

1	$z' = \frac{k^2}{z^*}$	
	$\arg(z') = \arg\left(\frac{k^2}{z^*}\right)$ $= \arg(k^2) - \arg(z^*)$ $= 0 - (-\arg(z))$ $= \arg(z)$ <p>Therefore the line PP' passes through the origin O</p>	
	O, P, P' are collinear and O, Q, Q' are collinear. Let $\angle OPQ = \theta$.	
	Area of triangle $OP'Q'$ is 4 times the Area of triangle OPQ .	
	$\frac{1}{2} OP' OQ' \sin \theta = 4 \left(\frac{1}{2} OP OQ \sin \theta \right)$ $\left \frac{k^2}{z^*} \right \left \frac{k^2}{w^*} \right = 4(2)(3)$ <p>Then $\left \frac{k^2}{z} \right \left \frac{k^2}{w} \right = 2^3 \cdot 3$ since $z^* = z$ and $w^* = w$</p> $\frac{k^4}{(2)(3)} = 2^3 \cdot 3$ $k^4 = 2^4 \cdot 3^2$ $k = \pm 2\sqrt{3}$	
2	<p>(a) $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8$ $L_1 = 1, L_2 = 3, L_3 = 4, L_4 = 7, L_5 = 11, L_6 = 18$ $L_5 = 11$ and $F_4 + F_6 = 3 + 8 = 11$ $\therefore L_5 = F_4 + F_6$ (verified)</p> <p>(b) To prove: $F_{m+n} = F_{m-1}F_n + F_mF_{n+1}$, $m \geq 2$ (m fixed), $n \in \mathbb{Z}^+$. For $n = 1$, LHS = F_{m+1}. RHS = $F_{m-1}F_1 + F_mF_2 = F_{m-1} + F_m = F_{m+1} = \text{LHS}$, implying the statement holds for $n = 1$. For $n = 2$, LHS = F_{m+2}.</p>	

	$\begin{aligned} \text{RHS} &= F_{m-1}F_2 + F_mF_3 = F_{m-1} + 2F_m = F_m + (F_{m-1} + F_m) \\ &= F_m + F_{m+1} \\ &= F_{m+2} = \text{LHS}, \end{aligned}$ <p>implying the statement holds for $n = 2$.</p> <p>Assume statement holds for $n = k - 1$ and $n = k$ for some $k \in \mathbb{Z}, k \geq 2$.</p> <p>That is, we have</p> $F_{m+k-1} = F_{m-1}F_{k-1} + F_mF_k \text{ and } F_{m+k} = F_{m-1}F_k + F_mF_{k+1}.$ <p>For $n = k + 1$, $F_{m+k+1} = F_{m+k-1} + F_{m+k}$</p> $\begin{aligned} &= F_{m-1}F_{k-1} + F_mF_k + F_{m-1}F_k + F_mF_{k+1} \text{ by (IH)} \\ &= F_{m-1}(F_{k-1} + F_k) + F_m(F_k + F_{k+1}) \\ &= F_{m-1}F_{k+1} + F_mF_{k+2} \end{aligned}$ <p>implying that statement holds for $n = k + 1$.</p> <p>So $F_{m+n} = F_{m-1}F_n + F_mF_{n+1}$ for a fixed $m \geq 2$ and for all $n \in \mathbb{Z}^+$.</p>	
	<p>(c) Let $m = n$,</p> $\begin{aligned} F_{2n} &= F_{n-1}F_n + F_nF_{n+1} \\ &= F_n(F_{n-1} + F_{n+1}) \\ &= F_nL_n \end{aligned}$	
3	<p>(a)</p> $\begin{aligned} 1 - e^{i\theta} &= e^{\frac{1}{2}i\theta} \left(e^{-\frac{1}{2}i\theta} - e^{\frac{1}{2}i\theta} \right) = e^{\frac{1}{2}i\theta} \left(\cos\left(\frac{1}{2}\theta\right) - i\sin\left(\frac{1}{2}\theta\right) - \left(\cos\left(\frac{1}{2}\theta\right) + i\sin\left(\frac{1}{2}\theta\right) \right) \right) = -2ie^{\frac{1}{2}i\theta} \sin\frac{1}{2}\theta \\ (1 - e^{i\theta})^2 &= -4e^{i\theta} \sin^2\frac{\theta}{2}. \end{aligned}$	
	<p>(b)</p> $\begin{aligned} C + iS &= 1 - \binom{2n}{1}e^{i\theta} + \binom{2n}{2}e^{2i\theta} - \binom{2n}{3}e^{3i\theta} + \dots + e^{2ni\theta} \\ &= (1 - e^{i\theta})^{2n} \\ &= \left(-4e^{i\theta} \sin^2\frac{\theta}{2} \right)^n \\ &= (-4)^n e^{in\theta} \sin^{2n}\frac{\theta}{2} \end{aligned}$	

	<p>Equating real parts, $C = (-4)^n \cos n\theta \sin^{2n} \frac{1}{2} \theta$</p> <p>Equating imaginary parts, $S = (-4)^n \sin n\theta \sin^{2n} \frac{1}{2} \theta$</p>	
	<p>$w^6 = 1 \Rightarrow w = e^{\frac{2k\pi}{6}i} = e^{\frac{k\pi}{3}i}, k = 0, \pm 1, \pm 2, 3.$</p> <p>$(1-w)^6 = (1-e^{i\theta})^6 = \left(-4e^{i\theta} \sin^2 \frac{1}{2} \theta\right)^3 = -64e^{3i\theta} \sin^6 \frac{1}{2} \theta$</p> <p>When $\theta = 0$, $(1-w)^6 = 0$</p> <p>When $\theta = \pm \frac{\pi}{3}$, $(1-w)^6 = -64(-1)\left(\frac{1}{2}\right)^6 = 1$</p> <p>When $\theta = \pm \frac{2\pi}{3}$, $(1-w)^6 = -64(1)\left(\pm \frac{\sqrt{3}}{2}\right)^6 = -27$</p> <p>When $\theta = \pi$, $(1-w)^6 = -64(-1)(1)^6 = 64$</p>	
4	$x_{r+1} = \mathbf{M}x_r \Rightarrow \begin{pmatrix} u_{r+1} \\ u_r \\ u_{r-1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} u_r \\ u_{r-1} \\ u_{r-2} \end{pmatrix}$	
	$\Rightarrow \begin{pmatrix} 4u_r - u_{r-1} - 6u_{r-2} \\ u_r \\ u_{r-1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} u_r \\ u_{r-1} \\ u_{r-2} \end{pmatrix}$	
	<p>ie $\mathbf{M} = \begin{pmatrix} 4 & -1 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$</p>	
	$\begin{pmatrix} 4 & -1 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 4 & -1 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 4 & -1 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 27 \\ 9 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix}$	

	$\therefore \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix}$ are eigenvectors of \mathbf{M} (verified)	
	Take $\mathbf{P} = \begin{pmatrix} 1 & 4 & 9 \\ -1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	
	$\mathbf{M} \begin{pmatrix} 1 & 4 & 9 \\ -1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 9 \\ -1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	
	MP=PD	
	M=PDP⁻¹	
	$\begin{pmatrix} u_{r+1} \\ u_r \\ u_{r-1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} u_r \\ u_{r-1} \\ u_{r-2} \end{pmatrix} = \mathbf{M}^2 \begin{pmatrix} u_{r-1} \\ u_{r-2} \\ u_{r-3} \end{pmatrix} = \dots = \mathbf{M}^{r-2} \begin{pmatrix} u_3 \\ u_2 \\ u_1 \end{pmatrix} = \mathbf{PD}^{r-2}\mathbf{P}^{-1} \begin{pmatrix} u_3 \\ u_2 \\ u_1 \end{pmatrix}$	
	Using GC, $\mathbf{P}^{-1} = \begin{pmatrix} \frac{1}{12} & -\frac{5}{12} & \frac{1}{2} \\ -\frac{1}{3} & \frac{2}{3} & 1 \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{2} \end{pmatrix}$ $\begin{pmatrix} u_{r+1} \\ u_r \\ u_{r-1} \end{pmatrix} = \begin{pmatrix} 1 & 4 & 9 \\ -1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^{r-2} & 0 & 0 \\ 0 & 2^{r-2} & 0 \\ 0 & 0 & 3^{r-2} \end{pmatrix} \begin{pmatrix} \frac{1}{12} & -\frac{5}{12} & \frac{1}{2} \\ -\frac{1}{3} & \frac{2}{3} & 1 \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -7 \end{pmatrix}$	

	$= \begin{pmatrix} 1 & 4 & 9 \\ -1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^{r-2} & 0 & 0 \\ 0 & 2^{r-2} & 0 \\ 0 & 0 & 3^{r-2} \end{pmatrix} \begin{pmatrix} -4 \\ -6 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 1 & 4 & 9 \\ -1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 4(-1)^{r-1} \\ -3 \cdot 2^{r-1} \\ 3^{r-1} \end{pmatrix}$ $= \begin{pmatrix} 4(-1)^{r-1} - 3 \cdot 2^{r+1} + 3^{r+1} \\ 4(-1)^r - 3 \cdot 2^r + 3^r \\ 4(-1)^{r-1} - 3 \cdot 2^{r-1} + 3^{r-1} \end{pmatrix}$	
	$\Rightarrow u_r = 4(-1)^r - 3 \cdot 2^r + 3^r$ for $r \geq 1$.	
5(i)	$f(0.5) = -0.21372 < 0$	
	$f(1) = 0.84147 > 0$	
	Hence, there is a change in sign in the interval $(0.5, 1)$.	
	$f'(x) = \cos x + \frac{1}{x}$	
	For $0.5 \leq x \leq 1$, $\cos x > 0$ and $\frac{1}{x} > 0 \Rightarrow f'(x) > 0$	
	Hence, f is a strictly increasing function. Therefore there is exactly one root in the interval $(0.5, 1)$.	
(ii)	$x_1 = \frac{\frac{1}{2} f(1) + \left f\left(\frac{1}{2}\right)\right }{ f(1) + \left f\left(\frac{1}{2}\right)\right }$	
	$= \frac{\frac{1}{2}(0.21372) + 0.84147}{0.84147 + 0.21372} = 0.601$ correct to 3 s.f.	
	$f''(x) = -\sin x - \frac{1}{x^2} = -\left(\sin x + \frac{1}{x^2}\right)$	
	For $0.5 \leq x \leq 1$, $\sin x > 0$ and since $\frac{1}{x^2} > 0$ for all real x ,	
	$f''(x) < 0 \Rightarrow$ curve is concave downwards and from (i), f is a strictly increasing function, the estimate is an overestimation.	
(iii)	Since 0.601 is an overestimate of the root, the root lies outside of the interval $(0.601, 1)$ and hence it is unwise to do so.	

	<u>Alternative Method</u>	
	$f(0.601) = 0.056307 > 0$	
	$f(1) = 0.84147 > 0$	
	Hence, there is no sign change in the interval $(0.601, 1)$.	
	Therefore the root does not lie in this interval and hence it is unwise to do so.	
(iv)	Let $F(x) = \sin^{-1}\left(\ln \frac{1}{x}\right)$	
	$F'(x) = \frac{1}{\sqrt{1 - \left(\ln \frac{1}{x}\right)^2}} \left(-\frac{1}{x^2} \right) = -\frac{1}{x\sqrt{1 - (\ln x)^2}}$	
	$x = 0.5, F'(x) = \left -\frac{1}{0.5\sqrt{1 - (\ln 0.5)^2}} \right \approx 2.77 > 1$	
	Therefore the iterative formula $x_{n+1} = \sin^{-1}\left(\ln \frac{1}{x}\right)$ may not work.	
(v)	Using $x_{n+1} = e^{-\sin x_n}$,	
	$x_1 = e^{-\sin 0.5} \approx 0.61914$	
	$x_2 \approx 0.55971$	
	$x_3 \approx 0.58805$	
	$x_4 \approx 0.57422$	
	$x_5 \approx 0.58090$	
	$x_6 \approx 0.57766$	
	$\therefore x \approx 0.58$	
	Consider $f(x) = \sin x + \ln x = \sin x + \ln(1 + (x-1))$	
	$\approx x + x - 1 = 2x - 1$	
	$f(x) = 0 \Rightarrow 2x - 1 \approx 0 \Rightarrow x \approx 0.5$	
	Hence, $x = 0.5$ is an appropriate initial estimate.	
	<u>Alternative method</u>	
	$x = e^{-\sin x} \approx e^{-x} \Rightarrow x \approx 1 - x$	
	$\Rightarrow x \approx 0.5$	
	Hence, $x = 0.5$ is an appropriate initial estimate.	

	Section B Statistics [50 marks]	
6(i)	$F(t) = \int_0^t \frac{\pi}{2} \sin(\pi x) \, dx$	
	$= \left[-\frac{1}{2} \cos(\pi x) \right]_0^t$	
	$= \frac{1}{2} [1 - \cos(\pi t)]$	
	$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2} [1 - \cos(\pi x)], & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$	
(ii)	$P(Y \leq y) = P\left(\frac{1}{2}(1 - \cos(\pi X)) \leq y\right)$	
	$= P(\cos(\pi X) \geq 1 - 2y)$	
	$= P\left(X \leq \frac{1}{\pi} \cos^{-1}(1 - 2y)\right)$	
	$= \frac{1}{2} \left[1 - \cos\left(\pi \left(\frac{1}{\pi} \cos^{-1}(1 - 2y)\right)\right) \right]$	
	$= \frac{1}{2} [1 - \cos(\cos^{-1}(1 - 2y))]$	
	$= \frac{1}{2} [1 - (1 - 2y)]$	
	$= \frac{1}{2} [2y] = y$	
	$f(y) = \frac{d}{dy} [F(y)]$	
	$= \frac{d}{dy} [y] = 1$	
	$0 \leq x \leq 1 \Rightarrow 0 \leq y \leq 1$	
	$\therefore Y \sim U(0,1)$, shown	
	Alternative Method	
	$\int_0^1 \frac{\pi}{2} \sin(\pi x) \, dx = 1$	
	$y = \frac{1}{2}(1 - \cos(\pi x)) \Rightarrow 1 = \frac{\pi}{2} \sin(\pi x) \frac{dx}{dy} \Rightarrow \frac{dx}{dy} = \frac{2}{\pi \sin(\pi x)}$	
	$x=0, y=0$ and $x=1, y=1$	
	$\int_0^1 \frac{\pi}{2} \sin(\pi x) \left(\frac{2}{\pi \sin(\pi x)} \right) dy = 1$	

	$\int_0^1 1 \, dy = 1 \Rightarrow f(y) = 1$	
	$\therefore Y \sim U(0,1)$, shown	

7	Let X and Y be the times taken to complete the test before and after drinking 2 cans of beer respectively.	
	Let $D = Y - X$ and μ_d be the population mean of D .	
	Assume that D follows a normal distribution.	
	$H_0 : \mu_d = 0$ $H_1 : \mu_d > 0$	
	From GC, $\bar{d} = 0.195, s^2 = 0.0383947368$	
	Under H_0 , $T = \frac{\bar{D}}{\frac{S}{\sqrt{20}}} \sim t(19)$	
	Using GC, $p\text{-value} = 0.000137$	
	Since $p\text{-value}$ is very small, H_0 will be rejected even at very small level of significance i.e. 0.02%. There is very significant evidence that drinking 2 cans of beer impairs reaction time.	

8	(a) H_0 : There is no difference between Programme A and Programme B H_1 : Programme A is better than Programme B	
	Let X be the number of better performers from programme A out of 12 pairs. Under H_0 , $X \sim B\left(12, \frac{1}{2}\right)$	
	From the result, $x = 9$ Using GC, $P(S \geq 9) = 1 - P(S \leq 8) = 0.073 > 0.05$	
	We do not reject H_0 and conclude that there is insufficient evidence at 5% significance level that programme A is a better training programme than programme B.	

	(b) With the additional information, we are able to rank the pairing based on the absolute difference of their scores as follows:																																								
	<table><tr><td>Pairing</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr><tr><td>Rankings</td><td>8</td><td>5</td><td>9</td><td>10</td><td>12</td><td>2</td><td>4</td><td>7</td><td>11</td><td>1</td><td>3</td><td>6</td></tr><tr><td>Sign of the differences</td><td>+</td><td>+</td><td>-</td><td>+</td><td>+</td><td>+</td><td>-</td><td>+</td><td>+</td><td>-</td><td>+</td><td>+</td></tr></table>	Pairing	1	2	3	4	5	6	7	8	9	10	11	12	Rankings	8	5	9	10	12	2	4	7	11	1	3	6	Sign of the differences	+	+	-	+	+	+	-	+	+	-	+	+	
Pairing	1	2	3	4	5	6	7	8	9	10	11	12																													
Rankings	8	5	9	10	12	2	4	7	11	1	3	6																													
Sign of the differences	+	+	-	+	+	+	-	+	+	-	+	+																													
	<p>Q = sum of the negative ranks = $9 + 4 + 1 = 14$ P = sum of the positive ranks = 64</p> <p>$T = \min (P, Q) = 14$</p>																																								
	<p>Checked Wilcoxon table in MF26, For $n = 12$, 1 tailed test at 5% sig level, critical region is $T \leq 17$</p>																																								
	<p>Since the observed value of T is 14, which lies in the rejection region, hence we reject H_0 , there is sufficient evidence at 5% significance level to conclude that programme A is a better training programme than programme B.</p>																																								

9(i)	Let \hat{p}_e be the sample proportion for the prizes 'caught' by the experienced players.	
	Width of 95% CI $= 2(1.95996)\sqrt{\frac{\hat{p}_e(1-\hat{p}_e)}{200}}$	
	$= 2(1.95996)\sqrt{\frac{\frac{1}{4} - \left(\hat{p}_e - \frac{1}{2}\right)^2}{200}}$	
	Maximum value of $\frac{1}{4} - \left(\hat{p}_e - \frac{1}{2}\right)^2$ is $\frac{1}{4}$	
	\Rightarrow largest possible width $= 2(1.95996)\sqrt{\frac{\frac{1}{4}}{200}} = 0.1386$, correct to 4 decimal places	
	$\frac{m}{200} = \frac{1}{2} \Rightarrow m = 100$	
(ii)	$\frac{m}{200} = \frac{0.2365 + 0.3635}{2}$	
	$m = 60$	

(iii)	Let \hat{p}_n be the sample proportion for the prizes ‘caught’ by the novice players.	
	95% CI for p_n is $\hat{p}_n \pm z \sqrt{\frac{\hat{p}_n(1-\hat{p}_n)}{n}} = 0.225 \pm 1.95996 \sqrt{\frac{(0.225)(0.775)}{200}}$	
	$= (0.225 - 0.057873, 0.225 + 0.057873)$	
	$= (0.167127, 0.282873)$	
	$= (0.1671, 0.2829)$	
	We are 95% confident that the confidence interval will contain p_n . OR 95% of the confidence intervals constructed will contain p_n .	
(iv)	It is necessary to assume the probability that a novice player will ‘catch’ a prize is constant for every novice player and the probability that an experienced player will ‘catch’ a prize is constant for every experienced player.	
(v)	The two confidence intervals have some overlap, so it means that there is insufficient evidence to support the claim as the true value of p_e may be lower than that for the true value of p_n .	

10	<p>(i) $n_1 + n_2 + n_3 + n_4 = N$ If A and B are independent, then $P(A B) = P(A)$</p> $\frac{n_1}{n_1 + n_2} = \frac{n_1 + n_3}{n_1 + n_2 + n_3 + n_4}$ $n_1(n_1 + n_2 + n_3 + n_4) = (n_1 + n_2)(n_1 + n_3)$ $n_1^2 + n_1n_2 + n_1n_3 + n_1n_4 = n_1^2 + n_1n_2 + n_1n_3 + n_2n_3$ $n_1n_4 = n_2n_3$	
	<p>(ii) It is incorrect to deduce from the fact that $48 \times 10 \neq 34 \times 8$ that the quality and shift are not independent because the above is just a sample whereas the result $n_1n_4 = n_2n_3$ is deduced from the population.</p>	

(iii)

		Shift		
		Day	Night	
Quality	Good	$48 - m$	$34 + m$	82
	Defective	$8 + m$	$10 - m$	18
		56	44	100

H_0 : Quality and shift are independent

H_1 : Quality and shift are not independent

Under H_0 , Expected freq = $\frac{(\text{Row total})(\text{Column total})}{\text{Grand total}}$

Observed Freq O_i	Expected Freq E_i
$48 - m$	$\frac{82 \times 56}{100} = 45.92$
$34 + m$	36.08
$8 + m$	10.08
$10 - m$	7.92

At 10% level of significance, reject H_0 if $\chi^2_{calc} > \chi^2_{10\%}(1) = 2.706$

$$\chi^2_{calc} = \sum_{i=1}^4 \frac{(O_i - E_i)^2}{E_i}$$
$$= \frac{(48 - m - 45.92)^2}{45.92} + \frac{(34 + m - 36.08)^2}{36.08} + \frac{(8 + m - 10.08)^2}{10.08} + \frac{(10 - m - 7.92)^2}{7.92} > 2.706$$

$$\frac{(2.08 - m)^2}{45.92} + \frac{(2.08 - m)^2}{36.08} + \frac{(2.08 - m)^2}{10.08} + \frac{(2.08 - m)^2}{7.92} > 2.706$$

$$(2.08 - m)^2 (0.275) > 2.706$$

$$|2.08 - m| > 3.137$$

$$2.08 - m > 3.137 \quad \text{or} \quad 2.08 - m < -3.137$$

$$m < -1.057 \text{ (rejected) or } m > 5.217$$

$$\therefore 6 \leq m \leq 10$$

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11(i)	The average rate at which the customers enter the store is a constant throughout the hour.	
	A customer entering the store is independent of any other customer entering the store.	
(ii)	Let X be the random variable denoting the number of customers entering the store in an hour. $X \sim \text{Po}(\lambda)$	
	$P(X = m) \geq P(X = m-1)$	
	$\frac{\lambda^m e^{-\lambda}}{m!} \geq \frac{\lambda^{m-1} e^{-\lambda}}{(m-1)!}$	
	$\frac{\lambda}{m} \geq 1 \Rightarrow m \leq \lambda$	
	$P(X = m) \geq P(X = m+1)$	
	$\frac{\lambda^m e^{-\lambda}}{m!} \geq \frac{\lambda^{m+1} e^{-\lambda}}{(m+1)!}$	
	$\frac{\lambda}{m+1} \leq 1 \Rightarrow m \geq \lambda - 1$	
	$\therefore \lambda - 1 \leq m \leq \lambda$	
	if λ is an integer, $m = \lambda, \lambda - 1$	
(iii)	$P(X = m) \geq 0.2$	
	$\frac{\lambda^m e^{-\lambda}}{m!} \geq 0.2$	
	if λ is an integer, $\frac{\lambda^\lambda e^{-\lambda}}{\lambda!} \geq 0.2$ or $\frac{\lambda^{\lambda-1} e^{-\lambda}}{(\lambda-1)!} \geq 0.2$	
	Using GC, $\lambda \leq 3$, where $\lambda \in \mathbb{Z}^+$	
(iv)	Let Y be the random variable denoting the time elapsed (in hours) until a new customer enters the store.	
	$Y \sim \text{Exp}(3)$	
	$P(Y > 5 Y > 2) = P(Y > 3)$	
	$= e^{-9}$ or 1.23×10^{-4}	
	Alternative Method	
	$X_1 + X_2 + \dots + X_5 \sim \text{Po}(15)$, $X_1 + X_2 \sim \text{Po}(6)$	
	$P(X_1 + X_2 + \dots + X_5 = 0 X_1 + X_2 = 0)$	

	$= \frac{P(X_1 + X_2 + \dots + X_5 = 0)}{P(X_1 + X_2 = 0)}$	
	$= 1.23 \times 10^{-4}$	