National Junior College
2016 – 2017 H2 Mathematics
Differentiation Techniques **Assignment Solutions**

1(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\ln\left(\sin^2 x\right)\right]$

$= x\left(\frac{2\cos 2x}{\sin 2x}\right)+\ln\left(\sin 2x\right)$ [M1]

$= 2x\cot 2x+\ln\left(\sin 2x\right).$ [A1]

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\mathrm{e}^{\tan kx}\right)$

$= x^2\left(k\cdot\sec^2 kx\cdot\mathrm{e}^{\tan kx}\right)+\left(2x\right)\mathrm{e}^{\tan kx}$ [M1]

$= x\mathrm{e}^{\tan kx}\left(kx\sec^2 kx+2\right).$ [A1]

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin^{-1}x}{1-x^2}\right)$

$= \frac{2x\sin^{-1}x}{\left(1-x^2\right)^2}+\frac{1}{\sqrt{1-x^2}}\left(\frac{1}{1-x^2}\right)$ [M1]

$= \frac{2x\sin^{-1}x}{\left(1-x^2\right)^2}+\frac{\sqrt{1-x^2}}{\left(1-x^2\right)^2}$

$= \frac{2x\sin^{-1}x+\sqrt{1-x^2}}{\left(1-x^2\right)^2}.$ [A1]

2(a) $x^y=\cos x$.

Taking "ln" on both sides, we get:
$y\ln x=\ln\left(\cos x\right)$. [M1]

Differentiate implicitly wrt x, we get:

$\frac{y}{x}+\frac{\mathrm{d}y}{\mathrm{d}x}\ln x=\frac{-\sin x}{\cos x}$ [M1]

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}\ln x=-\tan x-\frac{y}{x}$

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\tan x}{\ln x}-\frac{\ln\left(\cos x\right)}{x\left(\ln x\right)^2}.$ [A1]

2(b) $x^{y+1}=\mathrm{e}^{x+y}$.

Taking "ln" on both sides, we get
$(y+1)\ln x=x+y.$ [M1]

Differentiate implicitly wrt x, we get

$\ln x\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{y+1}{x}=1+\frac{\mathrm{d}y}{\mathrm{d}x}$ [M1]

$(\ln x-1)\frac{\mathrm{d}y}{\mathrm{d}x}=1-\frac{y+1}{x}$

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x-y-1}{x(\ln x-1)}.$ [A1]

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3. $\frac{\mathrm{d}x}{\mathrm{d}t}=2-\frac{2}{2t}=\frac{2t-1}{t},$

$\frac{\mathrm{d}y}{\mathrm{d}t}=2t-\frac{2}{t}=\frac{2}{t}\left(t^2-1\right).$ [M1]

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left[\frac{\mathrm{d}y}{\mathrm{d}t}\right]}{\left[\frac{\mathrm{d}x}{\mathrm{d}t}\right]}=\frac{\frac{2}{t}\left(t^2-1\right)}{\frac{2t-1}{t}}$ [A1]

$=\frac{2\left(t^2-1\right)}{2t-1}.$

Given that $\frac{\mathrm{d}y}{\mathrm{d}x}=2$,

$\frac{2\left(t^2-1\right)}{2t-1}=2$ [M1]

$t^2-1=2t-1$

$t^2-2t=0$

$t\left(t-2\right)=0$

$\Rightarrow t=0$ or $t=2$

Since $t>0$, $t=2$. [A1]