RAFFLES INSTITUTION
RAFFLES PROGRAMME 2024
YEAR 3 MATHEMATICS
TOPIC 2: GRAPHICAL SOLUTIONS OF EQUATIONS (MATHS 1)

WORKSHEET 2

Name: Class: 3 ( ) Date:

WORKSHEET 2: GRAPHICAL SOLUTIONS OF EQUATIONS

(NSM3 8th Edition Chapter 5, P.119)

ENDURING UNDERSTANDING(S)

Students will understand that

- the Cartesian coordinates system is a means of relating algebra and geometry,
- algebraic equations and graphs are different ways of representing the relationship between two or more quantities,
- algebraic equations can be solved graphically by finding the points of intersection of their graphs and
- graphs are tools which gives a visual representation of how variables are related in various real-life applications.

LEARNER OUTCOMES

At the end of the lesson, students will be able to

- draw proper tangents on graphs to solve given equations,
- solve equations and inequalities involving other functions such as cubic, reciprocal and exponential functions approximately by using graphical methods,
- find gradient at a point on a graph of other functions.

In this worksheet, you will learn how to use graphs of basic functions to solve equations. You will also learn how to use graphs of basic functions to find gradient, solve inequalities and solve further problems.

Important note: Graphs will be provided and hence there is no need to use graph paper to plot graphs.

(1) GRADIENT OF A CURVE

(NSM3 8th Edition Chapter 5, Section 5.4, P.134)

Recall:

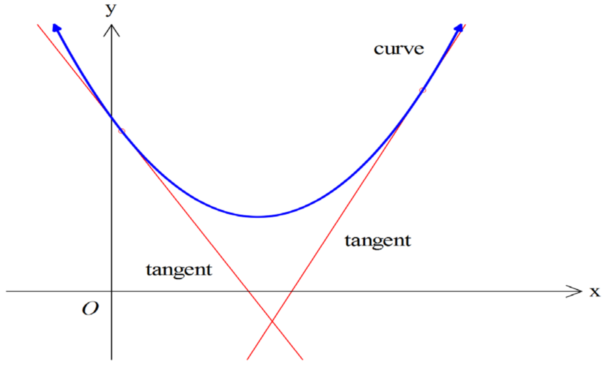
Gradient of a line gives an indication of the steepness or slope of a line.

Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, gradient of straight line $AB = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Any two points on the same straight line will give the same gradient and hence gradient of a straight line is a constant.

How about gradient of a curve?

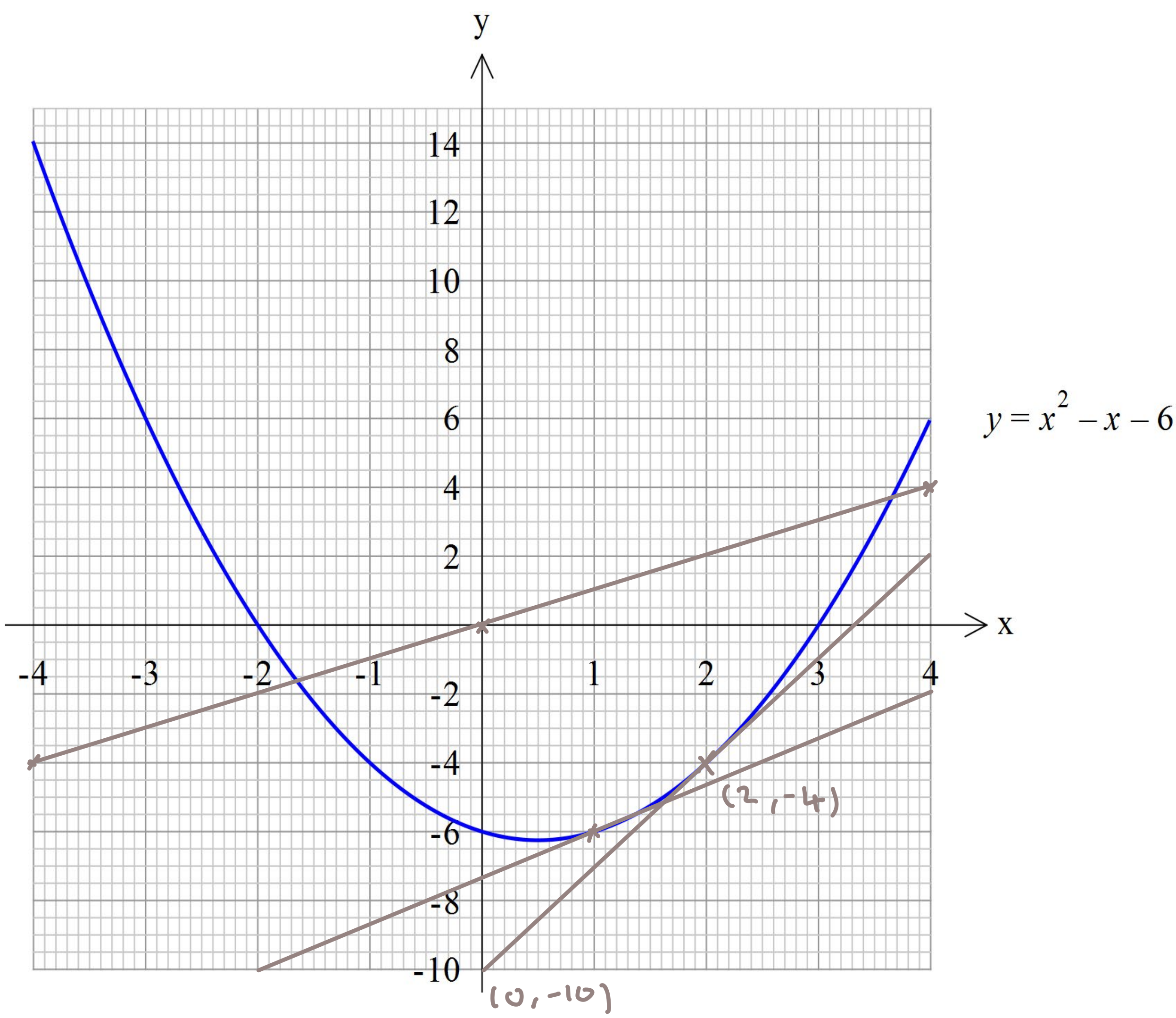
Definition of Tangent to a Curve: The **tangent** to a curve at point P is defined as the straight line that **touches** the curve at P.



Definition of Gradient of a Curve: Gradient of a curve at a given point is defined as the gradient of the tangent to the curve at that point.

Since the tangents to the curve are different at different points, gradient of a curve changes from point to point and hence it is not a constant. Gradients obtained by drawing tangents are only approximate values.

EG 1 The diagram shows the graph of $y = x^2 - x - 6$ for $-4 \leq x \leq 4$.



Use the graph to answer the following questions.

(a) Find the gradient of the curve at $x = 2$,

$$\frac{-4-(-10)}{2-0} = \frac{6}{2}$$

$$= 3$$ #

(b) By drawing a suitable tangent, find the x-coordinate of the point on the graph at which the gradient of the tangent is equal to 1.

x-coordinate $= 1$ #

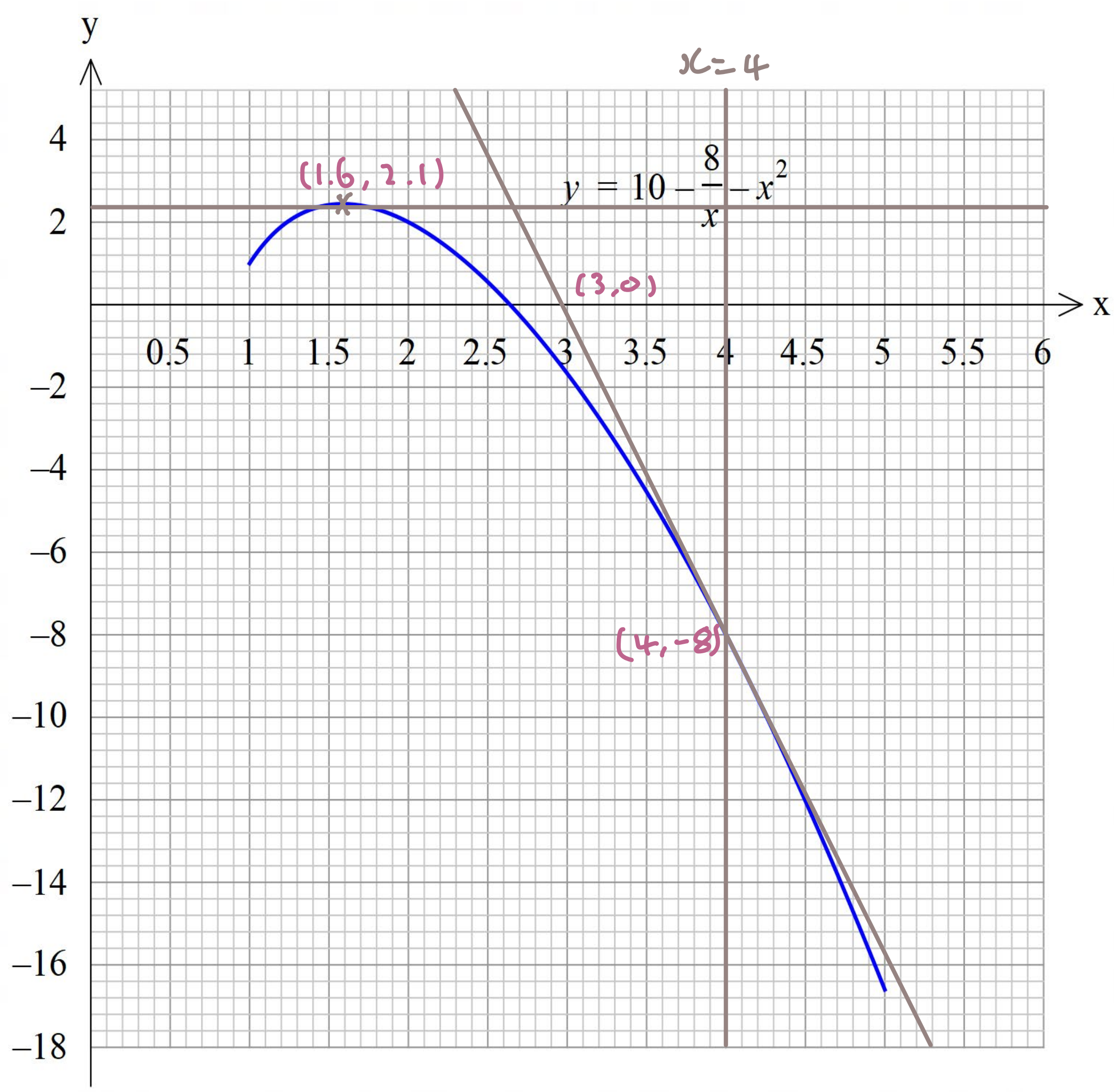
Notes:

1. *The tangent drawn should touch the curve at the given point.*
2. *The two points chosen to calculate the gradient should lie on the intersection of the grid lines on the graph paper.*
3. *As part of proper presentation for the calculation of gradient, the coordinates of the points chosen should be clearly indicated or a triangle should be drawn with the chosen points.*

HOMEWORK 1

LEVEL 2

1. The diagram below shows part of the graph of $y = 10 - \frac{8}{x} - x^2$ for $1 \le x \le 5$.



Use the graph to answer the following questions.

(a) Find the gradient of the curve at $x = 4$,

Draw $x = 4$

$\frac{0-(-8)}{3-4} = \frac{8}{-1}$

$= -8$ #

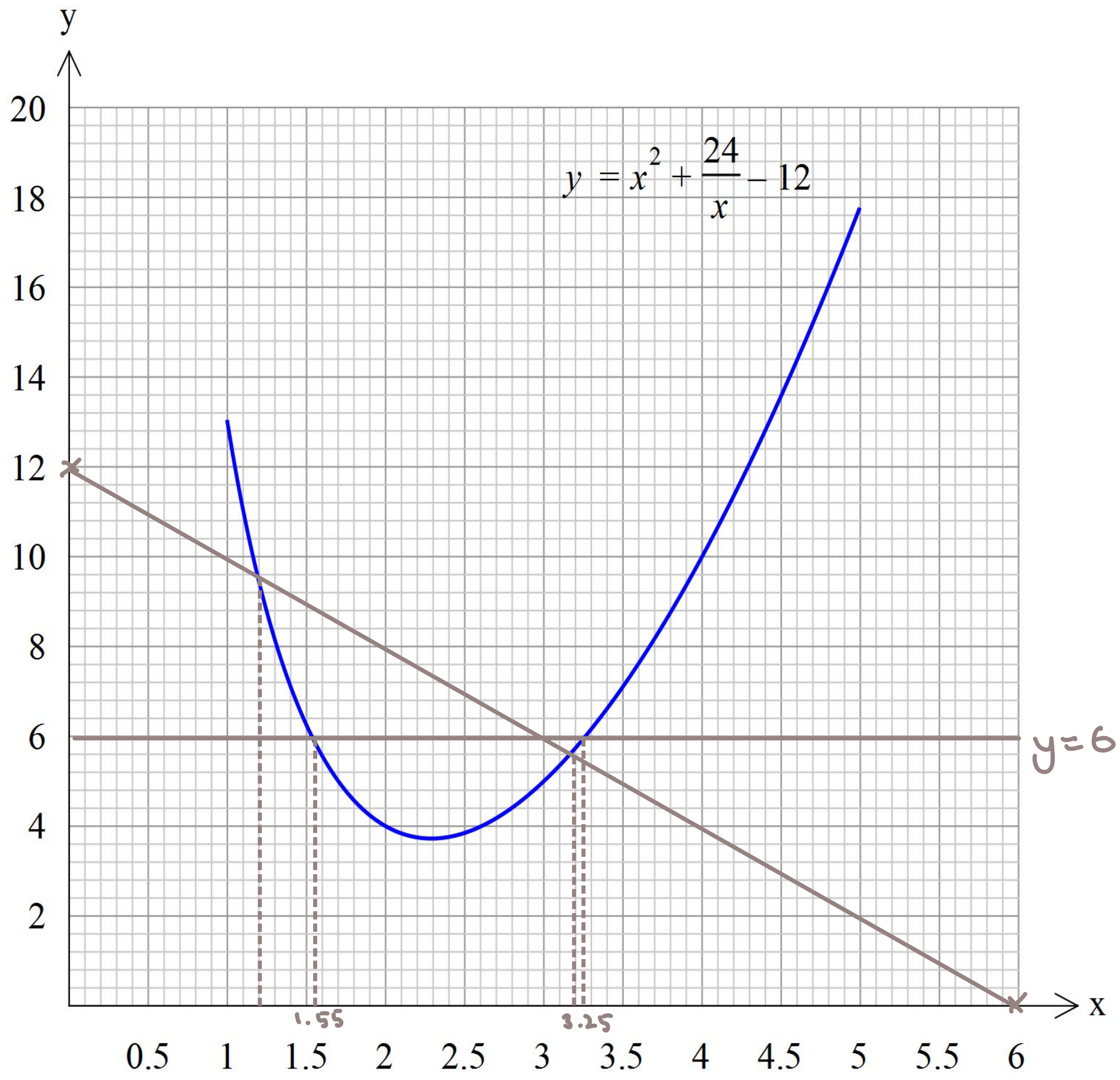
(b) Find the x-coordinate of the point on the curve where the gradient is zero.

[Ans: (a) –7.5 (b) x = 1.6]

$x = 1.6$ #

(2) GRAPHICAL SOLUTIONS OF EQUATIONS

EG 2 The diagram below shows part of the graph of $y = x^2 + \frac{24}{x} - 12$ for $1 \le x \le 5$.



(a) Use the graph to solve the equation $x^2 + \frac{24}{x} = 18$.

$x^2 + \frac{24}{x} = 18$

$x^2 + \frac{24}{x} - 12 = 6$

Draw $y = 6$

From graph, $x = 1.55$ or 3.25 #

(b) By adding a suitable straight line to the graph, solve the equation $x^3 + 2x^2 - 24x + 24 = 0$.

$x^2 + 2x - 24 + \frac{24}{x} = 0$

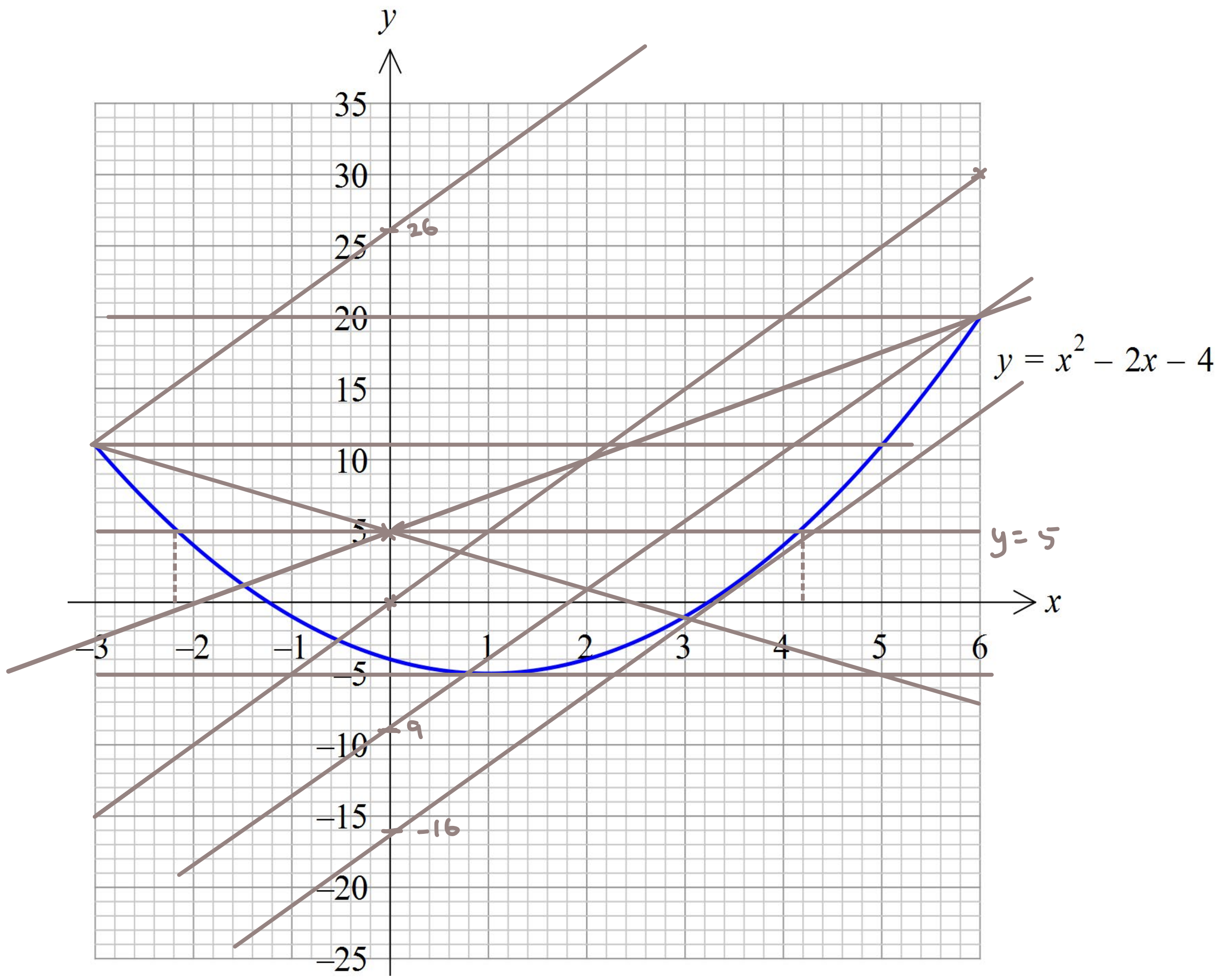
$x^2 + \frac{24}{x} = -2x + 24$

$x^2 + \frac{24}{x} - 12 = -2x + 12$

Draw $y = 12 - 2x$ #

From graph, $x = 1.2$ or 3.2

EG 3 The diagram shows the graph of $y = x^2 - 2x - 4$ for $-3 \leq x \leq 6$.



Use the graph to answer the following questions.

(a) (i) Find the x-coordinates of the points on the curve where $y = 5$.

$x = -2.2$ #

(ii) Write down and simplify the equation in x which has these values as its solutions.

$x^2 - 2x - 4 = 5$

$x^2 - 2x - 9 = 0$ #

(b) Find the range of values of k such that the equation $x^2 - 2x - 4 = k$ has

(i) two distinct solutions,

From graph,

$-5 < k \leq 11$ #

(ii) no solution.

From graph

$-5 > k > 20$ #

(c) Find the range of values of k such that equation $x^2 - 2x - 4 = 5x + k$ has only one solution.

$x^2 - 2x - 4 = 5x + k$

draw $y = 5x + k$

From graph,

$k = -16$ or $-9 < k \leq 26$ #

(d) Find the range of values of k such that equation $x^2 - 2x - 4 = kx + 5$ has two distinct solutions.

$x^2 - 2x - 4 = kx + 5$

Draw $y = kx + 5$

smallest $k = \frac{11-5}{-3-0}$

$= \frac{6}{-3}$

$= -2$

greatest $k = \frac{20-5}{6-0}$

$= \frac{15}{6}$

$= \frac{5}{2}$

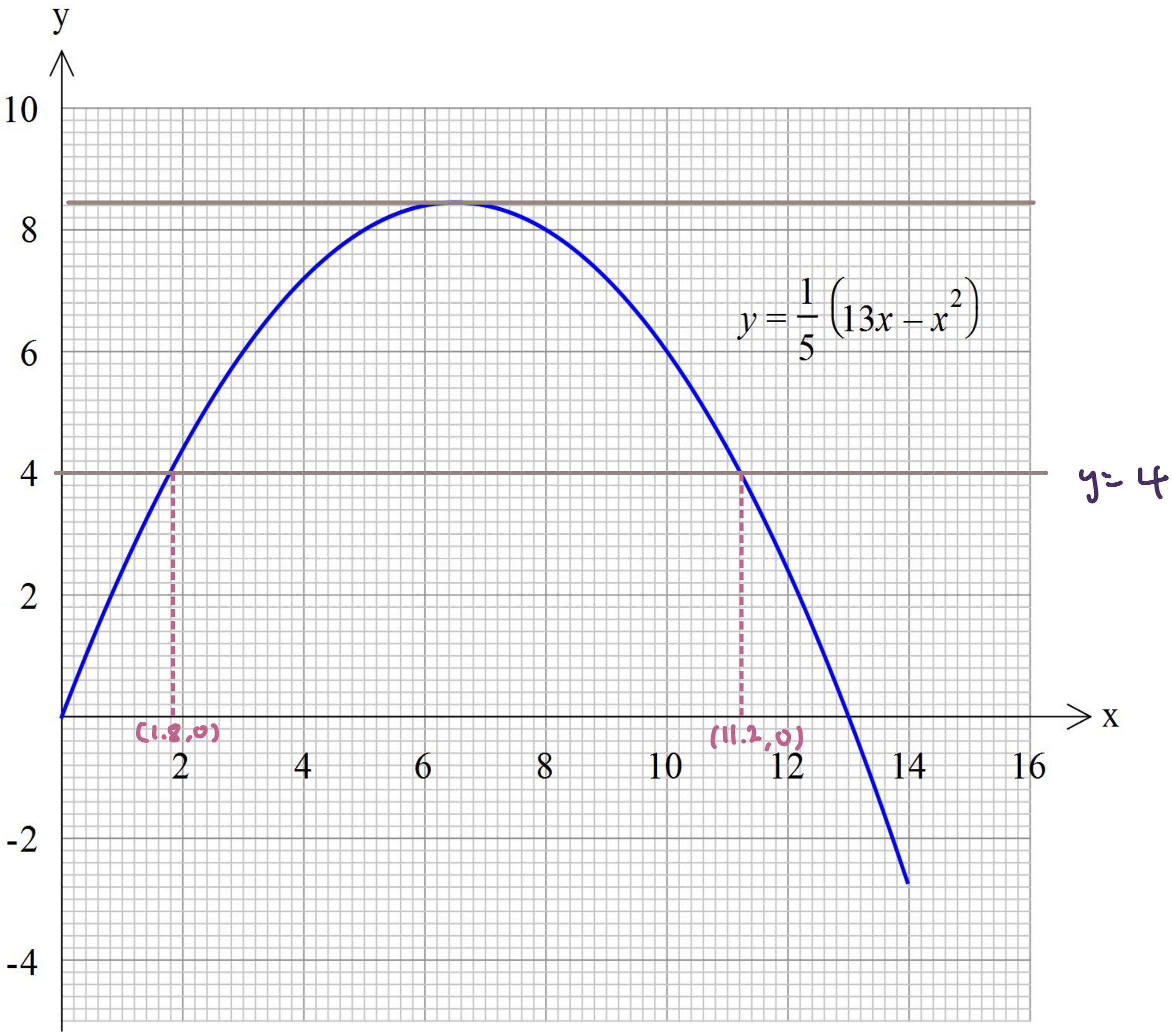
$= 2.5$

$\therefore -2 \leq k \leq 2.5$ #

HOMEWORK 2

LEVEL 2

1. The diagram below shows the graph of $y = \frac{1}{5}(13x - x^2)$ for $0 \le x \le 14$.



Use the graph to answer the following questions.

(a) (i) Find the x-coordinates of the points on the curve where $y = 4$.

Draw $y = 4$

$x = 1.8$ or $x = 11.2$ #

(ii) Write down and simplify the equation in x which has these values as its solutions.

$y = \frac{1}{5}(13x - x^2)$

Sub $y = 4$

$4 = \frac{1}{5}(13x - x^2)$

$20 = 13x - x^2$

$x^2 - 13x + 20 = 0$ #

(b) Find the range of values of k such that the equation $13x - x^2 = 10k$ has two distinct solutions.

[Ans: (a)(i) 1.8 or 11.2 (ii) $\mathbf{x^2 - 13x + 20 = 0}$ *(b) 0 ≤ k < 4.2]*

$13x - x^2 = 10k$

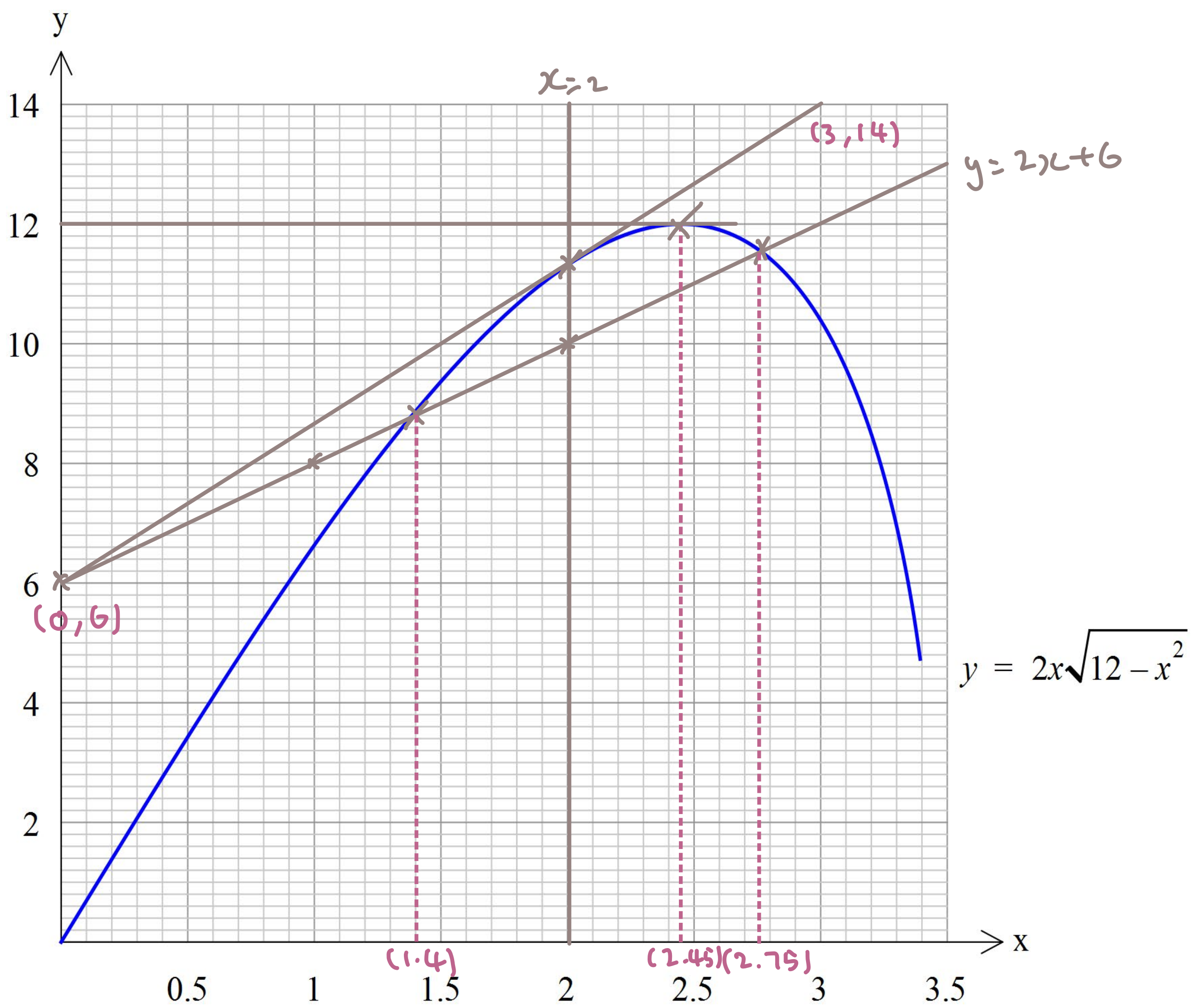
$\frac{1}{5}(13x - x^2) = \frac{10}{5}k$

Draw $y = 2k$

$0 \leq 2k < 8.4$

$0 \leq k < 4.2$

2 The diagram shows part of the curve $y = 2x\sqrt{12 - x^2}$ for $0 \leq x \leq 3.4$.



Use the graph to answer the following questions.

(a) Find the gradient of the curve at the point where $x = 2$ by drawing a tangent.

Draw $x=2$

$\frac{14-6}{3-0} = \frac{8}{3}$ #

(b) By drawing a suitable straight line, find the values of x for which $x\sqrt{12-x^2} = x+3$.

$x = 1.4$ or $x = 2.75$ #

$x\sqrt{12-x^2} = x+3$

$2x\sqrt{12-x^2} = 2x+6$

Draw $y=2x+6$

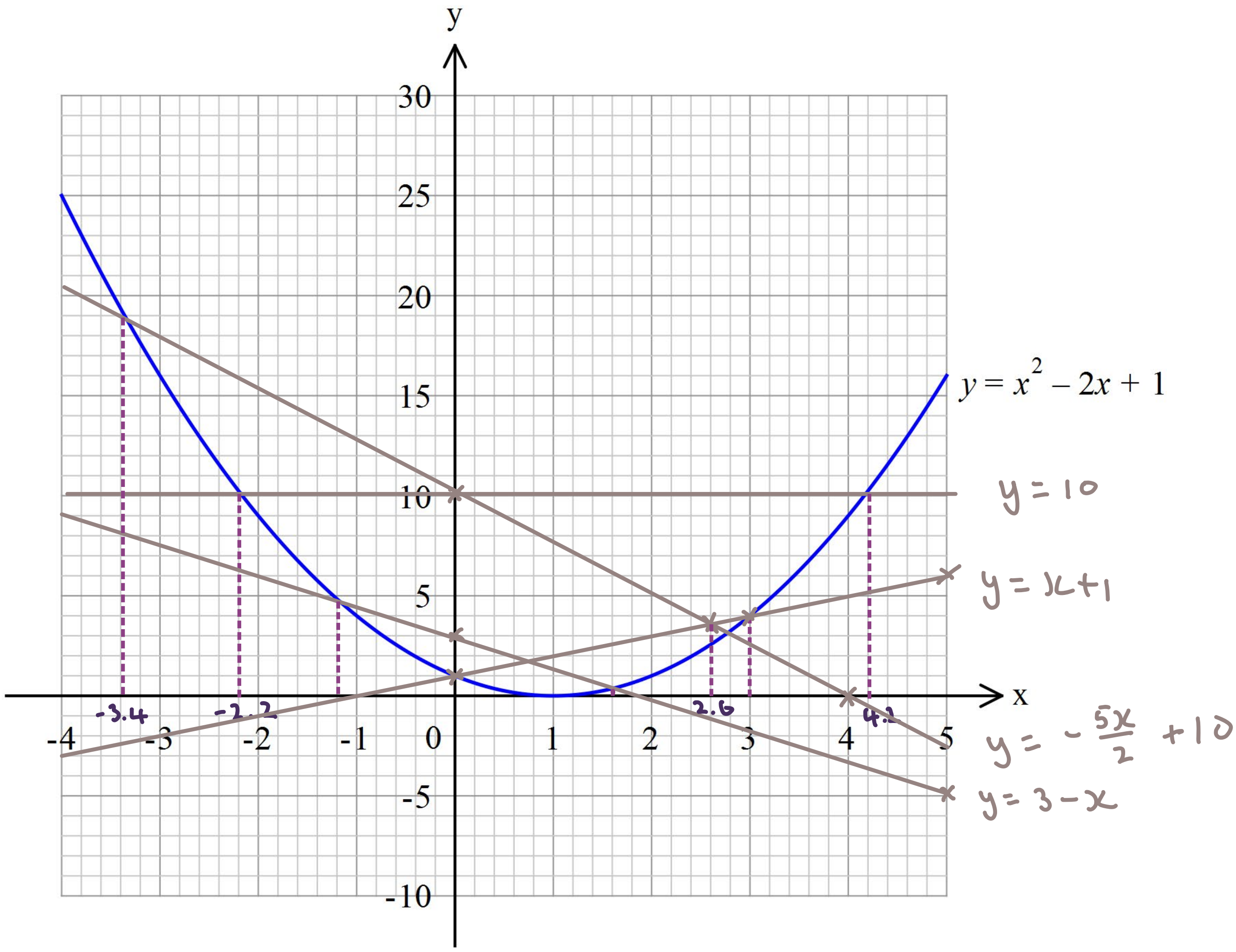
(c) Find the greatest possible value of y and the corresponding value of x in the range.

$y=12$ $x=2.45$ #

[Ans: (a) 2.83 (± 0.4) (b) x = 1.4 or 2.75 (c) y = 12, x = 2.45]

(3) GRAPHICAL SOLUTIONS OF INEQUALITIES

EG 4 The diagram shows the graph of $y = x^2 - 2x + 1$ for $-4 \leq x \leq 5$.



Using a graphical method, find range of values of x such that

(a) $x^2 - 2x + 1 < 10$, → below line

$x^2 - 2x + 1 < 10$

Draw $y = 10$

From graph,

$-2.2 < x < 4.2$ //

(b) $x^2 - 2x + 1 \geq x + 1$, above or touching line

$x^2 - 2x + 1 \geq x + 1$

Draw $y = x + 1$

From graph:

$-4 \leq x \leq 0$ or $3 \leq x \leq 5$ //

(c) $x^2 - x - 2 \leq 0$. below or touching

$x^2 - x - 2 \leq 0$

$x^2 - x - x - 2 + 3 \leq 0 - x + 3$

$x^2 - 2x + 1 \leq 3 - x$

Draw $y = 3 - x$

From graph,

$-1.2 \leq x \leq 1.6$ //

(d) $2x^2 + x - 18 > 0$ – above line

$2x^2 + x - 18 > 0$

$x^2 + \frac{x}{2} - 9 > 0$

$x^2 + \frac{x}{2}x - \frac{5x}{2} - 9 + 10 > -\frac{5x}{2} + 10$

$x^2 - 2x + 1 > -\frac{5x}{2} + 10$

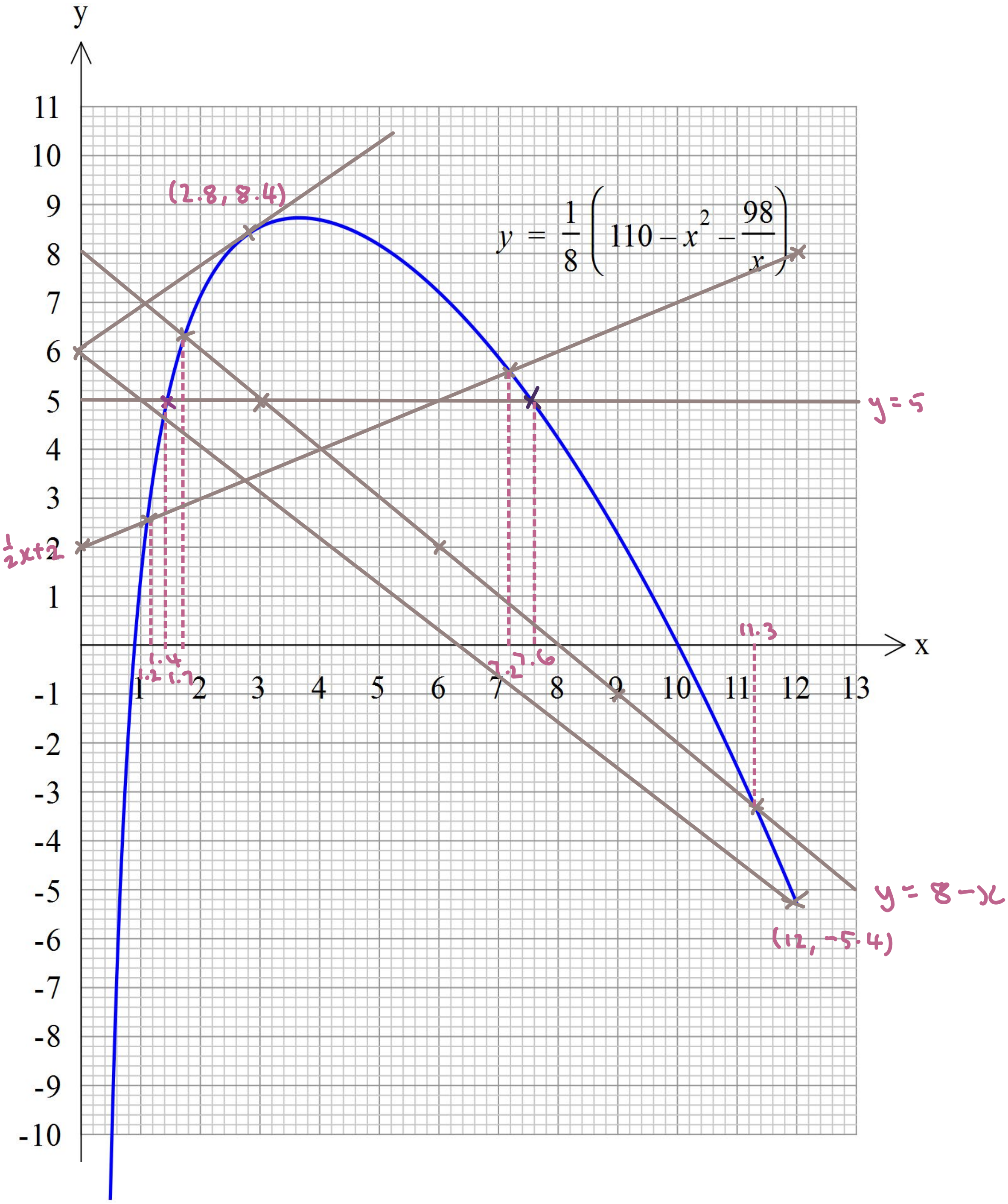
Draw $y = -\frac{5x}{2} + 10$

From graph:

$-4 \leq x < -3.4$ or $2.6 < x \leq 5$ //

Note: We will learn how to solve quadratic inequality using algebraic method later in the topic Quadratic Functions.

EG 5 The diagram shows the graph of $y = \frac{1}{8}(110 - x^2 - \frac{98}{x})$ for $0 < x \leq 12$.



(a) Use your graph to solve

(i) the equation $x^3 - 70x + 98 = 0$,

$y = \frac{1}{8}(110 - x^2 - \frac{98}{x})$

$x^3 - 70x + 98 = 0$ $(\div(-x))$

$-x^2 + 70 - \frac{98}{x} = 0$

$110 - x^2 - \frac{98}{x} = 40$

$\frac{1}{8}(110 - x^2 - \frac{98}{x}) = 5$

Draw $y = 5$

From graph,
$x = 1.4$, $x = 7.6$ //

x	3	6	9
y	5	2	-1

(ii) the equation $x^3 - 8x^2 - 46x + 98 = 0$,

$x^3 - 8x^2 - 46x + 98 = 0$ $(\div(-x))$

$-x^2 + 8x + 46 - \frac{98}{x} = 0$

$46 - x^2 - \frac{98}{x} = -8x$

$110 - x^2 - \frac{98}{x} = 64 - 8x$

$\frac{1}{8}(110 - x^2 - \frac{98}{x} = 8 - x$

Draw $y = 8 - x$

$x = 1.7$ or $x = 11.3$ #

(iii) the range of values of x for which $110 - x^2 - \dfrac{98}{x} \geq 4x + 16$.

$110 - x^2 - \frac{98}{x} \geq 4x + 16$

$\frac{1}{8}(110 - x^2 - \frac{98}{x}) \geq \frac{1}{2}x + 2$

Draw $y = \frac{1}{2}x + 2$

From graph,

$1.2 \leq x \leq 7.2$ #

(b) Find the range of values of k if the equation $\dfrac{1}{8}(110 - x^2 - \dfrac{98}{x}) = kx + 6$ has two distinct real roots.

$\frac{1}{8}(110 - x^2 - \frac{98}{x}) = kx + 6$

From graph

$k \geq \frac{6-(-5.4)}{0-12}$

$= -0.95$

$k < \frac{8.4 - 6}{2.8 - 0}$

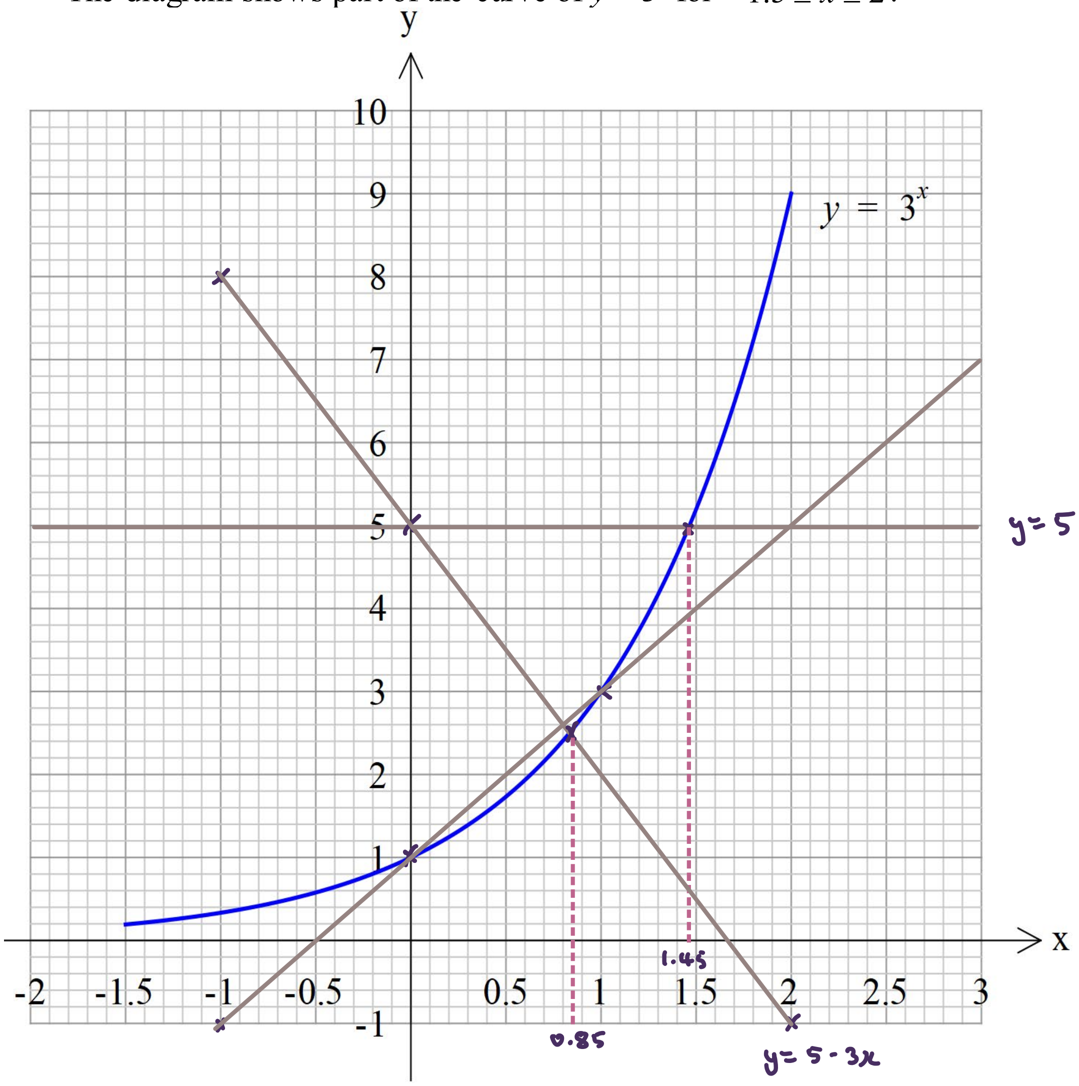
$= 0.857$

$\therefore -0.95 \leq k < 0.857$ #

HOMEWORK 3

LEVEL 2

1 The diagram shows part of the curve of $y = 3^x$ for $-1.5 \le x \le 2$.



(a) Use your graph to solve the following equations:

(i) $3^x = 5$,

Draw $y = 5$

From graph,

$x = 1.45$ #

(ii) $3^x + 3x = 5$.

$3^x + 3x = 5$

$3^x + 3x - 3x = 5 - 3x$

Draw $y = 5 - 3x$

From graph,

$x = 0.85$ #

(b) Using a graphical method, solve the inequality $3^x - 2x - 1 < 0$.

$3^x - 2x - 1 = 0$

$3^x = 2x + 1$

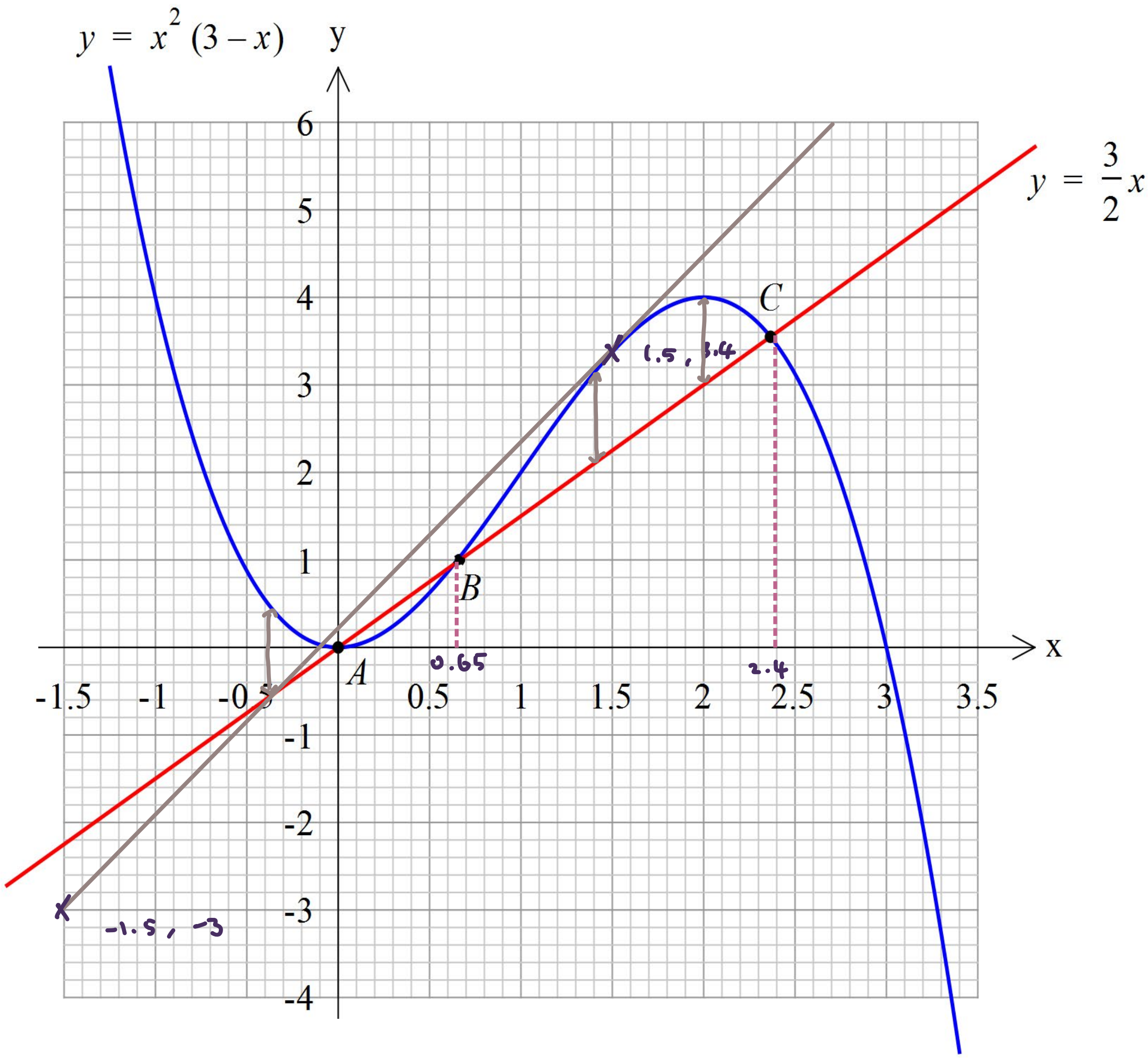
Draw $y = 2x + 1$

From graph

$0 < x < 1$ #

[Ans: (a)(i) x = 1.45 (ii) x = 0.8 (b) 0 < x < 1]

2 The diagram shows part of the graphs of the curve $y = x^2(3-x)$ and the straight line $y = \frac{3}{2}x$. They intersect at the points *A*, *B* and *C* as shown.



Use the graph to answer the questions below.

(a) Write down the cubic equation in the form $2x^3+ax^2+bx+c=0$ which has the x-coordinates of A, B and C as its solutions. What are the solutions of this equation?

$x^2(3-x)=\frac{3}{2}x$

$6x^2-2x^3=3x$

$2x^3-6x^2+3x$

$x=0$ or $x=0.65$ or $x=2.4$ #

(b) Solve the inequality $2x^3-6x^2+3x<0$.

$2x^3-6x^2+3x<0$

$6x^2-2x^3>3x$

$x^2(3-x)>\frac{3}{2}x$

Draw $y=\frac{3}{2}x$

From graph,

$0.65<x<2.4$

$x<0$ #

(c) Find the gradient of the curve at the point $x = 1.5$.

$\frac{3.4-(-3)}{1.5-(-1.5)}=\frac{6.4}{3}$

$=2.13$ #

(d) Without adding any additional lines, determine the number of solutions of the equation $x^2(3-x)-\frac{3}{2}x=1$, explaining carefully how you arrived at your answer.

3. The line passes at 3 points in the curve.

we want to find how many parts in the graph where the y-value of the curve $y=x^2(3-x)$ is exactly one unit more than the y-value of the line $y=\frac{3}{2}x$.

By observation, there are 3 such positions in the graph.

Thus there're 3 solutions to the equation.

[Ans: (a) $x=0, 0.65$ **or** 2.35 *(b)* $x<0$ **or** $0.65<x<2.35$ *(c) 2.25* (± 0.2) *(d) 3]*

FOR YOUR INTEREST

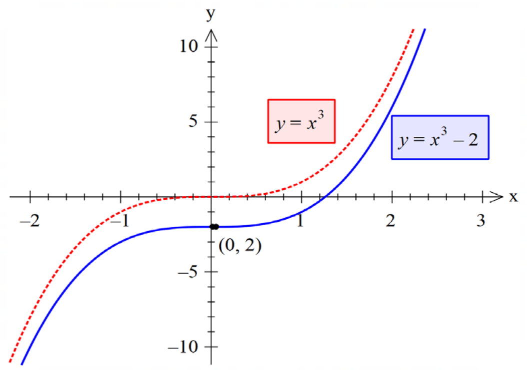
TRANSFORMATION OF GRAPHS

k is a constant and $k > 0$

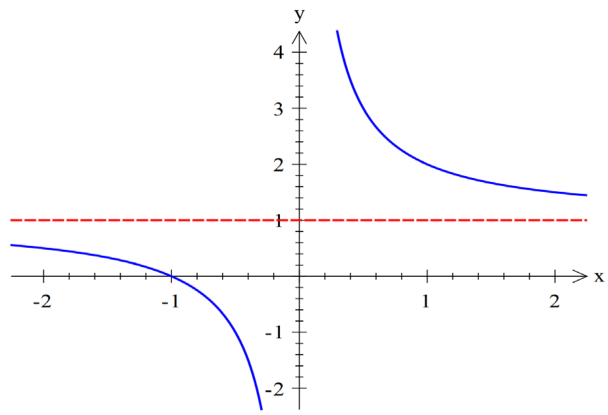
Transformation	Graph of	Obtained from graph of $y=$ f(x) by
Translation	$y = \mathrm{f}(x) \pm k$	shifting vertically upwards/downwards by k units
	$y = \mathrm{f}(x \pm k)$	shifting to the left/right by k units
Reflection	$y = -\mathrm{f}(x)$	reflecting in x-axis
	$y = \mathrm{f}(-x)$	reflecting in y-axis
Scaling	$y = k\mathrm{f}(x)$	shifting the point (x, y) to (x, ky)
	$y = \mathrm{f}(kx)$	Shifting the point (x, y) to $\left(\dfrac{x}{k}, y\right)$

EG 1 **(Translation)**

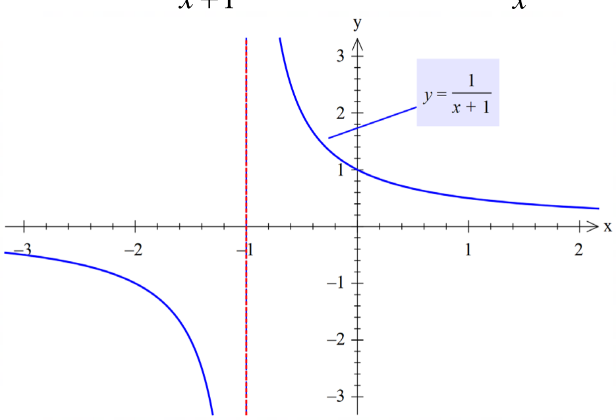
(a) $y = x^3 - 2$ (Shift graph of $y = x^3$ downwards by 2 units)



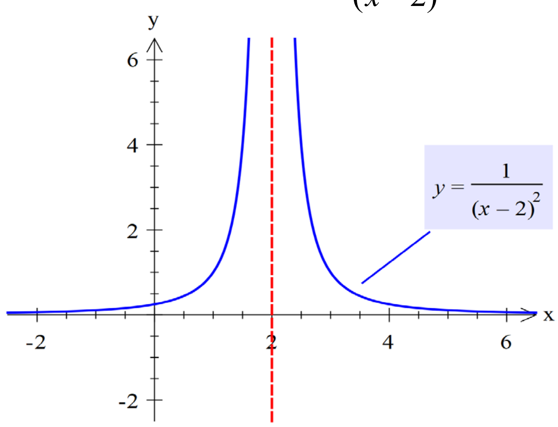
(b) $y = \dfrac{1}{x} + 1$



(c) $y = \dfrac{1}{x+1}$ (Shift graph of $y = \dfrac{1}{x}$ to the left by 1 unit)

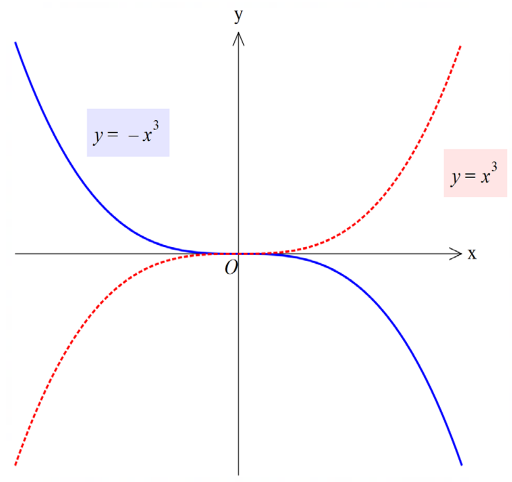


(d) $y = \dfrac{1}{(x-2)^2}$.

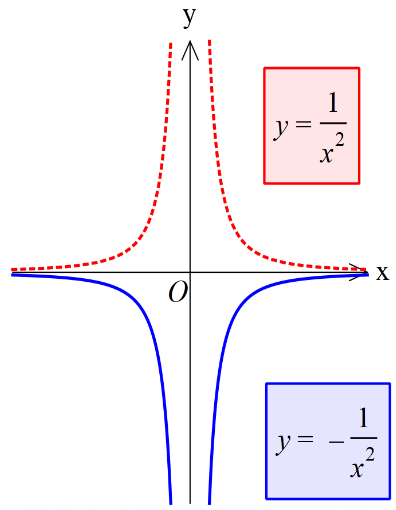


EG 2 **(Reflection)**

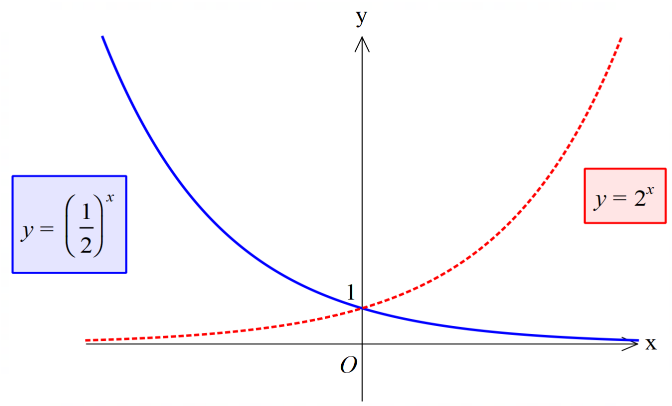
(a) $y = -x^3$ (reflect graph of $y = x^3$ in x-axis)



(b) $y = -\frac{1}{x^2}$.



(c) $y = 2^{-x}$ (reflect the graph of $y = 2^x$ in y-axis)

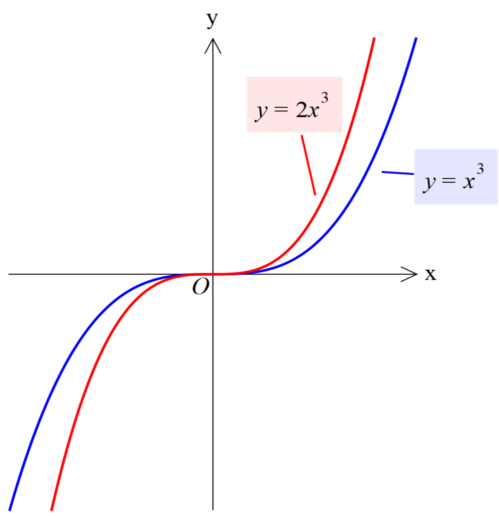


EG 3 **(Scaling)**

Sketch the following graphs on the same diagram.

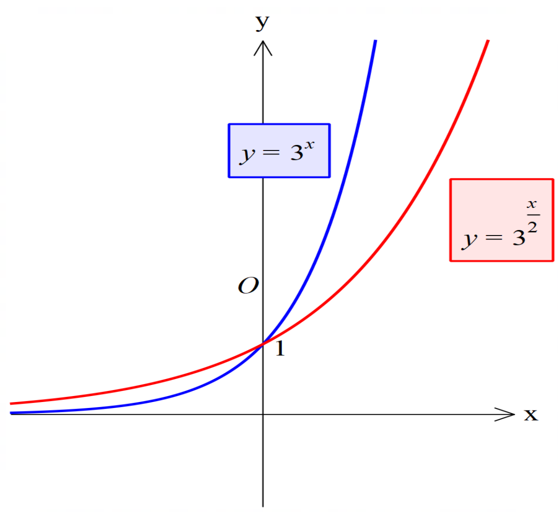
(a) $y = x^3$ and $y = 2x^3$

(Shift (x, y) on $y = x^3$ to $(x, 2y)$)



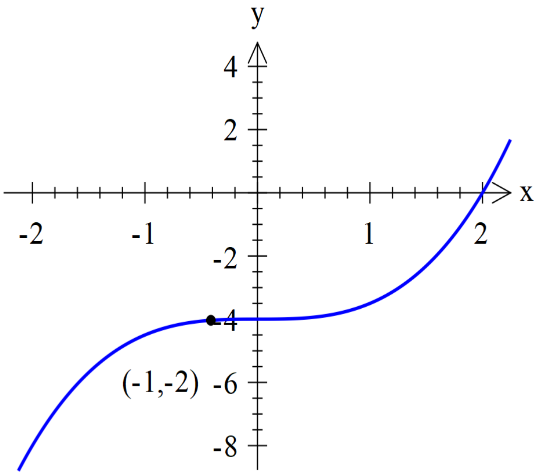
(b) $y = 3^x$ and $y = 3^{\frac{1}{2}x}$

(Shift (x, y) on $y = x^3$ to $(2x, y)$)



EG 4 Sketch each of the following graphs.

(a) $y = \frac{1}{2}x^3 - 4$



(b) $y = 1 - \frac{4}{x^2}$