Chapter 5 WORK, ENERGY AND POWER

Content

- Work
- Energy conversion and conservation
- Potential energy and kinetic energy
- Power

Learning Outcomes

Candidates should be able to:

(a)	show an understanding of the concept of work in terms of the product of a force and displacement in the direction of the force.		
(b)	calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = p \Delta V$.		
(c)	give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation.		
(d)	show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems.		
(e)	derive, from the equations for uniformly accelerated motion in a straight line,		
	the equation $E_k = \frac{1}{2}mv^2$.		
(f)	recall and use the equation $E_k = \frac{1}{2}mv^2$.		
(g)	distinguish between gravitational potential energy, electric potential energy and elastic potential energy		
(h)	deduce that the elastic potential energy in a deformed material is related to the area under the force-extension graph.		
(i)	show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems.		
(j)	derive, from the definition of work done by a force, the equation $E_p = mgh$ for		
	gravitational potential energy changes near the Earth's surface.		
(k)	recall and use the formula $E_{p} = mgh$ for gravitational potential energy changes		
	near the Earth's surface.		
(I)	define power as work done per unit time and derive power as the product of force and velocity in the direction of the force.		

5.1 WORK

The way we use the term "work" in physics is different from the way it is used in everyday life. Work is done by a force on a body when the force causes a displacement of the body in the direction of the force.

✓ Force exerted	✓ Force exerted	Force exerted
Weights displaced by force	X Wall not displaced by force	√ Wall displaced
√ Work is done	X No work done	√ Work is done

5.1.1 WORK DONE BY A CONSTANT FORCE



Note: The component of the force perpendicular to displacement, $F \sin \theta$, does not do any work.

Calculate the work done by the 10 N force in each case.



Work done by a force can be positive or negative. The significance of the sign for work done will be discussed in **5.2** Work Done and Energy.

5.1.2 WORK DONE BY A VARYING FORCE

Determining work done when force is not constant

If a body is displaced by a **varying force**, the work done by the force on the body is given by

$$W = \int F_{\rm s} ds$$

Where F_s is the component of the force in the direction of displacement.

Graphical representation

Graphically, the work done is given by the area under the F-s graph.



Example 2

Calculate the work done in each case.



5.1.3 WORK DONE BY AN EXPANDING GAS

Consider a gas enclosed in a chamber of cross sectional area *A* and fitted with a light frictionless piston.

If the gas expands at constant pressure p, such that the volume changes by an amount ΔV , the force exerted by the gas on the piston, F_{gp} , is

$$F_{gp} = pA$$

and the displacement of the piston, Δx , is given by

$$\Delta x = \frac{\Delta V}{A}$$

The work done by the gas on the piston, W_{gp} , is

$$W_{gp} = F_{gp} x$$
$$= (pA)(\frac{\Delta V}{A})$$

Hence,

 $W_{ap} = p\Delta V$

Determining work done by gas

If the pressure is not constant, the work done by the gas is given by:

 $W_{ap} = \int p \, dV =$ Area under p - V graph

Graphical representation

Graphically, W_{gp} is given by the area under the p-V graph.



Note: as the gas does positive work on the piston in expanding, the atmosphere is doing negative work on the piston.

5.1.4 WORK DONE AND ENERGY CHANGE

When work is done, energy is converted from one form to another as a result.

- <i>W</i> , negative work done
• Force is exerted on a body opposite to the direction of its motion (e.g. friction)
 Negative work is done on the body
Body loses energy

When body A does 100 J of work on body B, A loses 100 J of energy and B gains 100 J of energy. When body A does –100 J of work on body B, B loses 100 J of energy and A gains 100 J of energy.





Determine whether positive or negative work was done in each case.

Superman exerts a force of 3.0×10^4 N on a passenger jet to tow it down a runway at a *constant speed*. What is the work done on the passenger jet when Superman tows it a distance of 10 m?





5.2 ENERGY

Energy is an indirectly observed property of a body; that means it cannot be *directly measured* with any instrument, and must be determined from other measurements. It is the ability or capacity of the body to do work on another body (causing it to lose energy in the process).

Energy is a scalar quantity with no associated direction. Bodies in a system may possess the following types of energy.

5.2.1 POTENTIAL ENERGY

A body possesses potential energy when it has the potential to do work. Potential energy can be either positive or negative; the point of zero potential energy is used as a reference and depends on the definition of the specific type of potential energy.

Hence, when analysing the change in energy of a body or system between two points, it is often more useful to talk about the **change** in potential energy rather than the value of potential energy.

5.2.1.1 ELASTIC POTENTIAL ENERGY

When work is done by a force on an elastic body, it deforms and its shape changes. When the force is released, the deformation is restored. This allows the object to store **elastic potential energy** as a result of its **deformation**.

In a spring, the change in length caused by a force is known as the extension. Both compression and extension of the spring are able to store elastic potential energy. The elastic potential energy is given by

Elastic
$$E_{p}$$
 = Area under $F - x$ graph
= $\frac{1}{2}Fx = \frac{1}{2}kx^{2}$

(More detail can be found in Chapter 4 Forces.)

Example 4

Shade the area which represents the work done on the spring in each case.



5.2.1.2 GRAVITATIONAL POTENTIAL ENERGY NEAR SURFACE $(E_p = mgh)$

For a body to gain gravitational potential energy but not kinetic energy, it must move away from Earth (i.e. upwards) with a constant speed (i.e. zero acceleration). An **external agent** is required to exert an upward force on the body, and this force must be equal to weight (so that net force is zero).

$$F_{\text{ext}} - mg = 0$$

Near the Earth's surface, the gravitational field strength is **constant** at *g*, hence the body's weight and the external force F_{ext} are constant. If the body is lifted through a vertical displacement *h*, the work done on the body by the external force F_{ext} increases the gravitational potential energy of the body

$$\Delta E_{p} = W_{ext}$$
$$= F_{ext}h$$

Since $F_{\text{ext}} = mg$, $\Delta E_{\rho} = mgh$

Calculating gravitational potential energy

The gravitational potential energy of a body with mass m at a height h above a reference level is

gravitational
$$E_p = mgh$$

5.2.2 KINETIC ENERGY ($E_k = \frac{1}{2}mv^2$)

A body in motion possesses kinetic energy. We cannot observe kinetic energy directly, but we can observe the effects of gain or loss in kinetic energy.

Consider a body of mass *m* being accelerated **from rest** by a constant net force *F* on frictionless level ground. The work done by net force *F* on the body causes its kinetic energy E_k to increase. When the body has been displaced by a horizontal distance *s*,

 $\Delta E_{k} = Fs$ Since F = ma (according to Newton's Second Law), $\Delta E_{k} = mas$

Since net force F is constant and hence acceleration a is constant, we can substitute the term as using an equation of linear motion:

$$v^2 = u^2 + 2as$$

 $as = \frac{1}{2}v^2 - \frac{1}{2}u^2$



constant a



Thus, we can relate the change in kinetic energy ΔE_k to the initial velocity *u* and the final velocity *v*.

$$\Delta E_k = m \left(\frac{1}{2} v^2 - \frac{1}{2} u^2 \right)$$

The body was accelerated from rest (*u* = 0), hence the kinetic energy possessed by the body is given by $\Delta E_k = \frac{1}{2}mv^2$.

Calculating kinetic energy

The kinetic energy of a body with mass m travelling with velocity v is given by

 $E_k = \frac{1}{2}mv^2$

5.3 CONSERVATION OF ENERGY

Although energy can be converted from one form to another, the total amount of energy in the universe remains constant.

Principle of Conservation of Energy

Energy can neither be created nor destroyed, but it can be transformed from one form to another.

Because of this principle of nature, even though energy can be transferred in and out of a system, we can account for the total amount of energy remaining in the system.

System Model

Rather than consider the entire universe in our calculations, we typically consider energy changes within a *system*. A valid system may be

- a single body or particle,
- a collection of bodies or particles,
- all the matter within a fixed region of space (such as a cylinder that is open to the surroundings), or
- a body that breaks up into smaller parts.

Identifying the system allows us to check if external forces are present, and hence to determine if the total energy of the system is conserved.

Conversion of Energy and Conservation of Energy

Difference between Isolated System and Closed System

Consider a simple system which only has kinetic energy E_k and potential energy E_p .

If the system does not exchange energy or matter with its surroundings, it is an **isolated system**. The total energy possessed by the system remains unchanged if no **external work is done** on the system. Within the system, an increase in kinetic energy must result in an equal decrease in potential energy, and vice versa. We say that **the total energy of an isolated system is conserved**.

If the system is able to exchange energy *but not matter* with its surroundings, it is a **closed system**. Energy can be transferred via work done on the system, thus the total energy of a closed system need not be conserved.



Note: Transfer of energy can take place via a few processes: work done on the system, heat transfer into or out of the system, mechanical waves passing through the system, electrical transmission, and electromagnetic radiation. We only focus on work done on the system here.

Since energy is a scalar quantity, and the energy of a system can be determined, we can write an equation to relate the initial and final energy of the system.

Determining initial/final energy of a system					
Total initial energy of system + Net work done on system = Total final energy of system (potential + kinetic) (positive or negative) (potential + kinetic)					
OR					
Total $E_i + W_{net} = Total E_f$					
Determining change in total energy of a system					
$W_{net} = Total E_{f} - Total E_{i}$ $\Delta E_{total} = W_{net}$					
OR					
Change in total energy of system (final – initial) = Net work done on system (positive or negative)					
Problem Solving Skills Set (PS ³) Steps in determining change in energy of a system					
 Determine what constitutes the system (which bodies are part of the system?) Determine the initial state of the system Determine the final state of the system 					

4. Form an equation to relate the initial and final total energy $(E_p + E_k)$ of the system

Write the energy equation for each of the following scenarios.



A 2000 kg car coasts up a hill (without additional force) at A with a speed of 20 m s⁻¹. Its speed when it reaches B is 5.0 m s⁻¹ and the distance along the road from A to B is 40 m. Determine the average resistive force *f* along the road from A to B.

By conservation of energy:

 $\frac{1}{2}mv_i^2 + 0 - f(d) = \frac{1}{2}mv_f^2 + mgh$ $\frac{1}{2}(2000)(20^2) - f(40) = \frac{1}{2}(2000)(5^2) + (2000)(9.81)(10)$ 178800 = f(40)f = 4500 N (2 s.f.)



Example 6

M and N, two objects of mass 1.0 kg and 0.60 kg respectively, are connected by a light cord passing over a light, free-running pulley. From rest, N descends vertically while M moves over a smooth plane inclined at 30° to the horizontal.

Determine the final velocity of both objects when M has travelled a distance d = 2.0 m along the plane.

$$m_M g d \sin \theta - m_N g d + \frac{1}{2} (m_M + m_N) (v_f^2 - v_i^2) = 0$$
(1.0)(9.81)(2.0 \sin 30°) - (0.60)(9.81)(2.0) + $\frac{1}{2} (1.0 + 0.60) (v_f^2 - 0) = 0$

 $0.80v_f^2 = 1.962, v_f = 1.57 \text{ m s}^{-1}$



5.4 ENERGY LOSS AND EFFICIENCY

In the absence of any energy loss, the total input energy to a device is completely converted to useful work done. However, in practice, the useful energy output from the device is always less than the energy input – the difference is **energy loss**. We say that this energy is *lost* (not destroyed) to the environment because it did not do any useful work for us.

Definition of efficiency					
The energy efficiency η of a device is the ratio of useful energy output to the energy input.					
Determining efficiency of a process					
$n = \frac{\text{useful energy output}}{1 = \frac{1}{2}} = \frac{1}{2} \frac{1}{2$					
' energy input energy input power input					

The value of efficiency can never be greater than 1; in practical devices, it is always less than 1. For example, an automobile engine converts chemical energy released from the burning of gasoline into mechanical energy that moves the pistons in the engine and eventually the wheels, enabling the car to move. However, nearly 85% of the input energy from combustion is wasted as thermal energy that heats up the exhaust gases and engine, or used to overcome friction (doing work against friction) in the moving parts. Hence, only about 15% of the energy from combustion goes into useful work in automobiles.

Example 7

A small motor is used to raise a weight of 2.0 N through a vertical height of 0.80 m. The efficiency of the motor is 20%. Calculate the minimum energy input to the motor to raise the weight.

System: weight & Earth Initial point: before weight is raised Final point: after it is raised through 0.80 m

When minimum energy is provided, work done increases gravitational E_p without increase in E_k . $\Delta E_p + \Delta E_k = W_{useful}$

 $\Delta E_{p} + \Delta E_{k} = W_{useful}$ $(mg)h + 0 = W_{useful}$ $W_{useful} = (2.0)(0.80) = 1.6 \text{ J}$ Since the efficiency of the motor is 20%, $\eta = \frac{W_{useful}}{W_{input}} = 0.20$ $W_{input} = 8.0 \text{ J}$

5.5 POWER

In practical devices, we are often more concerned about the time taken to do a certain amount of work; a device that does too long to do the work required is not very useful to us. Thus it is meaningful to quantify the rate of work done using a physical quantity – **Power**.



A fire-pump delivers 20 kg of water per second, the water being initially at rest and leaving the nozzle at a speed of 12 m s⁻¹. The nozzle is 6.0 m above the source of water supply and the pump is 40% efficient. Calculate the input power to the pump.

System: The volume of water in the fire pump **Initial to final point:** over a duration of 1 s The useful power of the pump is being used to increase the energy of water

 $\Delta \boldsymbol{E}_{p} + \Delta \boldsymbol{E}_{k} = \boldsymbol{W}_{\text{useful}}$

$$\Delta E_{p} + \Delta E_{k} = W_{\text{useful}}$$

$$mg\Delta h + \frac{1}{2}m(v_{f}^{2} - v_{i}^{2}) = W_{\text{useful}}$$

$$W_{\text{useful}} = (20)(9.81)(6.0) + \frac{1}{2}(20)(12^{2})$$

$$= 2617.2 \text{ J}$$

$$\eta = \frac{W_{\text{useful}}}{W_{\text{input}}} = 0.40$$

$$W_{\text{input}} = 6500 \text{ N} (2 \text{ s.f.})$$

(a)

Example 9

A cyclist is travelling at a constant speed of 2.80 m s⁻¹ up a slope at an angle 2.0° above the horizontal.

Calculate the rate of change of gravitational potential energy of the cyclist if the total mass of the cyclist and bicycle is 80 kg.

<u>⊤2.0°</u> (b)

If the cyclist uses 85.0 W of power (≈1218 the total resistive force experienced by the cyclist.

The useful power of the cyclist is being used to increase her gravitational potential energy and do work against friction $P_{\rm input} - P_{\rm resistive} = P_{\rm output}$

(a) Rate of change of GPE,

$$V_y$$
 V
2.0° V_x

$$\frac{dE_p}{dt} = \frac{mg\Delta h}{t}$$
$$= mgv_y$$
$$= (80)(9.81)(2.80\sin 2.0^\circ)$$
$$= 76.7 \approx 77 \text{ W} (2 \text{ s.f.})$$

(b) By conservation of energy, $P_{\text{resistive}} = 85.0 - 76.7$ $F_{\text{resistive}}v = 8.3$

$$F_{\text{resistive}} = \frac{8.3}{2.80} = 3.0 \text{ N}$$

APPENDIX: POWER USE OF HOME DEVICES AND APPLIANCES

NEA Energy Label

In Singapore, all air-conditioners, refrigerators, clothes dryers and television must have an energy label. This energy label is issued by the National Environment Agency (NEA)¹, and indicates the appliance's energy efficiency rating, (estimated) annual energy consumption, and annual energy cost.



NEA Energy Label

NEA also has a list of registered appliances that carry the Energy Label. From that list, the annual energy cost for air-conditioners can range from \$241 to \$2,301, depending on the cooling capacity and energy efficiency rating of the radiator unit!

In 2003, Singapore was ranked 14th in total energy consumption per capita in a year worldwide with 6455.7 kgoe/a (kg of oil per annum). 10 years later, in 2013, this had dropped to 4833.4 kgoe/a (ranked 21st worldwide). We have also moved from oil-fired steam turbine plants (37% efficiency) to Combined Cycle Gas Turbine (CCGT) plants burning natural gas, which are cleaner and more efficient (up to 53%).

In 2016, 96% of our power production comes from CCGTs, and 2.9% comes from burning waste products (including sewage). Households used 7,220.9 GWh (15.2%) of a total 47,513.8 GWh islandwide. Commercial buildings and services used 17,481.0 GWh (36.8%).²

Any further development in energy efficiency will rely on renewable energy sources, as well as energy conservation and reuse.

¹ The Energy Label, National Environment Agency. <u>http://www.nea.gov.sg/energy-waste/energy-efficiency/household-sector/the-energy-label</u>

² Singapore Energy Statistics 2016, Energy Market Authority. <u>https://www.ema.gov.sg/cmsmedia/Publications_and_Statistics/Publications/SES/2016/Singapore%20Energy%20Statistics%202016.pdf</u>

Power Consumption of Home Devices and Appliances

The following table lists the typical (estimated) power consumption of some common electronic devices and home appliances³:

Electronic devices

Smartphone (standby)	70 mW	
Smartphone (sending email)	430 mW (on Wifi) to 610 mW (on 3G)	
Smartphone (phone call)	1000 mW	
Smartphone (gaming)	Up to 3 W	
Home router	4 W to 18 W	
Laptop (2016 Macbook)	3 W (idle) to 20 W (load)	
Laptop (2016 Macbook Pro)	10 W (idle) to 65 W (load)	
LED TV (32-inch)	49 W (varies with brightness)	
LED TV (55-inch) ⁴	55 W (varies with brightness)	
Desktop computer	40 W (idle) to 400 W (typical heavy use)	

Home appliances

Cold cathode fluorescent light (CCFL) bulbs	13 W to 40 W (depending on bulb rating)	
LED bulbs	6 W to 14 W (at equivalent brightness)	
Ceiling fan	25 W (low speed) to 75 W (high speed)	
Refrigerator	40 W (low) to 300 W (high),	
	1 W (thermostat idle)	
Vacuum cleaner	200 W to 700 W	
Washing machine	500 W	
Toaster	700 W	
Electric kettle	800 W or more	
Clothes iron	1000 W or more	
Microwave	1500 W (typical)	
Shower heater	1500 W (typical)	
Air conditioner	900 W to 2000 W	
(at peak power)	(depending on number of fan coil units running)	

Why should appliance efficiency matter to you?

- 1. A less energy-efficient appliance uses more power to achieve the same effect. This means you are paying more without benefiting from it.
- 2. A less energy-efficient appliance produces more waste heat, which escapes into the room or house (unless the heat is vented outside). This makes the house more uncomfortable and means even more power used to cool the house down.

³ List of the Power Consumption of Typical Household Appliances, Daftlogic. <u>https://www.daftlogic.com/information-appliance-power-consumption.htm</u>

⁴ What you need to know about TV power consumption, CNET. <u>https://www.cnet.com/news/what-you-need-to-know-about-tv-power-consumption/</u>