Anglo - Chinese School

(Independent)

FINAL EXAMINATION 2019

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

MONDAY	**7th October 2019**	**1 hour 30 minutes**

ADDITIONAL MATERIALS:

Answer Paper (7 sheets)
Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS
The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1 *[Maximum mark: 7]*

(i) A triangle has the following lengths $\frac{4}{x+3}$ meters, $\frac{3x}{x^2-9}$ meters and $\frac{2}{x-3}$ meters, where $x > 3$. Find the perimeter of the triangle, P, as a single fraction in its simplest form. [4]

(ii) Given that the perimeter of the triangle is 2 meters, calculate the value of x. [3]

2 *[Maximum mark: 11]*

(a) Factorize completely $x^2 - 6xy + 9y^2 - 16z^2$. [3]

(b) (i) Given that $y = \sqrt{\frac{b(y^2 - x^2)}{a}}$, express x in terms of y, a and b. [3]

(ii) Find the value(s) of x when $a = \sqrt{2}$, $b = -3.125$ and $y = 7\frac{1}{3}$, leaving your answer correct to 4 significant figures. [2]

(c) If the length of the base of a triangle is increased by 17% and the length of its perpendicular height is decreased by 17%, find the percentage change, if any, in its area. [3]

3 *[Maximum mark: 8]*

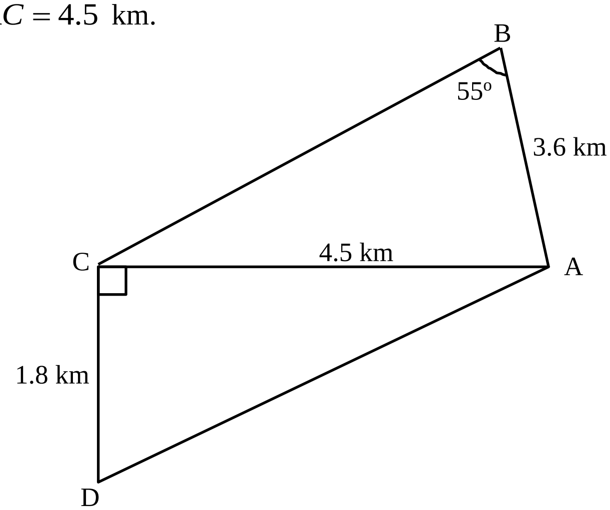
The line $x + y = 7$ intersects the curve $xy = 12$ at P and Q. Find

(a) the coordinates of P and Q, [4]

(b) the equation of the perpendicular bisector of P and Q. [4]

4 *[Maximum mark: 9]*

A, *B*, *C* and *D* are four points on a flat ground. *D* is due south of *C* and $\angle ABC = 55^o$. $AB = 3.6$ km, $CD = 1.8$ km and $AC = 4.5$ km.



Calculate

(a) $\angle CDA$, [2]

(b) the bearing of *D* from *A*, [2]

(c) $\angle BCA$, [2]

(d) the shortest distance from *B* to *AC*. [3]

5 *[Maximum mark: 13]*

(a) Find the range of values of m for which the curve $y = x^2 + mx + 5$ does not intersect the line $y + 2x + m = 0$ for all real values of x. [5]

(b) Find the number of roots for the equation $px^2 + qx - 4p = 0$, where $p \neq 0$. Justify your answer. [3]

(c) The equation $kx^2 - 7x + 6 = 0$, $k \neq 0$, has two distinct roots, α and β. If $\alpha = \frac{4}{3}\beta$, find the value of k. [5]

6 *[Maximum mark: 14]*

(a) Solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$ [4]

(b) Given that $p = 2^x$ and $q = 2^y$, express $\log_2 p^3 q$ in terms of x and y. [2]

(c) Find the value of *k* if $e^{3k+4} = \ln 79$. [3]

(d) If $4^{2x+1} = 3^{x-1}$, show that $\left(\frac{3}{16}\right)^x = 12$. Hence, solve for x. [5]

7 *[Maximum mark: 12]*

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 3x - 4 + \frac{2}{x}$. Some corresponding values of x and y are given in the following table.

x	0.25	0.5	1	1.5	2	3	4	5	6
y	4.75	1.5	a	1.83	3	5.67	8.5	11.4	14.3

(a) Calculate the value of a. [1]

(b) Taking 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical axis, draw the graph of $y = 3x - 4 + \frac{2}{x}$ for $0.25 \le x \le 6$. [4]

(c) By drawing a tangent, find the gradient of the curve at $(2,3)$. [2]

(d) By drawing a suitable line on the graph, solve the equation $x - 2 + \frac{2}{3x} = 0$. [3]

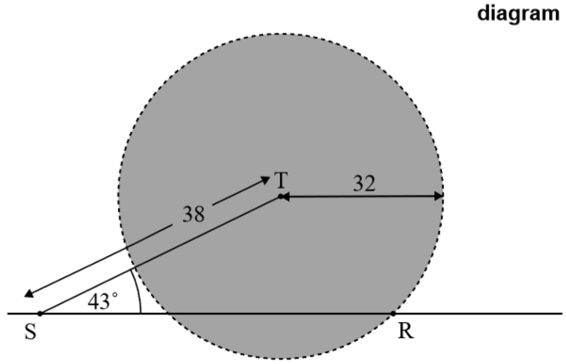
(e) Find the minimum value of y and its corresponding value of x. [2]

8 *[Maximum mark: 6]*

T is the foot of a communication tower that produces a signal that can reach cellular phones within a radius of 32 km. A straight road starting from S passes through the area covered by the tower's signal.

The following diagram shows a line representing the road and a shaded circle representing the area covered by the tower's signal. Point R is on the circumference of the circle and points S and R are on a straight road. Point S is 38 km from the foot of the tower, $\angle RST = 43^o$ and $SR = x$ km.

diagram not to scale



(a) Using the Cosine Rule, show that $x^2 - (76\cos 43^o)x + 420 = 0$. [2]

(b) Hence or otherwise, find the total distance along the road where the signal from the tower can reach cellular phones. [4]

---**END OF PAPER**---

Answers:

1i) $\frac{3(3x-2)}{(x-3)(x+3)}$
1ii) 5.58
2a) $(x-3y-4z)(x-3y+4z)$
2bi) $\pm\sqrt{\frac{y^2(b-a)}{b}}$
2bii) ± 8.838
2c) 2.89%
3a) (4, 3) and (3, 4)
3b) $y = x$
4a) 68.2
4b) 248.2
4c) 40.9
4d) 3.58
5a) $-4 < m < 4$
5b) 2 real roots
5c) $k = 2$
6a) 8
6b) $3x + y$
6c) -0.842
6d) -1.48
7a) 1
7c) 2.5
7d) 0.423 & 1.58
7e) (0.816, 0.899)
8b) 37.6