

CANDIDATE NAME: .....

CLASS: .....

INDEX NUMBER: .....

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**ADDITIONAL MATHEMATICS**

**4049/01**

Paper 1

**November 2024**

Secondary 4 Express

**2 hours 15 minutes**

Setter: itzpipey :)))

Candidates answer on the question paper.

No Additional Materials are required.

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**READ THESE INSTRUCTIONS FIRST**

Write your name, class, and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

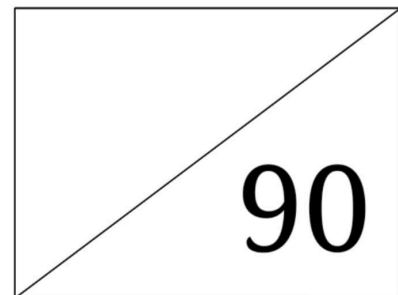
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.



# MATHEMATICAL FORMULAE

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## 1. ALGEBRA

### *Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### *Binomial expansion*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

### *Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

### *Formulae for $\triangle ABC$*

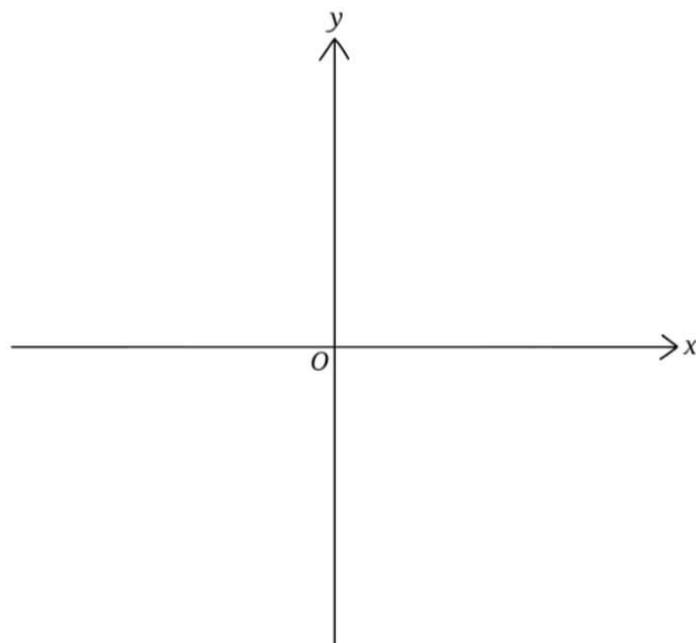
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1      Solve for  $a$  and  $b$  if  $\frac{\sqrt{24} - a\sqrt{6}}{\sqrt{8} - \sqrt{3}} = \frac{\sqrt{2} + b\sqrt{3}}{2}$ . [4]

- 2 The curve  $y = -x^3 + 5x - 3x \ln x$  has  $k$  stationary points. By sketching suitable curves on the axes below, find the value of  $k$ . You are to label all axial intercepts in their **exact** form. Please show your workings. [5]



3 (a) State the range of principal values for

(i)  $\sin^{-1} x$ , [1]

(ii)  $\cos^{-1} x$ . [1]

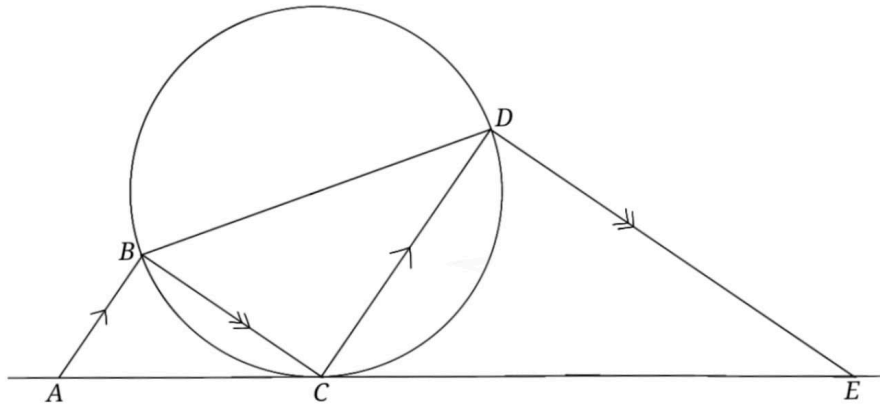
(b) You are given that  $\frac{\sin^4 x - \cos^4 x}{T(x)} = \sin 4x$ , where  $T(x)$  is a function of  $x$ . Show that  $T(x) = -\frac{1}{2} \operatorname{cosec} 2x$ . [3]

- 4 (a) By representing 7999999 in the form  $a^3 - b^3$ , explain why 7999999 is not a prime number. [3]

- (b) Represent  $\frac{48x^2 + 3x - 2}{14x^2 - 7x^3}$  as a sum of partial fractions. [5]

- 5 Deduce whether the curve  $y = e^{-\sin^2 x}$  is increasing or decreasing for  $0 < x < \frac{\pi}{4}$ . [4]

6



The diagram shows a triangle  $BCD$  inscribed in a circle, with  $BD$  as the diameter.  $A$  and  $E$  are points on a tangent of the circle at  $C$  such that  $AB$  is parallel to  $CD$  and  $BC$  is parallel to  $DE$ .

(a) Show that triangles  $ABC$ ,  $BCD$  and  $CDE$  are similar. [3]

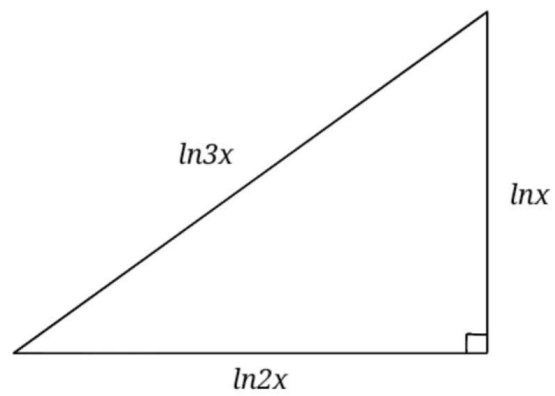
(b) Explain whether a circle could be drawn intersecting points  $A$ ,  $B$  and  $C$ , and state which line would be the diameter. [2]



- (c) Show that  $AB \times CD + BC \times DE = BD^2$ . [3]

- 7 The curve  $y = 12x^3 + 56x^2 + 57x - 26$  intersects the line  $y = 2x - 1$  at the point  $(\frac{1}{3}, \frac{1}{3})$ . Find the coordinates of the other intersection point(s) between the and line. [6]

8



A right triangle has side lengths of  $\ln x$ ,  $\ln 2x$  and  $\ln 3x$  units. Solve for  $x$ .

[6]

- 9      (a)      Sketch the curves  $y = -\sin x$  and  $y = -\cos \frac{x}{2} - 3$  for  $0 \leq x \leq 2\pi$ . [4]

- (b)      Find the area of the region bound by the two curves, the  $y$ -axis and the line  $x = \pi$ .  
Express your answer in the **exact** form. [4]

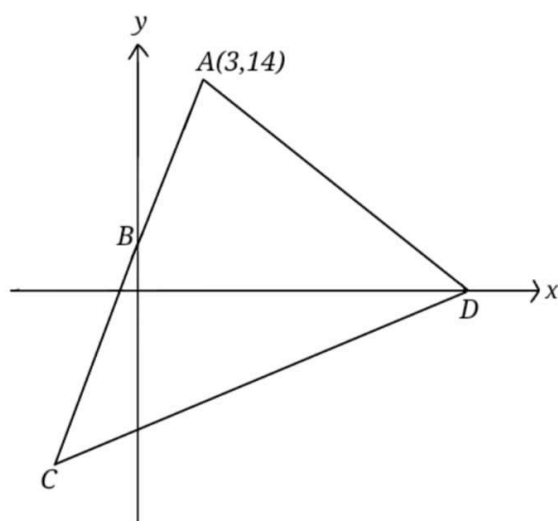
- 10 The number of views a K-pop music video gains,  $V$ , after  $t$  days can be modelled by  $V = Ae^{-k(t-1)} + 50\,000$ . The table below shows some information about the video.

Number of days that passed	Number of <b>total</b> views of the video (nearest whole number)
1	1050000
2	2098400
3	N

- (a) Find the values of  $A$ ,  $k$  and  $N$ . [5]

- (b) Suggest and explain the value that  $V$  gets closer to as time passes. [2]

11



The diagram shows points  $A(3, 14)$ ,  $B$ ,  $C$  and  $D$ .  $B$  lies on the  $y$ -axis while  $D$  lies on the  $x$ -axis.  $D$  is equidistant from  $A$  and  $B$  and the area of  $ABCD$  is  $492.75 \text{ units}^2$ .

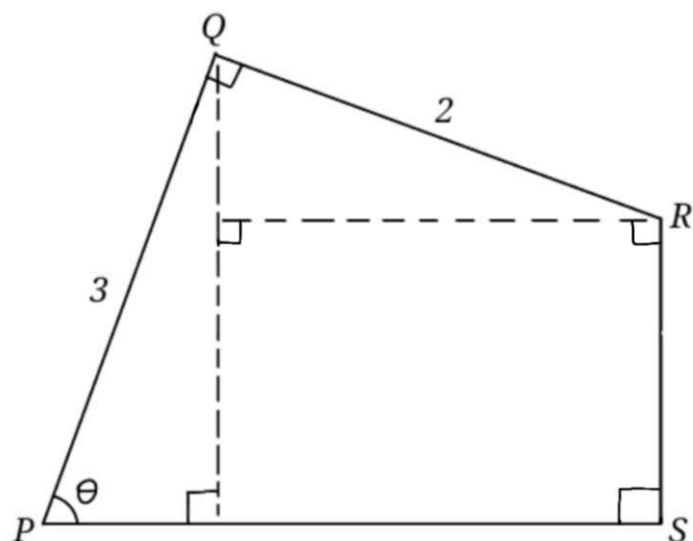
(a) Given that  $AB$  is perpendicular to the line  $4y + x = 3$ , find the coordinates of  $B$ . [2]

(b) Find the coordinates of  $D$ . [3]

(c) The equation of  $BC$  is  $y = \frac{17}{4}x + 2$ . Find the coordinates of  $C$ . [4]

(d) Explain why  $AC$  is **not** a straight line. [1]

12



The diagram shows the cross-sectional area of a garden shed, where  $PQ = 3$  metres and  $QR = 2$  metres. Angle  $QPS = \theta$  radians.

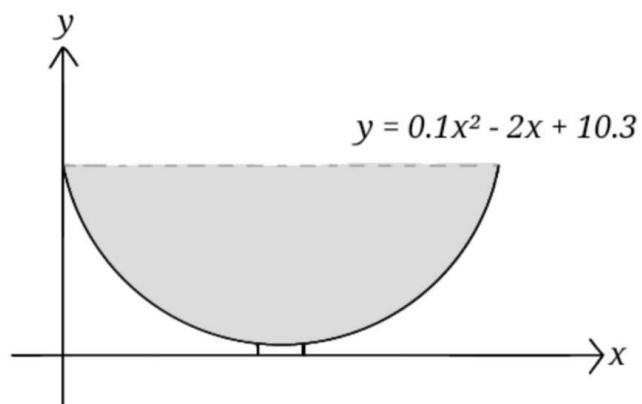
- (a) Show that the area of  $PQRS$ ,  $A = 3 + \frac{5}{4}\sin 2\theta - 3\cos 2\theta$ . [4]



- (b) Express  $\frac{5}{4}\sin 2\theta - 3\cos 2\theta$  in the form  $R\sin(2\theta - \alpha)$ , where  $R > 0$  and  $\alpha$  is acute. [2]

- (c) Find the maximum possible area of  $PQRS$  and the corresponding value of  $\theta$ . [3]

13



A bowl is set on a surface such that it can be modelled by the equation  $y = 0.1x^2 - 2x + 10.3$ , where the  $x$ -axis represents the surface and  $y$  represents the height of the bowl above the surface in centimetres.

(a) (i) State the highest height of the bowl above the surface. [1]

(ii) Find the diameter of the bowl. [2]

- (b) Express  $y$  in the form  $a(x - h)^2 + k$  and hence find the coordinates of the lowest point inside the bowl. [3]

- (c) Find the cross-sectional area of the bowl (the shaded region). [4]

**END OF PAPER**

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