Name:		Index Number:		Class:	

CATHOLIC HIGH SCHOOL
Preliminary Examination
Year 4 (Integrated Programme) – Solutions

MATHEMATICS Paper 1

5 OCTOBER 2020

2 hours

Candidates answer in the space provided on the Question Paper.

---

1 **(a)** Find the fraction which is exactly halfway between $\frac{3}{7}$ and $\frac{6}{7}$.

Answer $\frac{9}{14}$ [1]

(b) Calculate $\sqrt{3.595^2 \times 0.0875^3}$, correct your answer to 4 significant figures.

Answer 0.09305 [1]

---

2 Factorise fully $3ax + 12by - 2bx - 18ay$.

$$\begin{aligned} 3ax + 12by - 2bx - 18ay &= x(3a - 2b) + 6y(2b - 3a) \\ &= x(3a - 2b) - 6y(3a - 2b) \\ &= (x - 6y)(3a - 2b) \end{aligned}$$

Answer [2]

3 **(a)** Simplify $\left(\frac{2}{3}ab^2\right)^2 \div \frac{6b}{a^2}$.

$$\left(\frac{2}{3}ab^2\right)^2 \div \frac{6b}{a^2} = \frac{2}{27}a^4b^3$$

Answer …… [2]

(b) Given that $\left(2^{49} - 2^{50}\right)\left(2^0 - 2\right) = 4^x$, find the value of x without using calculator.

$$\left(2^{49} - 2^{50}\right)\left(2^0 - 2\right) = 4^x$$
$$2^{49}(1-2)(1-2) = 2^{2x}$$
$$x = 24\frac{1}{2}$$

Answer x = …… [2]

4 **(a)** Matthew bought a pair of limited edition sports shoes for \$199 and made a profit of 250% of his cost. Find his selling price.

$$\text{Selling Price} = (199)(350\%)$$
$$= \$696.50$$

OR

$$\text{Selling price} = (250\%)(199) + 199$$
$$= \$696.50$$

Answer \$ …… [2]

(b) A rectangular piece of fabric which measures 270 cm by 420 cm is cut into identical squares to make masks. Find the largest possible area of each square such that there is no leftover fabric.

$$270 = 2 \times 3^3 \times 5$$
$$420 = 2^2 \times 3 \times 5 \times 7$$
$$\text{Largest area} = (2 \times 3 \times 5)^2$$
$$= 900$$

Answer ……cm^2 [3]

5 Tom invested some money in a savings account for 2 years. The rate of compound interest was fixed at 2.88% per annum compounded quarterly.

At the end of the 2 years, there was \$9427 in his account.

How much did Tom invest in the account? Leave your answer to the nearest dollar.

$$A = P\left(1+\frac{\frac{2.88}{4}}{100}\right)^{2\times 4}$$

$$9427 = P\left(1+\frac{0.72}{100}\right)^{8}$$

$$P = \$8901$$

Answer \$..... [2]

6 Mr Lim bought a system 4 aircon for his new home which cost \$4259 on hire purchase. He paid a down payment of \$1500. The remaining sum is to be paid in monthly instalments over 3 years at 1.4% per annum. Find the amount of his monthly instalment.

$$\text{Interest} = \frac{(4259-1500)(1.4)(3)}{100} = \$115.878$$

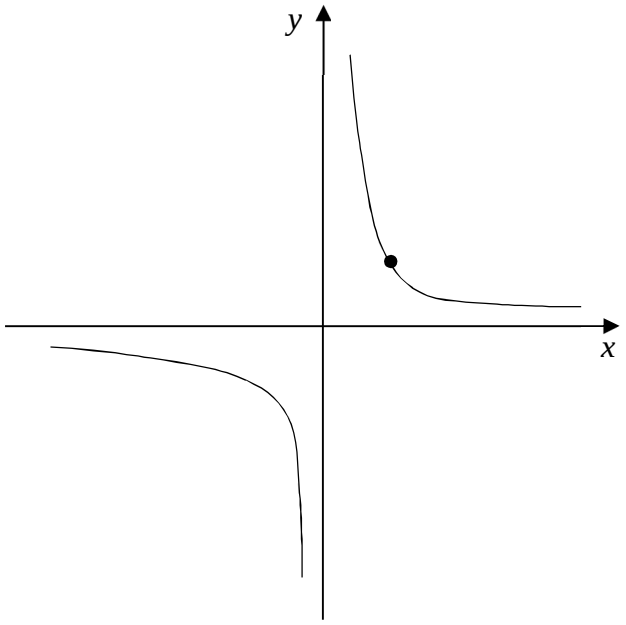
$$\text{Monthly instalment} = \frac{4529-1500+115.878}{36}$$

Answer \$..... [3]

7 The point (1, 1) is marked on each diagram in the answer space.

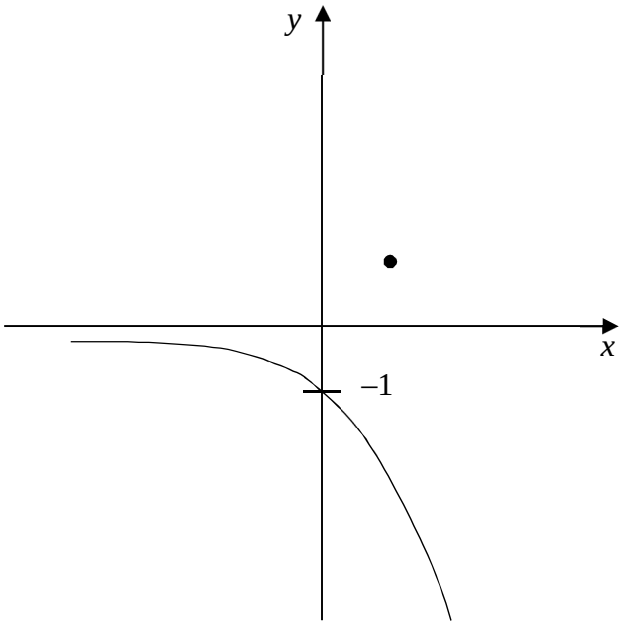
On these diagrams, sketch the graphs of

(i) $y = \frac{1}{x}$,



[1]

(ii) $y = -3^x$.



[1]

8 The scale of a map is 4 cm : 2.5 km.

(a) Write this scale in the form of 1 : *n*.

1 : 62500

Answer …….. [1]

(b) A plot of land is represented by an area 8.5 cm^2 of on the map. Calculate the actual area of lake in m^2, leaving your answers in standard form.

$1\text{cm}^2 : 390625\text{m}^2$

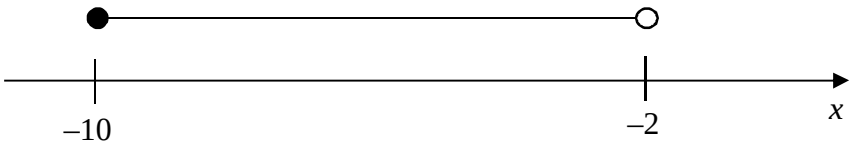
$8.5\text{cm}^2 : 3.32\times10^6\text{m}^2$

Answer ……m^2 [2]

9 **(a)** Solve the inequalities $-1\le\frac{x}{2}+4<3$ and represent the solutions on the number line provided.

$-1\le\frac{x}{2}+4<3$

$-10\le x<-2$ [B1]



Answer ……. [1]

(b) It is given that $-5\le y\le-1$ and *x* and *y* are integer values. Using the answers found in **(a)**, find the maximum value of

(i) x^2-y,

$x^2-y=(-10)^2-(-5)=105$

Answer ……. [1]

(ii) $\frac{y}{x}$.

$\frac{y}{x}=\frac{-5}{-3}=1\frac{2}{3}$

Answer ……. [1]

10 p is directly proportional to square root of q.

q is increased by 125%.

Find the percentage increase in p.

$$p = k\sqrt{q}$$

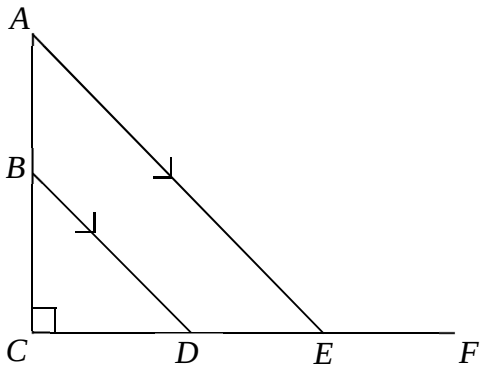
$$\% \text{ increase} = \frac{k\sqrt{2.25q} - k\sqrt{q}}{k\sqrt{q}} \times 100\%$$

$$= 50\%$$

Answer ……% [2]

11 In the diagram, ABC and $CDEF$ are straight lines and are perpendicular to each other. Lines BD and AE are parallel lines.

Given that $\tan D\hat{B}C = x$, find the value of $\cos A\hat{E}F$ in terms of x.

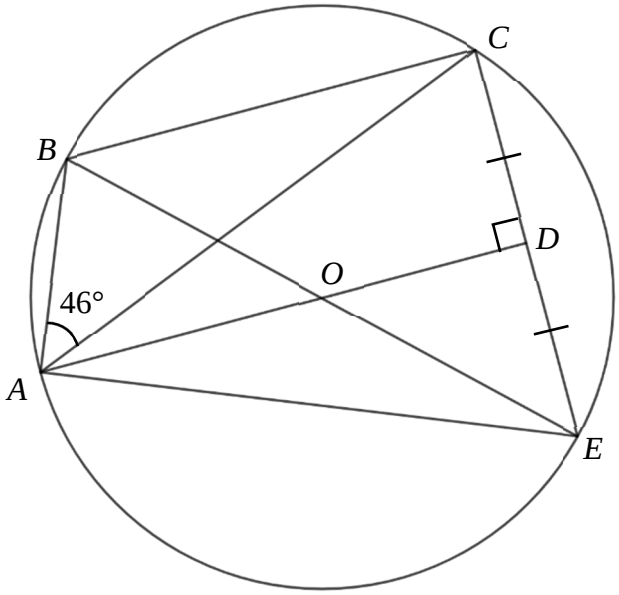


$$BD = \sqrt{1 + x^2}$$

$$\cos A\hat{E}F = -\frac{x}{\sqrt{1 + x^2}}$$

Answer …… [2]

12 The diagram shows a circle *ABCE*, with centre *O*. Lines *AOD* and *BOE* are straight lines. Line *AOD* is the perpendicular bisector of chord *CE*.



Given that angle *BAC* = 46°, find, giving reasons for each answer,

(a) angle *CEB*,

angle *CEB* = 46° (angles in same segment)

Answer ……° [1]

(b) angle *CAE*,

angle *CAE* = 90° – 46° = 44° (right angle in semi-circle)

Answer ……° [1]

(c) angle *ABC*.

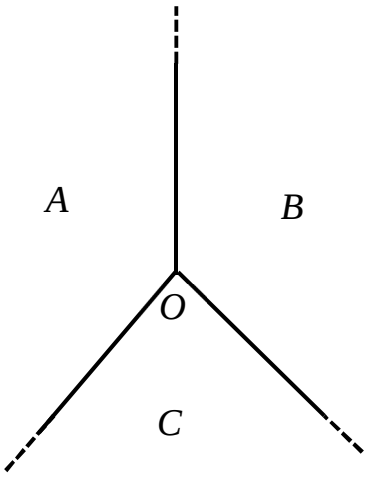
angle $AEC = \dfrac{180° - 44°}{2} = 68°$ (base angles of isosceles Δ)

Angle *ABC* = 180° – 68° = 112° (supplementary angles in opposite segments)

Answer ……° [2]

13 The diagram (not drawn to scale) shows one interior angle of each of three regular polygons *A*, *B* and *C*.

The polygon fit together at *O*. It is given that *C* is an equilateral triangle and the number of sides polygon *A* has is 1.5 times that of polygon *B*.



(a) Find the number of sides of polygon *B*.

$$\frac{180^\circ(a-2)}{a}+\frac{180^\circ(b-2)}{b}=360^\circ-\frac{180^\circ}{3}$$

$$\frac{a-2}{a}+\frac{b-2}{b}=2-\frac{1}{3}$$

$$\frac{1.5b-2}{1.5b}+\frac{b-2}{b}=\frac{5}{3}$$

$$\frac{3b-4}{3b}+\frac{b-2}{b}=\frac{5}{3}$$

$$3b-4+3b-6=5b$$

$$b=10$$

Answer …… [3]

(b) Find the exterior angle of polygon *A*.

$$a=10\times1.5=15$$

$$\text{Ext angle of A}=\frac{360^\circ}{15}=24^\circ$$

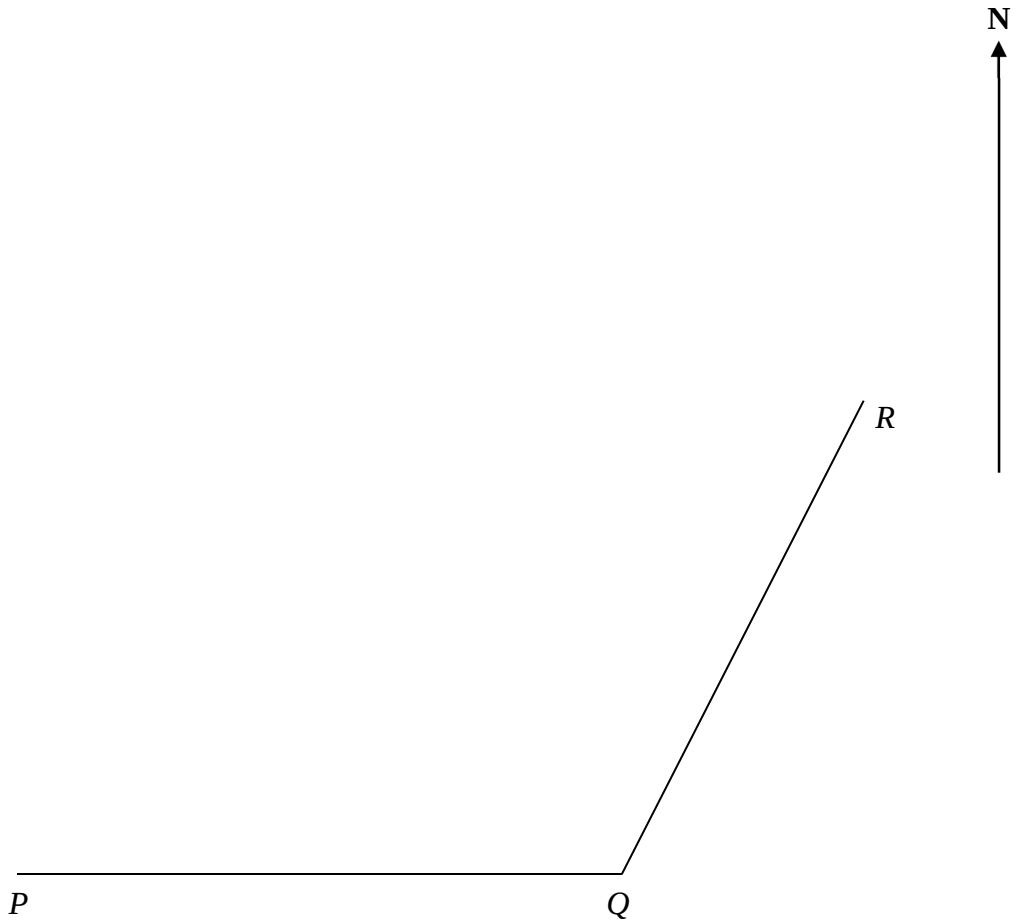
Answer ……° [1]

14 Three ships, *P*, *Q* and *R* are such that *Q* is due east of *P*.
A map is being drawn to a scale of 1 cm : 2 km. The positions of *P, Q* and *R* are shown below. Another ship, *S*, is 18 km from *R* and 20 km from *P*.

(a) Using a ruler and compass only, complete the map to show the position of ship *S*. [1]

(b) Measure the bearing of *S* from *R*. [1]

(c) A lighthouse is located
I equidistant from *P* and *Q*,
II equidistant from *PS* and *RS*.
Mark the location of the lighthouse with a cross on the map and label it "L".
[3]



15 A class of 15 boys and 20 girls took a test. The boys' marks are given in the table below.

Marks	11	12	14	16	17	18	20
Frequency	2	3	5	1	2	1	1

(a) Find the mean mark and standard deviation of the boys.

Mean = 14.4 , SD = 2.60

Answer Mean = ……. [1]

SD = ……. [1]

(b) The mean mark for the girls was 17. Find the mean mark of the class.

$$\text{Mean mark of class} = \frac{(14.4)(15)+(17)(20)}{35}$$

$$= 15.9$$

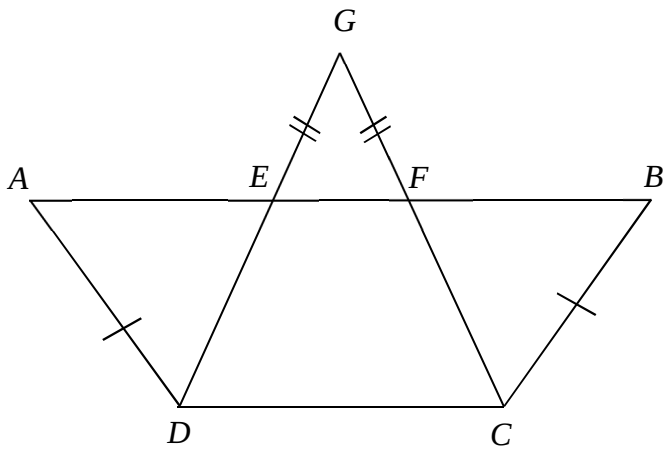
Answer …… [2]

(c) 15 boys from another class has a mean mark of 16.5 and standard deviation of 1.25. Using your answers from **(a)**, explain what this tells us about the second group of boys compared with the first.

The boys from the second class did better with a higher mean mark of 16.5 as compared to the first class mean mark of 14.4.

The marks of the boys from the second class has a narrower spread of results (their results are more clustered together) with a standard deviation of 1.25 as compared to the first class' standard deviation of 2.60.

16 The diagram below shows a trapezium *ABCD* where *AD* = *BC*. Lines *DE* and *CF* are produced to meet at point *G* such that *EG* = *FG*. The perimeter of triangle *GDC* is 3 times the perimeter of triangle *GEF*.



(a) Prove that triangle *AED* is congruent to triangle *BFC*.

Angle *GEF* = Angle *GFE* (base angles of isosceles triangle)

Angle *AED* = Angle *BFC* (vertically opposite angles to Angle *GEF* and Angle *GFE*)

Angle *EAD* = Angle *FBC* (angles in isosceles trapezium)

Given *AD* = *BC*, triangle *AFD* is congruent to triangle *BFE* (AAS)

(b) Given further that $AE = 2EF$, $GC = 9$ cm and $EF = 2$ cm, find

(i) the length of FC,

$$\frac{GF}{GC} = \frac{1}{3}$$
$$GF = \frac{1}{3}(9) = 3$$
$$FC = 6 \text{ cm}$$

Answer ……cm [2]

(ii) $\dfrac{\text{Area of triangle } GEF}{\text{Area of triangle } GDC}$,

$$\frac{1}{9}$$

Answer ……... [1]

(iii) $\dfrac{\text{Area of triangle } FBC}{\text{Area of trapezium } ABCD}$.

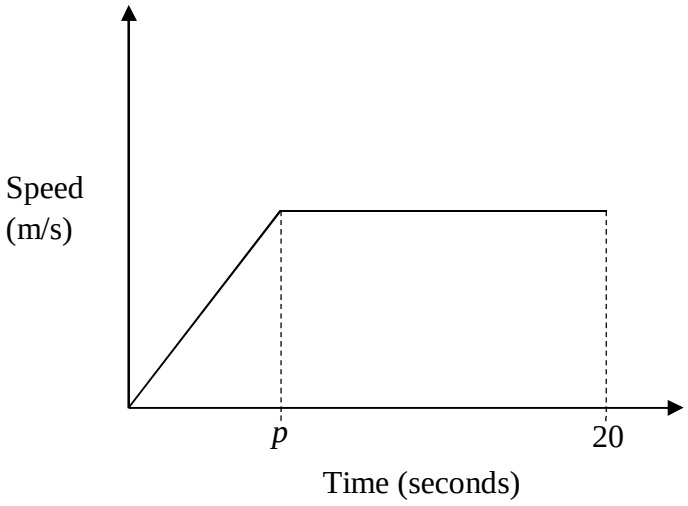
Finding any ratio of triangles of same height in trapezium:

$$\frac{\text{Area of } \Delta BFC}{\text{Area of } \Delta FEC} = \frac{1}{2} \quad \textit{or} \quad \frac{\text{Area of } \Delta EFC}{\text{Area of } \Delta FDC} = \frac{1}{3}$$
$$\frac{\text{Area of } \Delta BFC}{\text{Area of trapezium } ABCD} = \frac{4}{16} = \frac{1}{4}$$

Answer ……... [2]

17 The diagram shows the speed-time graph for a van's journey.

In the first p seconds, the van traveled a distance of 50 m with a constant acceleration of 4 m/s^2.

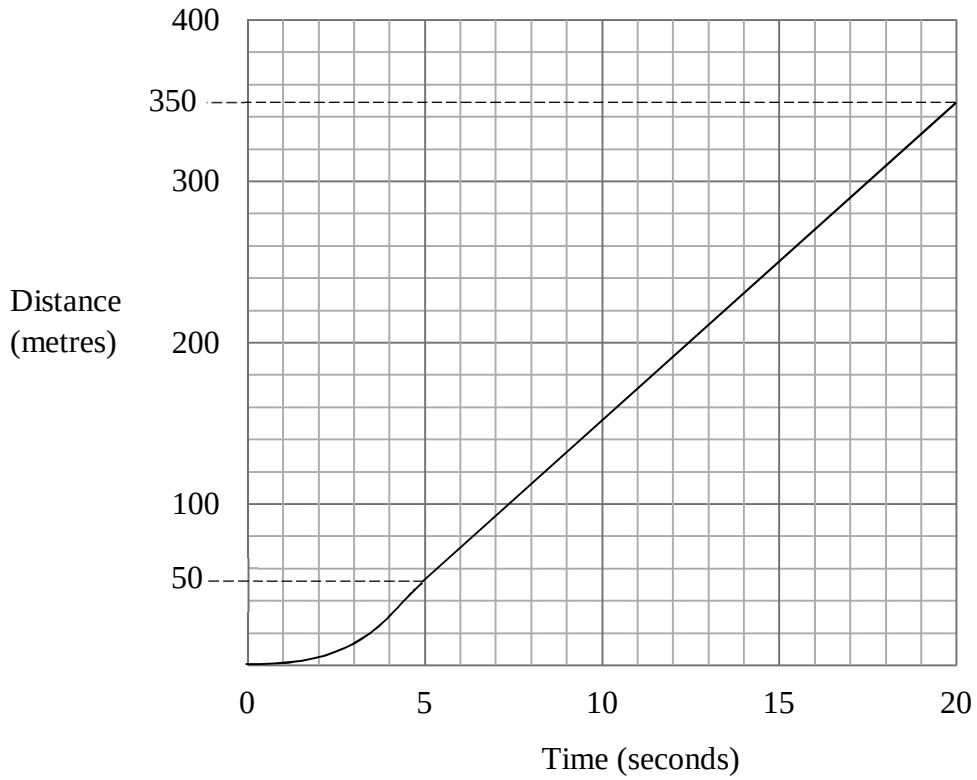


(a) Find the value of p.

$$\frac{1}{2}(4p)(p) = 50$$
$$p = 5$$

Answer p = …… [2]

(b) Use the grid below to sketch the distance-time graph for the journey.



[2]

(a) A car was traveling along the same road with a constant speed of 16 m/s. Explain your answers clearly with workings whether the van will overtake the car during this twenty seconds of journey.

The total distance traveled by the car in 20 sec = (16)(20) =320m, which is a shorter distance than the total distance that the van has traveled. Hence, the van will overtake the car during this 20sec journey.

18 The flight distance between San Francisco and Singapore is estimated to be 7340 nautical miles. The flight duration from San Francisco to Singapore is 16 h 40 m and the time in San Francisco is 15 hours behind the time in Singapore. [1 nautical mile = 1852 meters]

(a) Calculate the average speed, in km/h, of an aeroplane traveling from San Francisco to Singapore.

$$\frac{(7340)(1.852)\text{km}}{16\frac{2}{3}\text{h}}$$
$$= 816 \text{ km/h}$$

Answer ……km/h [2]

(b) An aeroplane leaves San Francisco on Friday 1st May at 21 15 local time. Calculate the date and local time at which the aeroplane would reach Singapore.

3rd May Sunday

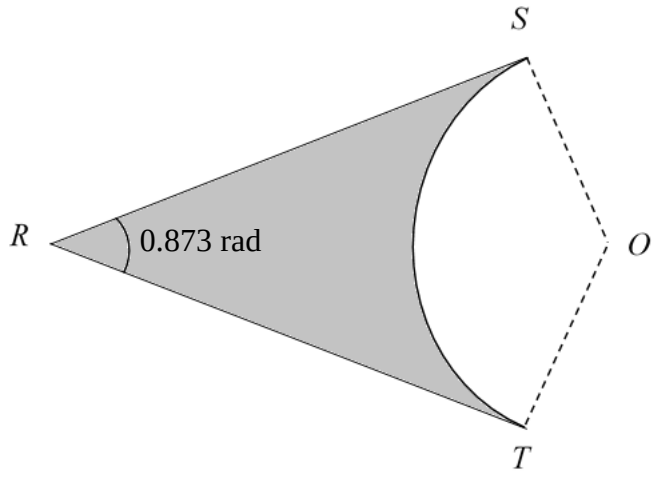
04 55

Answer Date ……

Day ……. [1]

Local Time : ……h [1]

19 *RS* and *RT* are tangents to sector *SOT* with centre *O*. Angle *SRT* is 0.873 radians and the perimeter of the shaded region is 52 cm.



(a) Show that the value of radius *OS* is 7.932 cm, correct to 4 significant figures.

Let the radius of sector be r

$$\tan 0.4365 = \frac{r}{RS}$$

$$RS = \frac{r}{\tan 0.4365}$$

$$2\left(\frac{r}{\tan 0.4365}\right) + (r)(2.2685) = 52$$

$$r = 7.93\text{cm}$$

Answer ……cm [3]

(b) Find the area of shaded region.

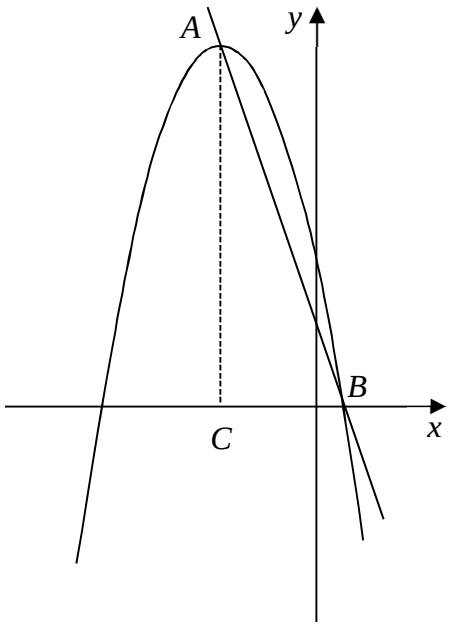
Area of shaded region

$$= \left(\frac{7.9319}{\tan 0.4365}\right)(7.9319) - \frac{1}{2}(7.9319)^2(2.2685)$$

$$= 63.5 \text{ cm}^2$$

Answer ……cm^2 [3]

20 The diagram shows a quadratic curve with a straight line $y = 9 - 9x$ intersecting the curve at its turning point, point A and its x–intercept, point B. The point C is vertically below point A on the x–axis. The area of triangle ABC is $91\frac{1}{8}$ units^2.



(a) Show that the x–coordinate of A is $-3\frac{1}{2}$. [4]

$B(1,\ 0)$

$\frac{1}{2}(1-x)(9-9x) = 91\frac{1}{8}$

$9 + 9x^2 - 18x = 182\frac{1}{4}$

$(2x-11)(2x+7) = 0$

$x = 5\frac{1}{2}$ (rej) or $x = -3\frac{1}{2}$

(b) Hence, find the equation of the curve.

$y = a\left(x + 3\frac{1}{2}\right)^2 + 40\frac{1}{2}$

$0 = a(4.5)^2 + 40\frac{1}{2}$

$a = -2$

$y = -2\left(x + 3\frac{1}{2}\right)^2 + 40\frac{1}{2}$

OR

$y = a(x+8)(x-1)$

When $x = -3\frac{1}{2}, y = 40\frac{1}{2}$,

$40\frac{1}{2} = a\left(-3\frac{1}{2} + 8\right)\left(-3\frac{1}{2} - 1\right)$

$a = -2$

$y = -2(x+8)(x-1)$

Answer $y =$ …… [3]