
  - This is an example where the mass changes with time and the velocity gained is constant.

To apply $F = \Delta v \dfrac{dm}{dt}$:

Resultant force acting on the sand = $F = \Delta v \dfrac{dm}{dt} = (0.050 - 0)(0.10) = 0.0050\,\text{N}$

- Suppose a horizontal stream of water hits a vertical wall and loses all its horizontal momentum.



Change in horizontal velocity of water, $\Delta v = 0 - (-v) = v$

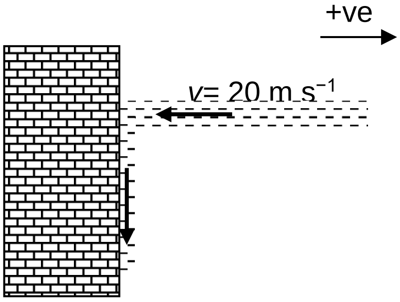
Horizontal force F acting on the water = $\Delta v \dfrac{dm}{dt} = v \dfrac{dm}{dt}$ (in the positive direction)

where $\dfrac{dm}{dt}$ is the <u>mass flow rate</u> of the water (in kg s^{-1}).

<u>Example 6</u>

A hose directs a horizontal jet of water with velocity 20 m s^{-1} onto a vertical wall. The cross-sectional area of the jet is 5×10^{-4} m^2.

If the density of water is 1000 kg m^{-3}, calculate the force acting on the water, assuming the water is brought to rest at the wall.



<u>Solution</u>

Mass flow rate of the water $\dfrac{dm}{dt} = \dfrac{d}{dt}(\rho V) = \rho \dfrac{d}{dt}(As) = \rho A \dfrac{ds}{dt} = \rho A v$

$= (1000)(5 \times 10^{-4})(20) = 10 \text{ kg s}^{-1}$

Force acting on the water $= \Delta v \dfrac{dm}{dt} = [0 - (-20)](10) = 200 \text{ N}$ (acting to the right).

1.2.5 Impulse

- **Impulse is defined as the product of force and time of impact.**

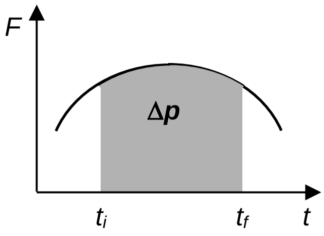
- Recall Newton's second law:

resultant force $F = \dfrac{dp}{dt}$

Integrating both sides give: $dp = F\,dt$

$$\int_{p_i}^{p_f} dp = \int_{t_i}^{t_f} F\,dt$$

$$\boxed{\Delta p = \int_{t_i}^{t_f} F\,dt}$$



where

Δp is the change in momentum of the body over a time interval $t_f - t_i$; and

$\int_{t_i}^{t_f} F\,dt$ is the impulse of the resultant force F acting over the time interval $t_f - t_i$.

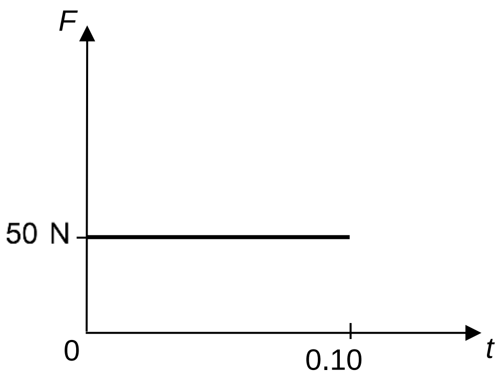
- Impulse is calculated as the integral of force over the time interval during which the force acts and is represented by the **area beneath a force-time graph**. Hence, impulse = $\int_{t_i}^{t_f} F\,dt$. It is also the product of the average net force and the time of impact.

- The impulse acting on an object that is free to move is equal to its **change in momentum.**

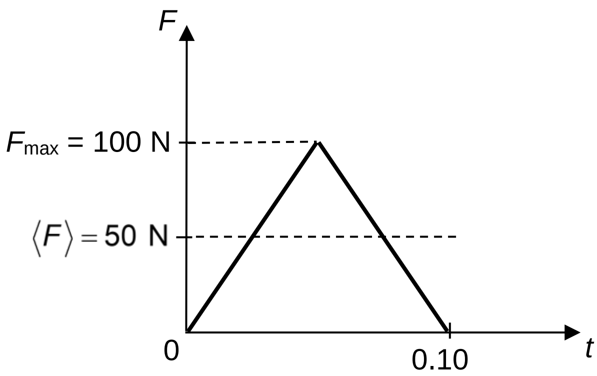
- The unit of impulse is N s or kg m s^{-1}.

- The $F-t$ graphs below show different ways that a resultant force can be applied on a body over a time interval $\Delta t = 0.10$ s.

Graph A shows a constant force of 50 N.
Graph B shows a force increasing from zero to 100 N and then decreasing to zero.



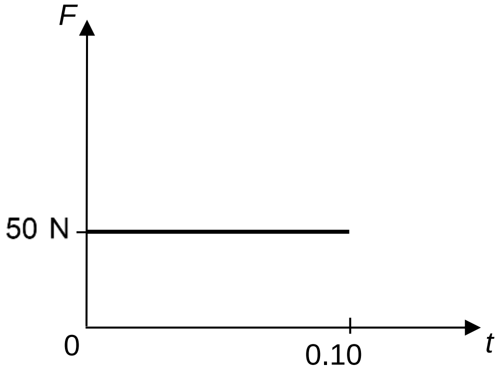
Graph A



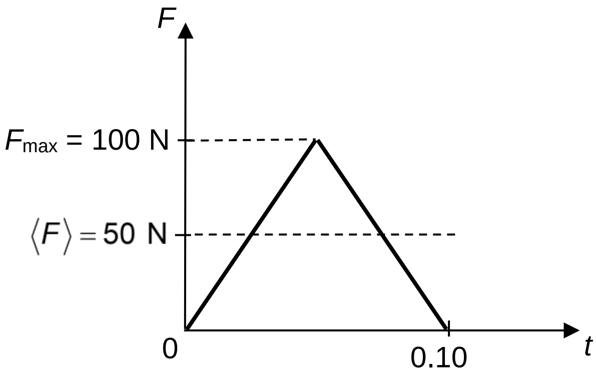
Graph B

- The area under the graph is equal for both graphs. Therefore, the change in momentum of the body is the same for both cases.

- For Graph A, the change in momentum Δp of the body $= 50 \times 0.10 = 5.0 \text{ kg m s}^{-1}$



- For Graph B, the change in momentum Δp of the body
$= \dfrac{1}{2}(100)(0.10) = 5.0 \text{ kg m s}^{-1}$ OR $\langle F \rangle \Delta t = 50 \times 0.10 = 5.0 \text{ kg m s}^{-1}$



Therefore, the change in momentum of a body can be expressed as

$$\Delta p = \langle F \rangle \Delta t$$

where

$\langle F \rangle$ is the ***average*** resultant force acting on the body; and

Δt is the time interval over which the average resultant force is applied.

$\langle F \rangle \Delta t$ is the impulse of the average resultant force $\langle F \rangle$ acting over the time interval Δt .

Example 7

A car of mass 1200 kg is accelerated by a uniform resultant force of 3000 N for a time of 5.0 s. What is the gain in the momentum of the car?

Solution

The resultant force of 3000 N acting on the car is uniform. Therefore the average resultant force $\langle F \rangle$ acting on the car is 3000 N.

Change in momentum of the car $\Delta p = \langle F \rangle \Delta t$

$= 3000 \times 5.0$

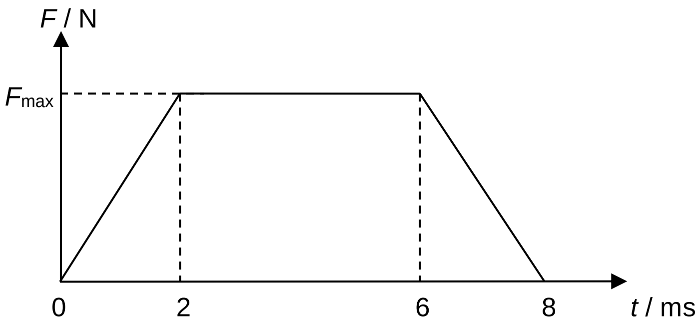
$= 1.5 \times 10^4$ kg m s^{-1}

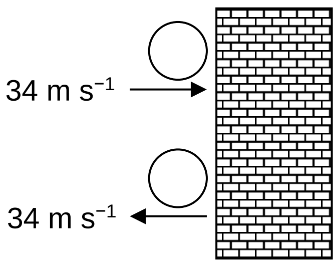
in the same direction as the resultant force

Example 8

The graph shows the variation with time of contact force during the collision of a 58 g tennis ball with a wall. The initial velocity of the ball is 34 m s^{-1} perpendicular to the wall; it rebounds with the same speed, also perpendicular to the wall.

What is F_{max}, the maximum value of the contact force during the collision?





Solution

Taking the direction away from the wall to be the positive direction $\xleftarrow{+ve}$

Change in momentum of ball $\Delta p = m(v_f - v_i) = (0.058)[(34 - (-34)] = 3.94$ kg m s^{-1}

For the F - t graph, $\Delta p = \int F\, dt$

$$3.94 = \frac{1}{2}\left[8 + (6-2)\right] \times 10^{-3}(F_{\max})$$

$$3.94 = 0.006F_{\max}$$

$$F_{\max} = 657 \text{ N} \text{ (i.e. away from the wall)}$$

1.3 Newton's third law of motion

- Forces come in pairs. If a hammer exerts a force on a nail, the nail exerts an equal but opposite force on the hammer. If you lean against a brick wall, the wall pushes back on you.

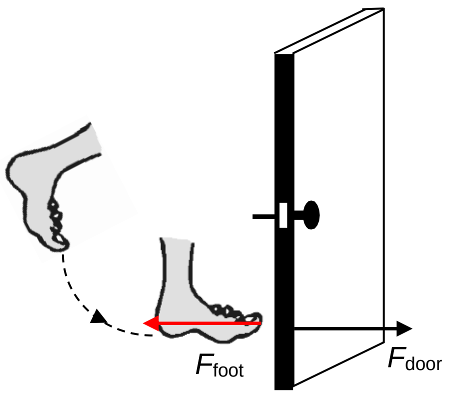
- **Newton's third law of motion** states that:

If body A exerts a force on body B, body B will exert the same type of force of equal magnitude but opposite in direction on body A.

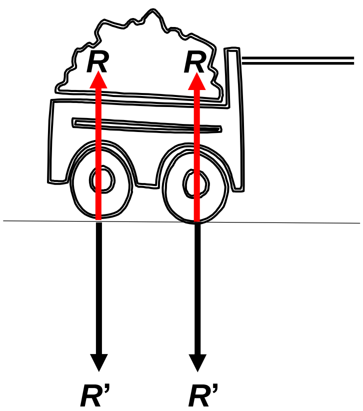
- These two forces are often referred to as an **action-reaction pair.**

  1. They act on two different objects.
  2. They must be of the same type.
     (Same type means that if one is a gravitational force, the other must also be gravitational, or if one is a tension, the other force must also be tensional.)
  3. They have the same magnitude.
  4. They act in opposite directions.
  5. They always exist in pairs, regardless of whether the objects are stationary or moving.

- Examples of action-reaction pairs:



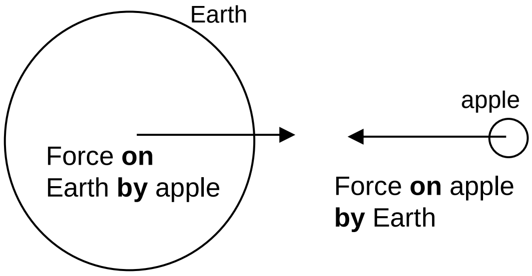
- The force F_{door}, exerted by the boy on the door accelerates the door (it flies open).
- At the same time, the door exerts an equal but opposite force F_{foot}, on the boy, which decelerates the boy (his foot loses forward velocity).
- The boy will be painfully aware of the 'reaction' force to his 'action', particularly if his foot is bare.



R is the force acting on wheel, by ground.
R' is the force acting on ground, by wheel.

R and *R'* is an action-reaction pair.

i) an apple placed in space near the Earth



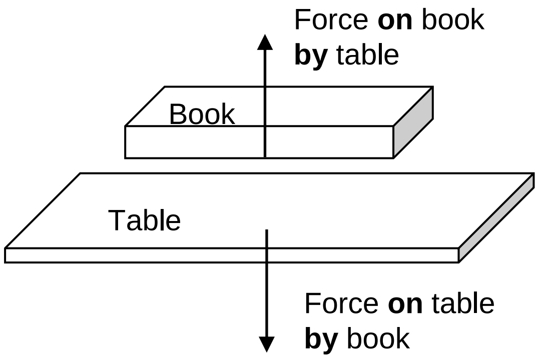
Notes:

The **force on apple by Earth** is the **weight** of the apple.

The apple is actually also attracting the Earth towards itself (**Force on Earth by apple**).

i) a stationary book on a table

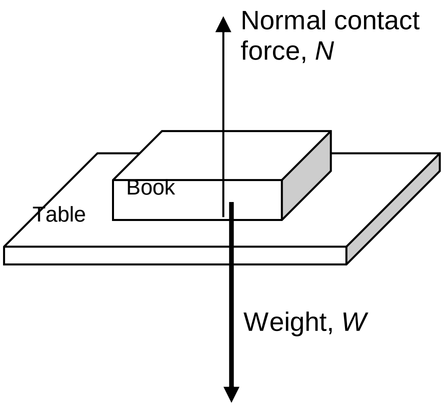
Free body diagram



Notes:

- By replacing the name of the force using ***"force on___ by ___"***, it will be clear which 2 forces are the action-reaction pair.
- The force **on** book **by** table is the force "felt" by the book.
- The force **on** table **by** book is the force "felt" by the table.

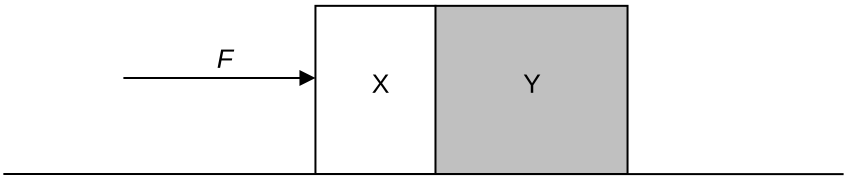
- For a book resting on a table, discuss if the normal contact force, N, and the weight, W, are Newton's 3rd law paired forces.



Are *N* and *W* action-reaction pair?

Example 9

Two blocks X and Y, of masses *m* and 3*m* respectively, are accelerated along a smooth horizontal surface by a force *F* applied to block X as shown.



Determine the force exerted by block X on block Y during this acceleration in terms of *F*?

Solution

Drawing separate force diagrams,

Let F_{XY} be the force exerted by Y on X.
Let F_{YX} be the force exerted by X on Y.

Taking right to be positive direction and applying Newton's second law to X gives:

$$F - F_{XY} = ma \quad \text{-------------- (1)}$$

Applying Newton's second law to Y gives:

$$F_{YX} = 3\,ma \quad \text{-------------- (2)}$$

where *a* is the acceleration of both blocks.

Dividing (2) by (1) yields $\dfrac{F_{YX}}{F - F_{XY}} = 3$

$$F_{YX} = 3F - 3F_{XY}$$

By Newton's third law, $|F_{XY}| = |F_{YX}|$.

$$\text{Hence } 4F_{YX} = 3F$$

$$F_{YX} = 0.75\,F$$

2 Linear Momentum and its Conservation

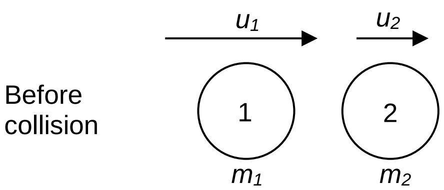
2.1 Principle of conservation of momentum

(h) State the principle of conservation of momentum.
(i) Apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required).
(j) Show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.
(k) Show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

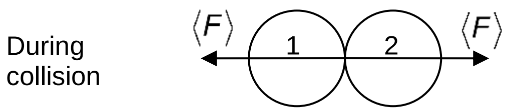
- Applying **Newton's third law** to closed collision systems will lead us to the **principle of conservation of momentum**.

- By a **system**, we mean a set of chosen objects which may interact with each other. A **closed (or isolated) system** is one in which the only (significant) forces are those between the objects in the system.

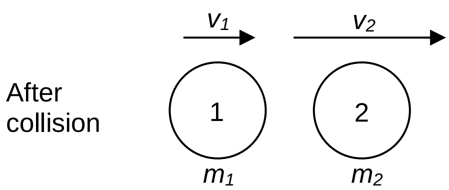
- Consider an object 1, of mass m_1 and velocity u_1, colliding with object 2, of mass m_2 and velocity u_2, moving in the same direction. (u_1 will have to be larger than u_2 in order for them to collide.)



- During the collision, an average force $\langle F \rangle$ is exerted by object 1 on object 2. By Newton's third law, an equal and opposite $\langle F \rangle$ is exerted by object 2 on object 1.



- We consider average force $\langle F \rangle$ in this case because the force may not be constant during the duration of contact.
- In the very short time interval Δt that both objects are in contact with each other, they will experience the same magnitude of impulse $\langle F \rangle \Delta t$ but opposite in direction.
- If object 1 moves with a reduced velocity v_1 after collision, object 2 will move with an increased velocity v_2 :



Taking right to be the positive direction:

Change in momentum of object 1: $-\langle F\rangle \Delta t=\Delta p_1=m_1v_1-m_1u_1$ -------------- (1)

Change in momentum of object 2: $\langle F\rangle \Delta t=\Delta p_2=m_2v_2-m_2u_2$ -------------- (2)

NOTE: Δp_1 is negative; Δp_2 is positive.

Substituting (2) into (1) gives:

$$m_1v_1-m_1u_1=-(m_2v_2-m_2u_2)$$
$$m_1v_1-m_1u_1=-m_2v_2+m_2u_2$$

Therefore $\boxed{m_1u_1+m_2u_2=m_1v_1+m_2v_2}$

Total momentum of the system before collision = Total momentum of the system after collision

- The **principle of conservation of momentum** states that:

The total momentum of a system is constant, provided no external resultant force acts on it.

- The principle of conservation of momentum can also be understood using the following equation:

$$\boxed{\Delta p_1+\Delta p_2=0}$$

- The principle of conservation of momentum is applicable to <u>any system</u> as long as there is no net external force acting on the system.

- The principle of conservation of momentum was proven by assuming that the <u>directions</u> of travel of objects 1 and 2 are the <u>same</u> before and after the collision, but the results are <u>valid</u> even if they were <u>travelling in opposite directions</u>.

- There are two common closed systems within the scope of this syllabus:
  - collisions and
  - disintegration.

2.2 Collisions

There are two types of collisions: elastic collisions and inelastic collisions.

2.2.1 (Perfectly) Elastic Collisions

- (Perfectly) elastic collisions are those in which the <u>**total kinetic energy is conserved**</u>.

- Truly elastic collisions can only occur in practice on an atomic scale i.e. collisions between atoms and molecules.

- By principle of conservation of momentum:

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2 \qquad \text{---- (1)}$$

- By conservation of K.E.:

K.E. of system before collision = K.E. of system after collision

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2 \qquad \text{---- (2)}$$

These two equations can be used to solve problems involving elastic collisions.
From equations (1) and (2), the following equation can be derived (refer to Appendix):

$$\boxed{u_1-u_2=v_2-v_1} \qquad \text{---- (3)}$$

(**NOTE**: that this is a **<u>vector</u>** equation)

which shows that the **relative speed of approach ($u_1 - u_2$) before collision is equal to the relative speed of separation ($v_2 - v_1$) after collision.**

Hence for **<u>(perfectly) elastic collisions</u>,** it is sufficient to use equations (1) and (3), instead of (1) and (2).

2.2.2 Inelastic Collisions

- Inelastic collisions are those in which **<u>total kinetic energy is not conserved</u>**: it may be converted to heat that is dissipated to the surroundings, and to a lesser extent, sound energy.

- In the real world inelastic collisions are the most common type of collision.

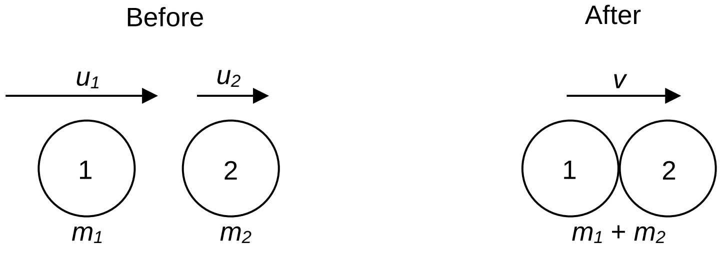
- By principle of conservation of momentum:

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2 \qquad \text{---- (1)}$$

Only equation (1) can be used for inelastic collisions to solve problems.

Equation (2) <u>cannot</u> be used since K.E. is not conserved.

- A special form of inelastic collision called **<u>completely / perfectly inelastic collision</u>** is one in which the **<u>two bodies stick together (or coalesce) after impact</u>** (e.g. a bullet being embedded in a target).



Taking right to be positive:

By principle of conservation of momentum:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

where
v is the **<u>common velocity</u>** after collision.

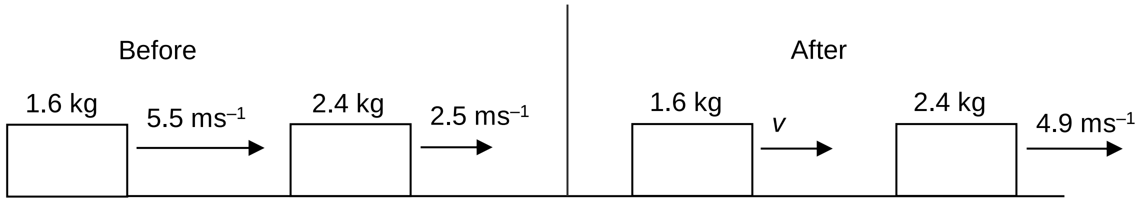
*Note that total momentum of the system is **always** conserved for both (perfectly) elastic and inelastic collisions.

Summarising the information in a table,

<table>
<tr><th>Interaction</th><th>Total Momentum</th><th>Total Kinetic Energy</th><th>Remarks</th></tr>
<tr><td>Elastic /
Perfectly elastic</td><td rowspan="3">conserved</td><td>conserved</td><td>the relative speed of approach
= relative speed of separation.
The objects <u>always separate</u> after interaction</td></tr>
<tr><td>Inelastic</td><td rowspan="2">not conserved
[some KE changed to other forms of energy]</td><td>The objects <u>always separate</u> after interaction</td></tr>
<tr><td>Perfectly inelastic</td><td>The objects <u>stick together (or coalesce)</u> after interaction.</td></tr>
</table>

Example 10

The blocks in the figure below slide without friction.
(a) What is the velocity v of the 1.6 kg block after the collision?
(b) Is the collision elastic?



Solution

By principle of conservation of momentum and taking right as positive:

(a) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(1.6)(5.5) + (2.4)(2.5) = (1.6)v + (2.4)(4.9)$

$v = 1.9 \text{ m s}^{-1}$

(b) K.E. before collision $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}(1.6)(5.5)^2 + \frac{1}{2}(2.4)(2.5)^2 = 31.7 \text{ J}$

K.E. after collision $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(1.6)(1.9)^2 + \frac{1}{2}(2.4)(4.9)^2 = 31.7 \text{ J}$

Since K.E. is conserved, the collision is elastic.

OR

$u_1 - u_2 = 5.5 - 2.5 = 3.0 \text{ m s}^{-1}$

$v_2 - v_1 = 4.9 - 1.9 = 3.0 \text{ m s}^{-1}$

Since relative speed of approach = relative speed of separation, the collision is elastic.

Example 11

Two trolleys X and Y are about to collide. The momentum of each trolley before impact is given in the figure below.

After the collision, the trolleys travel in opposite directions and the momentum of trolley X is 2 kg m s^{-1}. What is the momentum of trolley Y?

Solution

Before: After:

By conservation of linear momentum, taking right as positive:

$20 + (-12) = (-2) + p_y$

$p_y = 10 \text{ kg m s}^{-1}$

The momentum of trolley Y is 10 kg m s^{-1} to the right.

Example 12

A particle of mass m moving with speed u makes a head-on collision with an identical particle which is initially at rest. The particles coalesce (stick together) and move off with a common velocity.

(a) Determine the common speed of the particles after the collision.
(b) Determine the ratio of the kinetic energy of the system after the collision to that before it.
(c) Explain the difference in kinetic energy of the system before and after the collision.

Note: A **head-on** collision is one where the objects remain moving along the same straight line joining their centres of mass before and after collision. **(i.e. motions are collinear).**

Solution

(a) By principle of conservation of momentum and taking right positive,

$$mu + 0 = (m + m)v$$

where v is the common speed after collision.

$$\therefore\ v = 0.5u$$

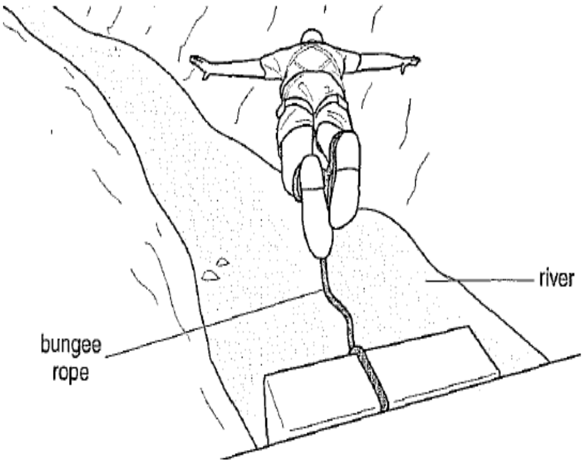
(b) Initial kinetic energy, $E_1 = \frac{1}{2}mu^2$

Final kinetic energy, $E_2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)\left(0.5u\right)^2 = \frac{1}{2}E_1$

$\Rightarrow$ Hence, the ratio of $\frac{E_2}{E_1} = 0.5$

(c) Half of the initial kinetic energy is converted to heat that is dissipated to the surroundings, and to a lesser extent, sound energy.

Example 13



The momentum of the person increases during free fall. Explain whether or not this is a violation of the principle of conservation of momentum.

Solution

This is **not** a violation because total momentum is conserved for a system of interacting bodies, and not just for one body. For the man alone, his momentum will change as there is an external resultant force (i.e. gravitational force) acting on him.

If we consider the man, Earth and bungee rope as the system, then the total momentum remains constant although the man's momentum changes.

As the man falls, the Earth moves upwards with a very tiny velocity (due to its large mass). Hence total momentum remains unchanged.

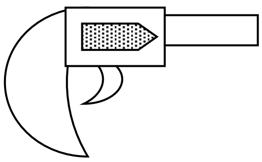
2.3 Disintegration

- Disintegrations are systems that are initially intact (with no external force acting on them) that subsequently disintegrate (break apart) into smaller pieces. The pieces are scattered in different directions. Since no net external force acts on these systems, we can apply the principle of conservation of momentum to such systems.

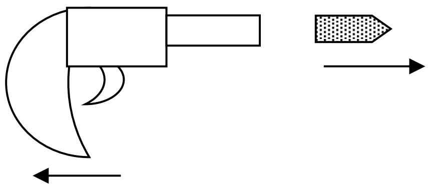
- However, total K.E. is obviously not conserved in this case – the system may start off with no K.E. at all but its fragments certainly possess K.E. upon disintegration. The K.E. may originate from the chemical reactions taking place within the systems or even from the change in mass of the reactants and products as in the case of spontaneous decay in nuclear reactions.

Examples of disintegrations

a. Before a gun is fired, the total momentum of the system, consisting of the gun and the shell is zero. The total momentum must remain at zero and so the gun recoils with a momentum equal and opposite to that of the shell immediately after firing.

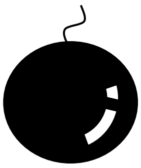


Before firing

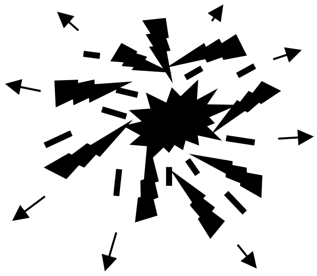


After firing

b. When a bomb explodes in mid-air, the total momentum of all the fragments just after the explosion must equal the total momentum of the complete bomb just before explosion.

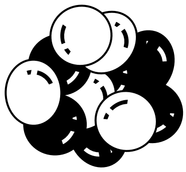


Before explosion

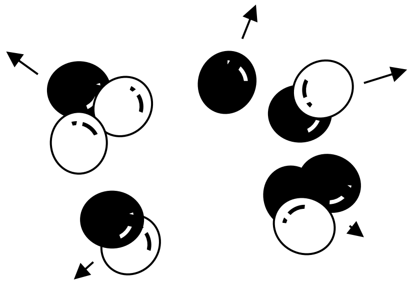


After explosion

c. When a stationary nucleus disintegrates, the total momentum of the resulting nucleus and the emitted particles must add up to zero. Apparent discrepancies led to the discovery of new fundamental particles.



Before disintegration



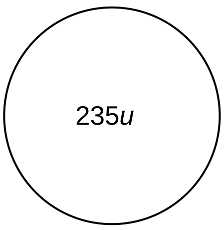
After disintegration

Example 14

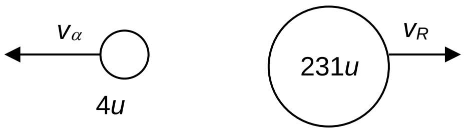
Uranium-235 disintegrates by emitting an alpha particle of mass 4*u* and leaving a residual nucleus of mass 231*u*, where *u* is the atomic mass unit.

Calculate the ratio of the kinetic energy of the alpha particle to that of the residual nucleus in this disintegration.

Before



After



Solution

By conservation of momentum and taking right to be positive:

$$(235u)\,0 = 231u\,(v_R) + 4u\,(-v_\alpha) \qquad \text{----------------} \quad (1)$$

Rearranging gives :

$$\frac{v_\alpha}{v_R} = \frac{231}{4} \qquad \text{----------------} \quad (2)$$

$$\frac{\text{K.E. of alpha particle}}{\text{K.E. of nucleus}} = \frac{\frac{1}{2}(4u)(v_\alpha{}^2)}{\frac{1}{2}(231u)(v_R{}^2)} = \frac{4}{231}\left(\frac{231}{4}\right)^2 = 57.8$$

Applications to Dynamics

What happens when you fire a gun in space?

Assuming an astronaut is floating freely in space, the gun will work just as it does on Earth. By Newton's 2nd law of motion, the bullet will continue moving unless it encounters another object in space. The astronaut will also begin to move, in the opposite direction to the bullet by Newton's 3rd law of motion.

http://www.businessinsider.sg/what-would-happen-fired-gun-space-2016-5/#7Rv7JCedDFqfL2H0.97

Safety features in vehicles

Car crashes are controlled by the laws of physics because moving cars have momentum. Even though cars are designed to crumple up and absorb impacts, it still poses a major risk to the driver and passengers. Cars have had seatbelts for decades, but they're a fairly crude form of protection.

The airbag inflates as soon as the car starts to slow down in an accident and deflates as your head presses against it. For the same change in momentum, the airbag lengthens the time of collision and reduces the average force acting on the head.
http://www.explainthatstuff.com/airbags.html

Skydiving without a parachute

Skydiver Luke Aikins lands safely after jumping 7620 m from an airplane without a parachute or wing suit as part of 'Stride Gum Presets Heaven Sent' on 30 July 2016 in Simi Valley, California. He reached speeds of 193 km h^{-1} (53.6 m s^{-1}) during the two-minute fall.

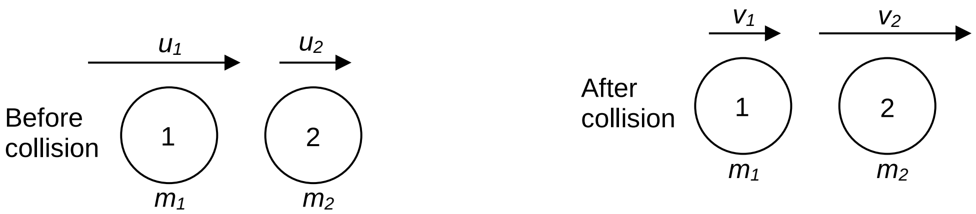
A second before impact, Aikins flipped onto his back. He landed in a polyethylene net measuring 30 x 30 m that was suspended above the ground by four cranes. The net cushioned his impact by increasing the time duration.
http://www.sciencefocus.com/article/physics/how-it-works-skydiving-without-parachute

References:

- *Nelkon & Parker, Advance Level Physics (6th Edition), Heinemann Educational (Pg 3-31)*
- *Halliday,Resnick, Walker, Fundamentals of Physics (Fouth Edition), John Wiley & Sons (Pg 97-130, 236-243, 255-284)*
- *Keith Gibbs, Advanced Physics (Second edition), Cambridge University Press (Pg 42-49)*
- *Douglas C. Giancoli, Physics (Sixth Edition), Pearson Prentice Hall (Pg 72-186)*

<u>Appendix</u>

For an **elastic collision**, show that relative speed of approach = relative speed of separation.



- By the principle of conservation of momentum,

  Momentum of system before collision = Momentum of system after collision

  $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{------- (1)}$$

- For a perfectly elastic collision, the kinetic energy of the system is conserved.

  K.E. of system before collision = K.E. of system after collision

  $$\frac{1}{2}m_1u_1^{\,2} + \frac{1}{2}m_2u_2^{\,2} = \frac{1}{2}m_1v_1^{\,2} + \frac{1}{2}m_2v_2^{\,2} \qquad \text{------- (2)}$$

From (1), $\quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad$ ---- (3)

From (2), $\quad m_1(u_1^{\,2} - v_1^{\,2}) = m_2(v_2^{\,2} - u_2^{\,2}) \qquad$ ---- (4)

Assuming $u_1 \neq v_1$ and $u_2 \neq v_2$, (4) ÷ (3) :

$$u_1 + v_1 = v_2 + u_2$$

$$\Rightarrow u_1 - u_2 = v_2 - v_1$$

which shows that the relative speed of approach $u_1 - u_2$ before collision is equal to the relative speed of separation $v_2 - v_1$ after collision