Anglo - Chinese School
(Independent)

FINAL EXAMINATIONS 2013

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2
With Answer Key

Tuesday **8th October 2013** **1 hour 30 minutes**

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. ***Please start each question on a new page.***

1 *[Maximum mark: 5]*

Solve $\frac{2}{3x}+\frac{3}{x-4}=\frac{5}{x+1}$, giving your answer correct to 2 decimal places.

2 *[Maximum mark: 8]*

The line $y+1=3x$ intersects the curve $2y^2+2x^2+y-x=11$ at P and Q. Find

(i) the coordinates of P and Q, [6 marks]

(ii) the distance PQ [2 marks]

3 *[Maximum mark: 9]*

(a) Simplify $\log_2\left(\frac{8}{25a^2}\right)-2\log_2\left(\frac{9a}{5}\right)+4\log_2\left(\frac{3a}{2}\right)$. [4 marks]

(b) Solve the equation $\log_4(x^2+32)+\log_2\frac{1}{x}=2+\log_2 3$. [5 marks]

4 *[Maximum mark: 11]*

(a) Calculate the values of *k* for which the equation $9x^2-2(kx+1)+k=0$ has equal roots. [4 marks]

(b) **(i)** Express $y=8x^2+8x+5$ in the form $y=p(x+q)^2+r$. [2 marks]

(ii) Hence, sketch the curve, indicating clearly on your sketch the axes intercept (s), if any, as well as the turning point. [3 marks]

(c) Sketch the curve $y=3^x$, indicating the point (1, 1) on your sketch. [2 marks]

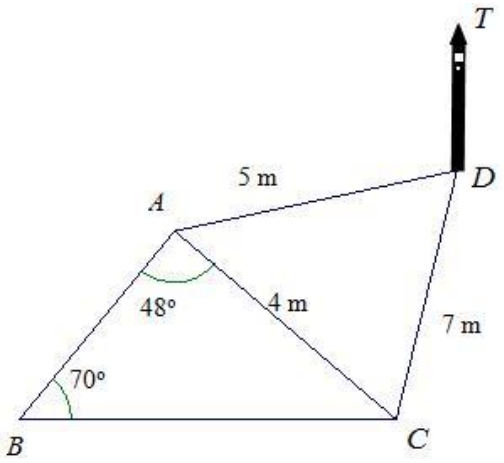
5 *[Maximum mark: 7]*

(a) The base of a triangle is $\left(3\sqrt{3}-\frac{2}{\sqrt{3}}\right)$ cm and its area is $\left(\sqrt{12}+4\right)$cm². Find the height of the triangle, giving your answer in the surd form. [3 marks]

(b) Given that $(1+a\sqrt{8})(\sqrt{32}-2)=30-b\sqrt{2}$, where a and b are integers, find the value of a and of b. [4 marks]

6 *[Maximum mark: 13]*

In the diagram, A, B, C and D are four points on level ground, such that AC = 4 m, AD = 5 m, CD = 7 m, $\angle BAC = 48°$ and $\angle ABC = 70°$.



(a) Find

(i) the length of BC, [2 marks]

(ii) the angle ACD, [3 marks]

(iii) the area of ΔACD, [2 marks]

(iv) the shortest distance from D to AC. [2 marks]

(b) A vertical tower, whose tip is T, is erected at D. Given that the angle of elevation of T from C is 55°, find the

(i) height of the tower, [2 marks]

(ii) the greatest angle of elevation of T, when viewed from a point along AC. [2 marks]

7 *[Maximum mark: 9]*

(a) Solve the equation $4(3^{y-2}) = 5^y$ [4 marks]

(b) Given the equation $e^{2x+1} - e^{x+2} - 2e^3 = 0$ and that $x = \ln a$ is a solution, find the value of a. [5 marks]

8 *[Maximum mark: 8]*

Given the points $A(4 - 2p, 5)$, $B(p, -1)$ and $C(2, 3)$, find the positive value of p for which

(i) the area of ΔABC is 13 square units, [3 marks]

(ii) the midpoint of AB is equidistant from C and the point (-2, -3). [5 marks]

9 *[Maximum mark: 10]*

Answer the whole of Question 9 on a sheet of graph paper.

It is known that variables x and y are related by the equation $ya^x = b + 5$, where a and b are unknown constants. Observed values of the two variables are shown in the following table.

x	- 1	0	1	2	3	4
y	22.5	7.5	2.5	0.83	0.28	0.09

(i) Using the scale of 2 cm to one unit on the horizontal axis and 4 cm to one unit on the vertical axis, plot the graph of lg y against x and draw a straight line graph. [4 marks]

(ii) Use your graph to estimate the value of a and of b. [4 marks]

(iii) From the graph, estimate the value of y when $x = 1.2$. [2 marks]

End of Paper

Answer Key

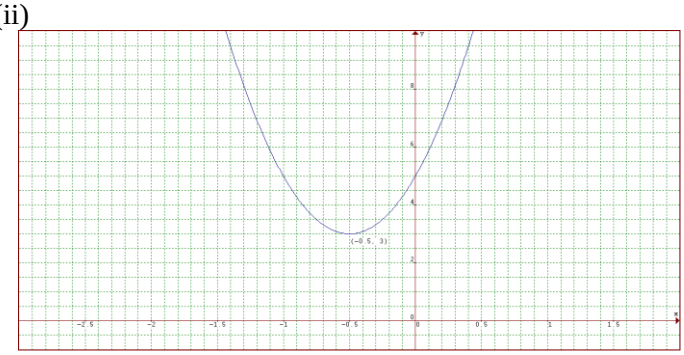
1. 15.62 or 0.13

2. (i) $\left(-\frac{1}{2}, -2\frac{1}{2}\right)$ and $(1,\ 2)$ (ii) 4.74 units

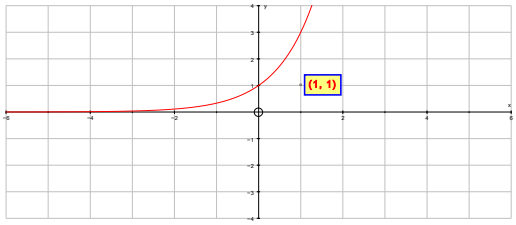
3. (a) -1 (b) 0.473

4. (a) 3 or 6 (b) (i) $y = 8(x+\frac{1}{2})^2 + 3$

(ii)



(c)



5. (a) $\frac{4(3+2\sqrt{3})}{7}$ cm (b) $a = 2,\ b = 4$

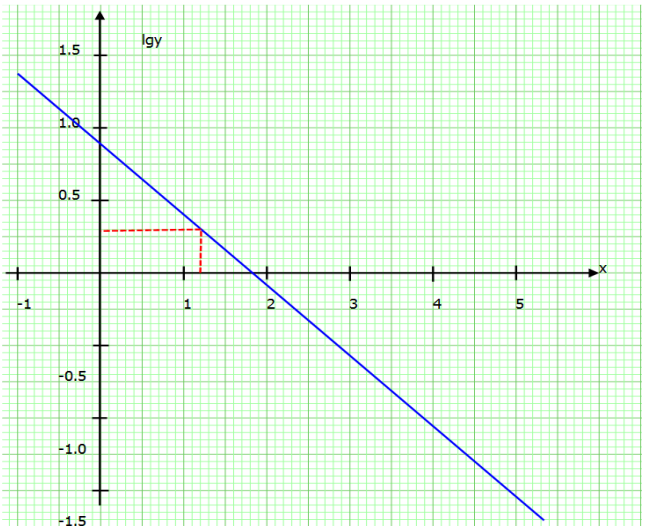
6. (a) (i) 3.16 m (ii) 44.4° (iii) 9.80 m² (iv) 4.90 m

(b) (i) 10.0 m (ii) 63.9°

7. (a) -1.59 (b) 2*e* or 5.44

8. (i) 5 (ii) 10

9. (i)



(ii) $a = 3.06$; $b = 2.50$ (iii) 2.00