

(Independent)



**FINAL EXAMINATION 2023** 

# YEAR THREE EXPRESS

## ADDITIONAL MATHEMATICS

## PAPER 2

4049/02

Wednesday

11 October 2023

1 hour 30 minutes

Candidates answer on the Question Paper. No additional materials are required.

### READ THESE INSTRUCTIONS FIRST

Write your index number in the space at the top of this page. Write in dark blue or black pen. You may use an HD pencil for any diagrams or graphs. Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 60.





[Turn over

#### 1. ALGEBRA

Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^{n} = a^{n} + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{2} + \dots + \binom{n}{r}a^{n-r}b^{r} + \dots + b^{n},$$

where *n* is a positive integer and 
$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$$

#### 2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\cos ec^2 A = 1 + \cot^2 A$$

*Formulae* for  $\triangle ABC$ 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

### Answer all the questions.

1. It is given that *a* and *b* are the roots of the quadratic equation  $x^2 - 2x - 1 = 0$  and that a > b. Show that  $\frac{a}{b} = -3 - 2\sqrt{2}$ . [5]

- 2. A circle with centre O passes through the points P(-1,7) and Q(0,8).
  - (a) State the relationship between the perpendicular bisector of PQ and the point O. [1]
  - (b) Find the coordinates of O, given that the line y = 2x 2 passes through the centre of the circle.



(c) Hence find the equation of the circle.

[2]

- 3. The polynomial f(x) is given by  $f(x) = 9x^3 30x^2 23x 4$ .
  - (a) Factorise f(x) completely.

[4]

(b) Hence, prove that the equation  $9x\sqrt{x} - 23\sqrt{x} = 4 + 30x$  has only one real root. Find the solution. [3] 4. (a) Solve the equation  $2^{x+1} + 2^{-x} = 3$ .

(b) The equation of a curve is  $y = 2x^2 - 4ax + b$  where *a* and *b* are non-zero constants. Explain why y > 0 if  $b > 2a^2$ . [4]

[3]

5. (a) Find an expression for x, in terms of e, for which  $\ln(3-x) = \ln x + 3$ . [3]

**(b)** Solve the equation  $\log_5 x + \log_{25} x = 4$ .

[3]

6. (a) (i) Prove the trigonometric identity: 
$$\frac{\csc^2 A + 2\cot A}{\left(\cos A + \sin A\right)^2} = \csc^2 A$$
 [4]

(ii) Hence solve the equation:  $\csc^2 A + 2\cot A = 4(\cos A + \sin A)^2$  for  $0 \le x \le 360^\circ$ . [4]

6. (b) Given that  $\frac{1+\sin x + 2\cos x}{1+2\sin x + \cos x} = 1$  and x is acute, find the exact value of  $\cos x$ . [4]

7. (a) (i) Write down the first four terms in the expansion of  $(1-4x)^6$ . [2]

(ii) Hence find the coefficient of  $x^3$  in the expansion of  $(3+x^2)(1-4x)^6$ . [2]

7. (b) Explain why there is no constant term in the expansion of 
$$\left(2x + \frac{1}{x^3}\right)^{18}$$
. [3]

A solid was heated and left to cool in a container in a room. The difference between its temperature and 8. the surrounding room temperature at time t hours was  $T^{\circ}C$ . The table shows some recorded values of t and T.

t (hours)	5	10	15	22	25
$T(m{m{\mathcal{R}}})$	15.1	8.4	5.2	2.7	1.6

The variables t and T are related by the equation  $T = ae^{bt}$ , where a and b are constants.

On the grid opposite, draw a straight line graph of ln *T* against *t*. **(a)** [3] [4]

[1]

- Use your graph to estimate the value of *a* and of *b*. **(b)**
- With reference to (a), explain why the value of T cannot be zero. (c)



#### Answers

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2.	(i)	The perpendicular bisector of the chord $PQ$ passes through $O$ , the centre of the circle.					
	(ii)	<i>O</i> (3,4)					
	(iii)	$(x-3)^{2} + (y-4)^{2} = 25$					
3.	(a)	$(x-4)(3x+1)^2$	(b)	<i>x</i> = 16			
4.	(a)	x = -1 or 0					
5.	(a)	$x = \frac{3}{\left(e^3 + 1\right)}$	(b)	<i>x</i> = 73.1			
6.	(aii)	A = 30°,150°,210°,330°	(b)	$\cos x = \frac{1}{\sqrt{2}}$			
7.	(ai)	$1 - 24x + 240x^2 - 1280x^3 + \dots$	(aii)	-3864			
8.	(b)	$a \simeq 28.5, \ b \simeq -0.116$					
	(c)	$T = 0 \Longrightarrow \ln T$ is undefined. Hence the value of T cannot be zero.					