Statistics Tutorial 1: Permutations and Combinations

Basic Mastery Questions

1.   There are two bags. The first bag contains balls numbered 1 to 40, the second contains balls numbered 1 to 10.
      (a)   How many ways could one ball be chosen?
      $$40 + 10 = 50$$

      (b)   How many ways could one ball be chosen from each bag?
      $$40 \times 10 = 400$$

      (c)   Considering only the bag with balls numbered 1 to 40:
             (i)   How many ways could all 40 balls be arranged in a straight line?
             $$40!$$

             (ii)   How many ways could 3 balls be drawn from the bag, if the balls are returned to the bag after each draw, and the order matters?
             $$40 \times 40 \times 40 = 40^3 = 64\,000$$

             (iii) How many ways could 6 balls be drawn from the bag, if the balls are not replaced, and the order matters?
             $$^{40}P_6 = \frac{40!}{34!} = 2\,763\,633\,600$$

             (iv)   How many ways could 10 balls be drawn from the bag such that the first and last balls are even, if the balls are not replaced and the order matters?
             $$20 \times 19 \times {}^{38}P_8 = 7.49 \times 10^{14} \ (3\text{sf})$$

      (d)   How many ways could 6 balls be drawn from the first bag, and 1 ball be drawn from the second bag if the balls are drawn without replacement, and the order of the draw does not matter?
      $$\binom{40}{6}\binom{10}{1} = 38\,383\,800$$

2.   J74/P1/1
      Four men, two women and a child sit at a round table. Find the number of ways of arranging the seven people if the child is seated
      (a)   between the two women
      (b)   between two men.

      (i) $4!2! = 48$

      (ii) $\binom{4}{2}4!2! = 288$