Chapter 8

Oscillations

A metronome, which is basically an upside down pendulum, is a device that produces regular ticks (beats). It dates back to the early 19th century. A metronome is used by some performing musicians for practice in maintaining a consistent tempo; it gives the composer an approximate way of specifying the tempo. From its inception, however, the metronome has been a highly controversial tool, and there are musicians who reject its use altogether.

\- http://en.wikipedia.org/wiki/Metronome

Oscillations

Candidates should be able to:

(a) describe simple examples of free oscillations.

(b) investigate the motion of an oscillator, using experimental and graphical methods.

(c) show an understanding of and use the terms amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency.

(d) recall and use the equation $a = -\omega^2 x$ as the defining equation of simple harmonic motion.

(e) recognise and use $x = x_0 \sin \omega t$ as a solution to the equation $a = -\omega^2 x$.

(f) recognise and use the equations $v = v_0 \cos \omega t$ and $v = \pm \omega \sqrt{x_0^2 - x^2}$.

(g) describe, with graphical illustrations, the changes in displacement, velocity and acceleration during simple harmonic motion.

(h) describe the interchange between kinetic energy and potential energy during simple harmonic motion.

(i) describe practical examples of damped oscillations with particular reference to the effects of the degree of damping and to the importance of critical damping in applications such as a car suspension system.

(j) describe practical examples of forced oscillations and resonance.

(k) describe graphically how the amplitude of a forced oscillation changes with driving frequency near to the natural frequency of the system, and understand qualitatively the factors which determine the frequency response and sharpness of the resonance.

(l) show an appreciation that there are some circumstances in which resonance is useful and other circumstances in which resonance should be avoided.

RELEVANT E-LEARNING WEBSITES

Applets

1) Spring-Mass SHM http://ngsir.netfirms.com/j/Eng/springSHM/springSHM_js.htm
2) Mass Spring Lab https://phet.colorado.edu/en/simulation/mass-spring-lab

Online lessons

3) University of Salford Online Lessons: http://www.acoustics.salford.ac.uk/feschools/waves/shm.htm#motion
4) PHYSCLIP Online Lessons
   a. Mechanics: SHM (http://www.animations.physics.unsw.edu.au/mechanics/chapter4_simpleharmonicmotion.html)
   b. Waves and Sound: Oscillations (http://www.animations.physics.unsw.edu.au/waves-sound/oscillations/)
5) Schoolphysics: http://www.schoolphysics.co.uk/age16-19/Mechanics/Simple%20harmonic%20motion/
6) How Radio works http://electronics.howstuffworks.com/radio8.htm

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Playlist of teaching videos and lecture examples:

https://youtube.com/playlist?list=PL_b5cjrUKDlYadbhfFX4Z8AwqHX1vdl9l

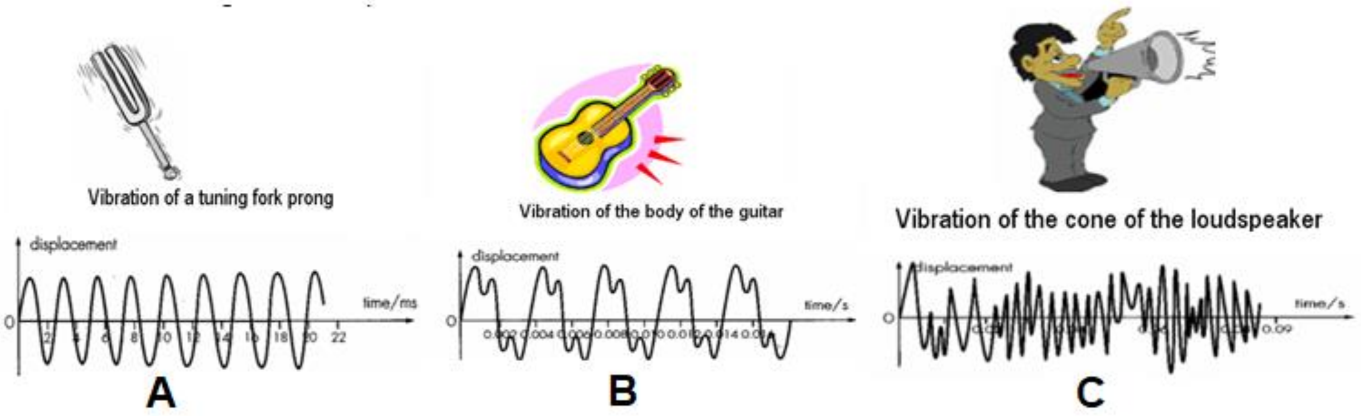
8.0 Introduction

Oscillations are very common in everyday life such as the motion of a clock pendulum, the vibrations of strings in musical instruments, the vertical oscillations of a car after passing over a bump on the road or even the vibrations of atoms in a lattice.

Oscillations can occur when displacing an object that is in a state of stable equilibrium from its equilibrium position results in a restoring force acting on the object directed towards the equilibrium position. The motion is said to be *periodic* if the displacement of a body in oscillatory motion repeats itself at equal intervals. The full range of its motion that is repeated is referred to as a cycle and the time taken for the body to go through a cycle is the period of the oscillation.

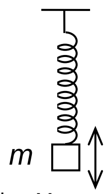
<u>Example 8.1:</u> Periodic Motion

Which of the following cases are periodic oscillations?

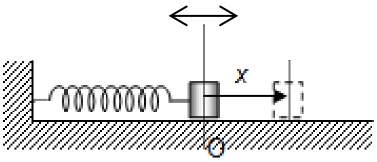


8.1 Simple Harmonic Motion

The free oscillation of a mass attached to a spring is a very special example of a periodic oscillation that we call simple harmonic motion (SHM). It is special because the displacement of the mass about its equilibrium position varies sinusoidally. We will prove this analytically but first, let us look at an experiment that shows the sinusoidal variation of its displacement with time.



Spring-Mass system in vertical SHM



Spring-Mass system in horizontal SHM

Spring-Mass System

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (*)$$

where T = period of oscillation
m = mass of the oscillating object
k = spring constant

(*) See Appendix 2 for the derivation of this relationship.

We can obtain a graph of the variation in the displacement of the mass hanging from a spring via the following experiment. We attach a pen to the mass such that its motion is traced onto a vertical sheet of graph paper that is pulled to the left at a constant rate.

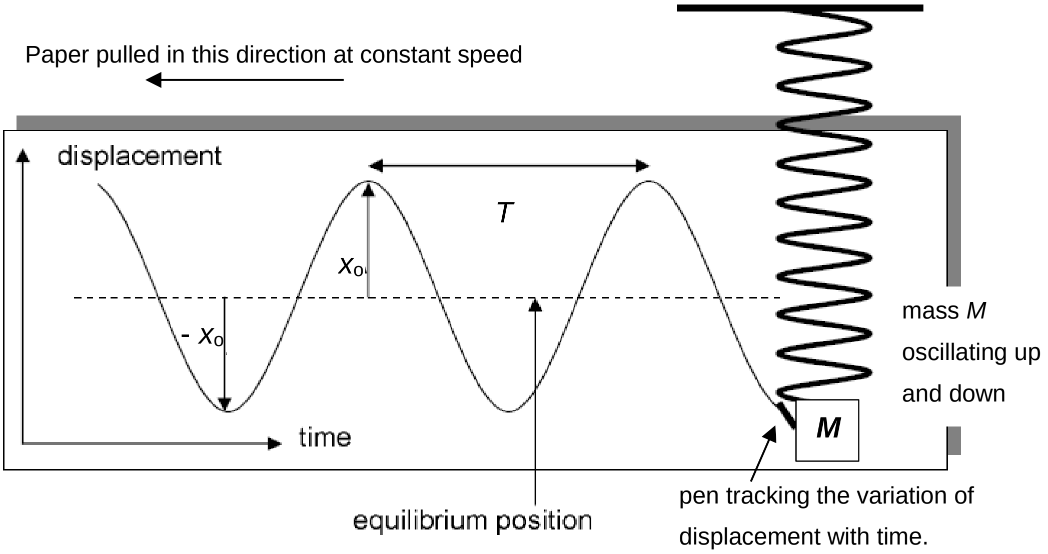


Fig. 8.1. Tracking the oscillation of a vertical spring-mass system

watch MIT Physics Demo -- Spray Paint Oscillator https://youtu.be/P-Umre5Np_0

8.1.1 Terms used to describe SHM (with reference to Fig 8.1)

Terms	Definitions
Displacement, x	The displacement x is the linear distance of the mass from its equilibrium position (x = 0) in a specified direction. It is a **vector**; the + / – sign tells the direction of displacement from equilibrium. If you set the direction for displacement above equilibrium to be positive, then displacement below the equilibrium would be negative and *vice versa*.
Equilibrium Position (x = 0)	When the oscillating mass is at equilibrium position, the **net force acting on the mass is zero.**
Amplitude, x_o	It is the **magnitude** of the maximum displacement of the mass from the equilibrium position. It is a scalar.
Period, T	It is the time taken for the mass to complete one cycle of oscillation. S.I. unit: second.
Frequency, f	It is the number of cycles per unit time: $f = \frac{1}{T}$. S.I. unit: hertz (Hz). 1 Hz = 1 s^{-1}.
Angular frequency, ω	It is the rate of change of the ***phase angle*** (introduced in Section 8.2) with respect to time: $\omega = \frac{d\theta}{dt}$. S.I. unit: rad s^{-1}. A cycle of oscillation is associated with a phase angle of 2π rad, $\omega = \frac{2\pi}{T} = 2\pi f$.

8.1.2 SHM - Displacement-time (*x*-*t*) relationship

From Fig. 8.1, we see that the time variation of the displacement of the mass obtained in the experiment is remarkably similar to a sine graph as shown in Fig 8.2.

The equation of such a graph can be written as follows,

$$x = x_0 \sin \omega t$$

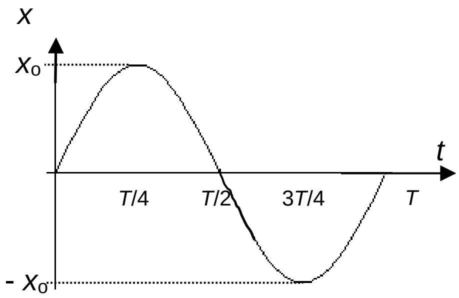
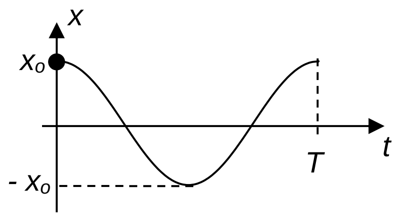
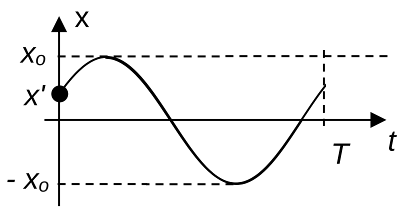


Fig. 8.2. Displacement-time graph of mass

where $\omega = \frac{2\pi}{T}$ is known as the ***angular frequency*** and x_o is the ***amplitude*** of the oscillation.

We can see from this graph that when we start timing, the displacement of the mass is zero, which means that the mass is at its equilibrium position at that instant. If the mass is not at the equilibrium position but at some other location at the instant when we start timing, we can adjust the equation easily to fit the graph obtained in the experiment.

Position of mass when timing starts ($t = 0$)	**Displacement-time (*x*- *t*) Graph**	**Displacement-time Equation**
Mass is *released from rest* at *maximum displacement* from equilibrium position.		$x = x_0 \cos \omega t$ where x_o is the amplitude.
Mass is moving with *initial velocity v* at *displacement x'*		$x = x_0 \sin(\omega t + \phi_1)$ or $x = x_0 \cos(\omega t + \phi_2)$ where ϕ is the initial phase angle (not in syllabus)

Note: ωt is in radians. Remember to set your calculator to **RADIAN mode** before calculation!

8.1.3 SHM - Velocity-time (*v*-*t*) relationship and acceleration-time (*a*-*t*) relationship

From the topic of kinematics, we know that from the displacement-time relationship of an object, we can obtain its corresponding velocity-time and acceleration-time relationship by differentiating the equations or analysing the gradients of the tangents at the various points on the graphs accordingly (see Fig. 8.3).

We can similarly do so for SHM.

The table illustrates two examples.

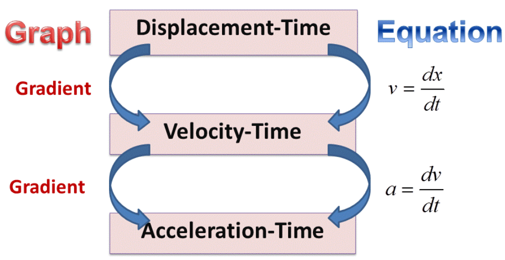
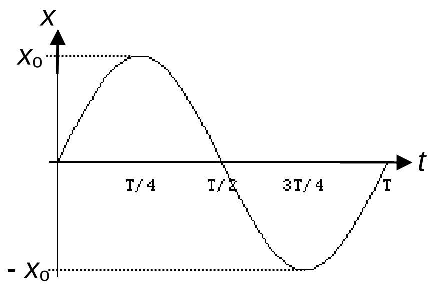
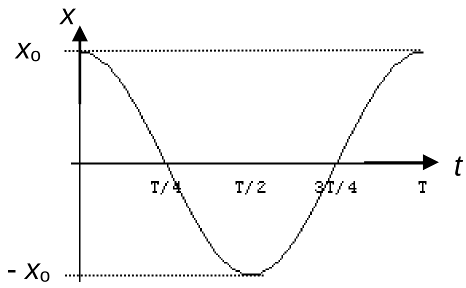
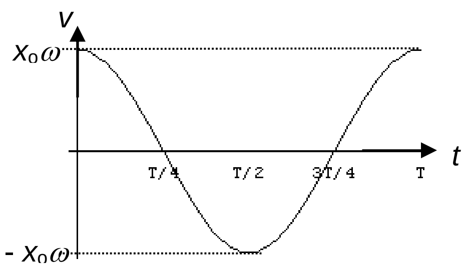
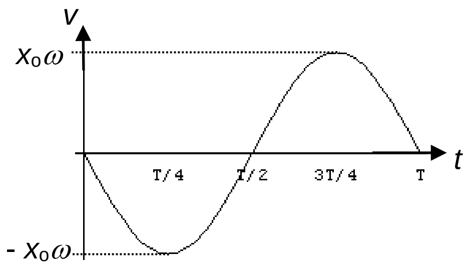
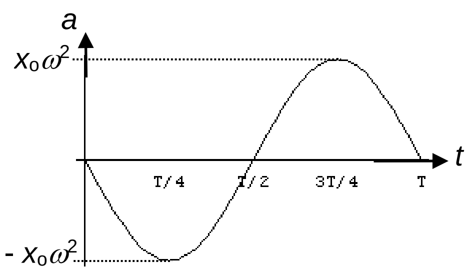
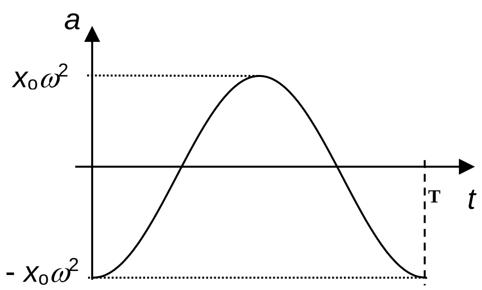


Fig 8.3

t = 0 (initial conditions)	**Example 1: $x = 0$, v is at maximum value**	**Example 2: $x = x_o$, $v = 0$**
***x-t* graph**		
Equation	$x = x_0 \sin \omega t$	$x = x_0 \cos \omega t$
***v-t* graph**		
Equation	$v = \frac{dx}{dt} = \omega x_0 \cos \omega t$	$v = \frac{dx}{dt} = -\omega x_0 \sin \omega t$
***a-t* graph**		
Equation	$a = \frac{dv}{dt} = -\omega^2 x_0 \sin \omega t$	$a = \frac{dv}{dt} = -\omega^2 x_0 \cos \omega t$

From the graphs and the equations above, together with the fact that $-1 \leq \sin\theta \leq 1$ and $-1 \leq \cos\theta \leq 1$,

we can see that:

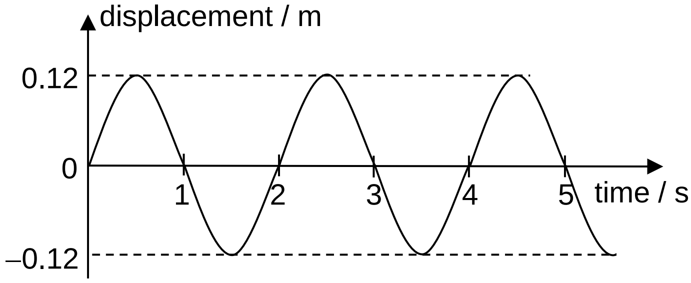
(1) Maximum value of velocity, v_o : $v_0 = \omega x_0$

(2) Maximum value of acceleration, a_o : $a_0 = \omega^2 x_0$

where ω is the angular frequency and x_o is the amplitude of the oscillation.

<u>Example 8.2</u>: Describing SHM in terms of time

The pendulum bob in a particular clock oscillates so that its displacement from a fixed point is as shown.



(a) By taking the necessary readings from the graph, determine for this simple harmonic oscillator

(i) the amplitude x_o

(ii) the angular frequency, ω

(iii) the maximum velocity v_o and

(iv) the maximum acceleration a_o

(b) Hence, write down the equation that describes the displacement-time relationship.

(c) Sketch labelled graphs, illustrating the corresponding time variation of the oscillator's

(i) velocity,

(ii) acceleration.

<u>Example 8.3:</u> Displacement-time Relationship for SHM

The displacement x (in m) of a body moving in SHM is given by the equation

$$x = 3.5 \sin 4.0t$$

a) What is the amplitude of the motion?

b) What is the angular frequency?

c) What will be the displacement 0.20 s after oscillation has begun?

d) Determine the velocity at time t = 0.50 s.

8.1.4 Definition of SHM – relationship between acceleration and displacement

Comparing the *x*-*t* and *a*-*t* relationship for both the examples in the table in section 8.1.3, we see that $a = -\omega^2 x$. This is actually the defining equation for SHM!

Definition of Simple Harmonic Motion

Simple harmonic motion is a periodic motion in which the **acceleration** of the body

- is **directly proportional** to the **displacement from its equilibrium point**, and
- is always in the **opposite direction to the displacement.**

$$a = -\omega^2 x$$

where ω is the angular frequency and x is the displacement from the equilibrium point.

Notes:

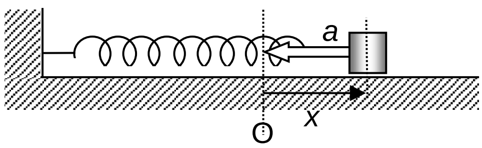
(1) The **negative sign** in the equation indicates that **the acceleration in SHM is always in the direction opposite to the displacement from the equilibrium point**; in other words, the acceleration is always directed towards the equilibrium point.

(2) Since $a = \frac{d^2x}{dt^2}$, we can write $\frac{d^2x}{dt^2} = -\omega^2 x$. This is a 2nd order differential equation and $x = x_0 \sin(\omega t + \phi)$ is a general solution where ϕ depends on the position of the oscillator when observation starts.

You are not required to know how to solve differential equations for the A-level Physics syllabus. However, by differentiating the expression $x = x_0 \sin(\omega t + \phi)$ twice, you can verify that $\frac{d^2x}{dt^2} = -\omega^2 x$.

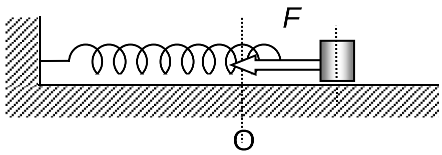
Case Study: SHM of Horizontal Spring-Mass System Explained Using Dynamics

Let us consider a mass oscillating at the end of a horizontal light spring. At the start of section 8.1, we said that the spring-mass system is an example of SHM. We shall now prove that indeed, the mass satisfies the defining equation of SHM. Ideally, the spring obeys Hooke's Law and the mass of the spring is negligible, the mass slides without friction on the horizontal surface. Point O is the equilibrium position of the mass, where the spring is unstretched.

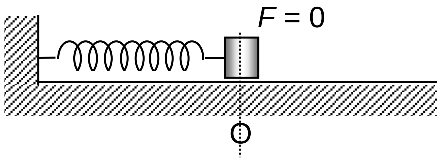


Let us now look at how the restoring force *F* exerted by the spring on the mass varies with displacement *x*:

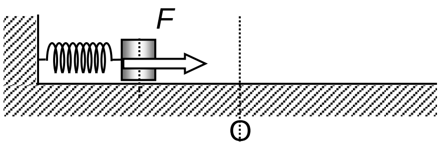
(a) when mass is displaced to the right $(x > 0)$, causing the spring to be stretched, *F* is towards the left.



(b) when the mass is at its equilibrium position $(x = 0)$ (spring is unstretched), *F* is zero.



(c) when the mass is displaced to the left $(x < 0)$, causing the spring to be compressed, *F* is towards the right.

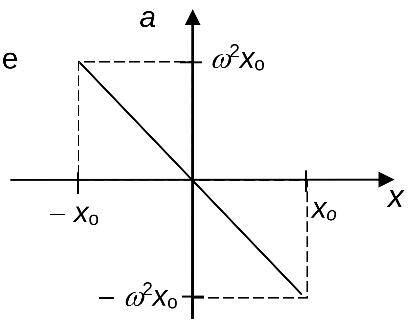


Hence, we see that

1. the **resultant restoring force is always towards the equilibrium position.**
2. the **direction of this resultant force is always opposite to that of the mass's displacement** at all points, except at equilibrium position, where $a = x = 0$.
3. According to Hooke's Law, $F = -kx$, where *k* is the spring constant. The restoring force, *F*, is proportional to the displacement, *x*, but directed opposite to it (as indicated by the negative sign).

   By Newton's 2nd Law, $F_{net} = ma$

   Since F_{net} is provided by the restoring force of the spring acting on the oscillating mass *m*,

   $$ma = -kx$$

   $$a = -\frac{k}{m}x$$

   Hence, the **acceleration is**

   - **opposite in direction to the displacement** and
   - **proportional to the displacement, with proportionality constant -*k* / *m*.**

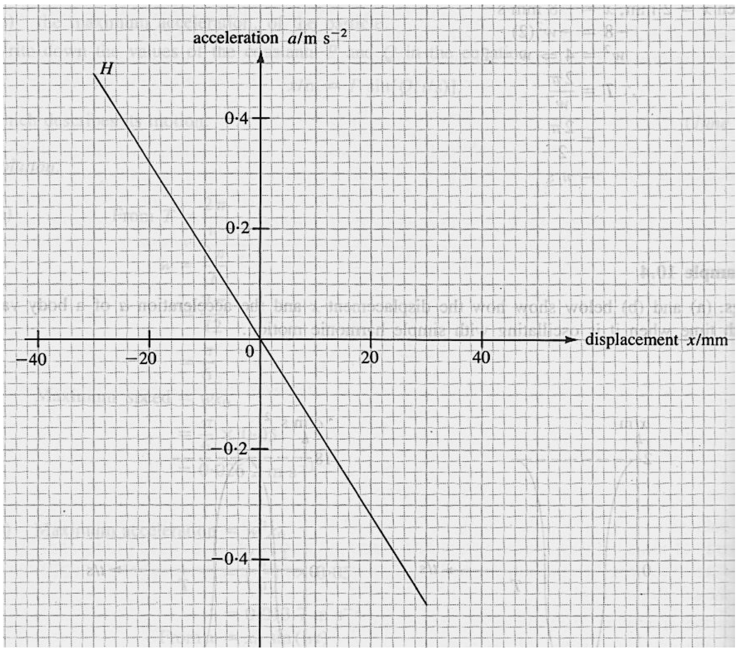
   which is the definition of SHM!

4. Comparing the expression above with the SHM equation $a = -\omega^2 x : \omega^2 = \frac{k}{m} \rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

   This is the expression we quoted for the period of the spring-mass system earlier. Note that the period of the oscillation depends on the physical properties of the system.

Example 8.4: Relationship between acceleration and displacement for SHM

The graph represents the motion of a particle.



Find

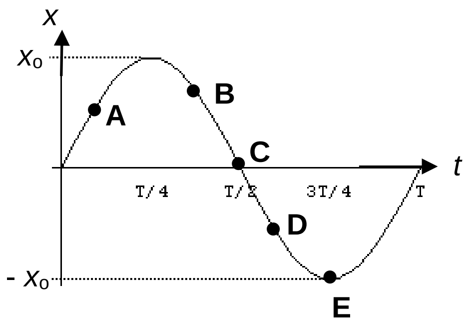
(a) the amplitude of oscillation,

(b) the angular frequency of oscillation,

(c) the period of oscillation.

Example 8.5: Relationship between acceleration and displacement for SHM

For each box in the table below, indicate whether the direction of the displacement *x,* velocity *v* and acceleration *a* of each point on the displacement time graph is "+", "-" or "0".



	A	B	C	D	E
x					
v					
a					

8.1.5 Simple Harmonic Motion: velocity-displacement (*v*-*x*) relationship

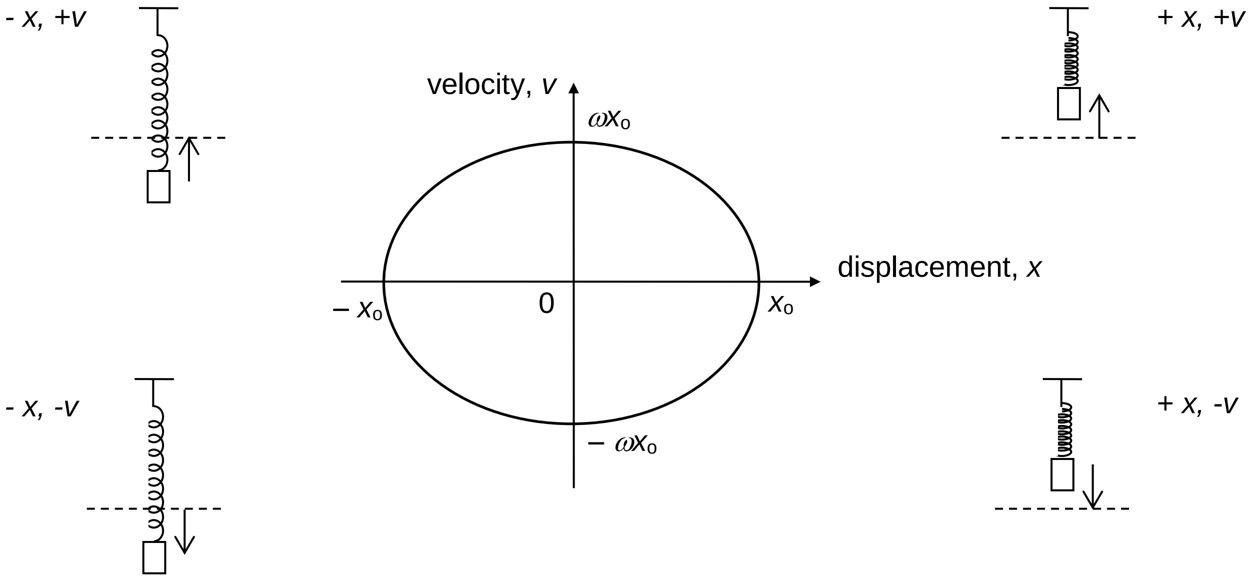
The velocity-displacement (*v*-*x*) relationship for SHM is given by

$$v = \pm\omega\sqrt{\left(x_0^2 - x^2\right)}$$

Note:

1) The equation is provided in the A-level Physics formulae list. It is applicable no matter where the body is when observation starts.
2) The "±" sign implies that the body in SHM always passes through <u>each position</u> back and forth with the same speed.
3) When the body is at maximum displacement $(x = x_0)$, $v = 0$.

   At the equilibrium position $(x = 0)$, the speed is maximum, $v_0 = \omega x_0$.
4) The *v*-*x* graph is an **ellipse** as shown.



Derivation of the relationship between velocity and displacement (not in syllabus)

Consider $x = x_0 \sin(\omega t + \phi)$..........(1)

Differentiate (1) with respect to t : $v = \dfrac{dx}{dt} = \omega x_0 \cos(\omega t + \phi)$........ (2)

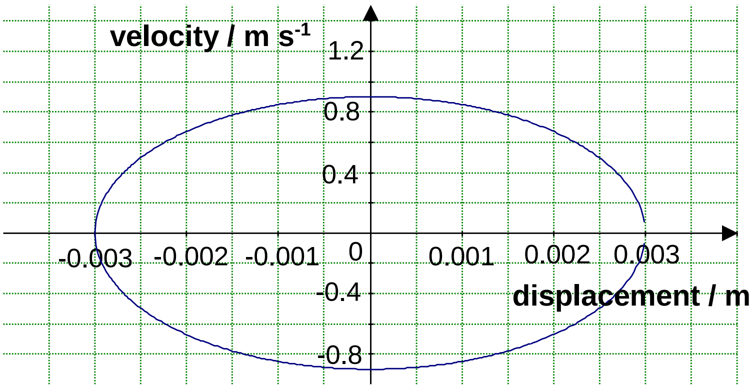
Given that $\sin^2\theta + \cos^2\theta = 1$, $\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

Substitute into equation (2): $v = \omega x_0\left[\pm\sqrt{1 - \sin^2(\omega t + \phi)}\right]$

$$\Rightarrow v = \pm\omega\sqrt{x_0^2 - x_0^2\sin^2(\omega t + \phi)}$$

$$\Rightarrow v = \pm\omega\sqrt{x_0^2 - x^2}$$

Example 8.6: **Relationship between velocity and displacement for SHM**



(a) Explain why there are two values of velocity for zero displacement.
(b) Explain why there are two values of displacement for zero velocity.
(c) Write down the equation, which describes the graph.

Solution:

Example 8.7: **Relationship between velocity and displacement for SHM**

An object moving with simple harmonic motion has an amplitude of 0.020 m and a frequency of 20 Hz. Calculate

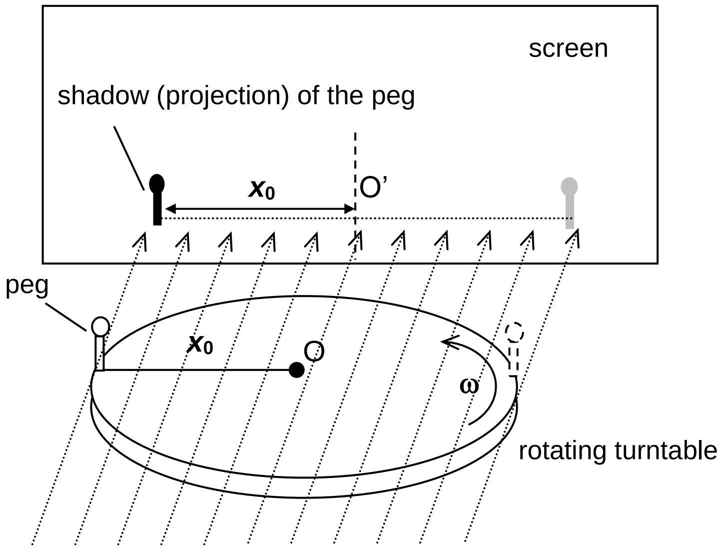
(a) the period of oscillation.
(b) the velocities at the equilibrium point and the position of maximum negative displacement.

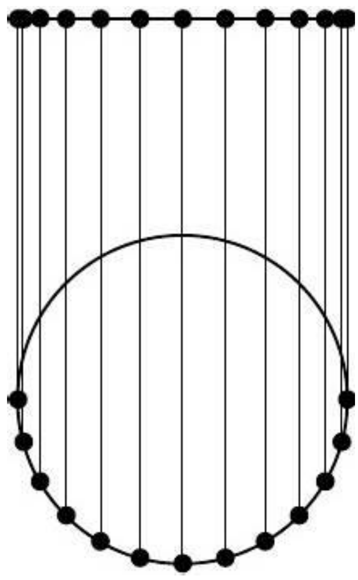
Solution:

8.2 Relationship between Simple Harmonic Motion and Uniform Circular Motion

We have thus far established that for SHM, $x = x_0 \sin \omega t$ where ω is a constant known as the angular frequency, it is also known as the rate of change in phase angle θ. Let us look at an experiment to gain a better understanding of the concept of phase angle.

Consider a peg on a turntable. Parallel light rays from a distant source are shone on the turntable. As the turntable rotates, the peg moves in uniform circular motion and the shadow of the peg against a vertical screen will move back and forth. As it turns out, we can prove that the motion of its shadow is SHM.





Some observations

1) The motion is symmetrical about the fixed point, O' on the screen.

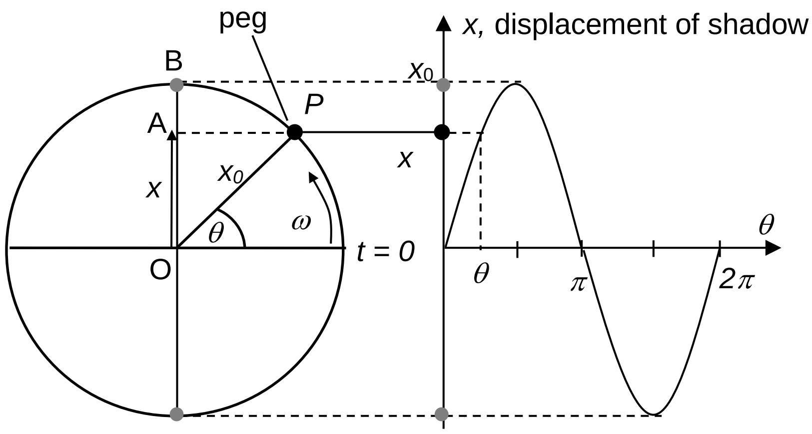
2) The time taken to complete one oscillation (period) is the same as the time taken to complete one cycle. Thus frequency of SHM = frequency of the circular motion.

3) The distance from the fixed point O' to the extremity (amplitude), is equal to the radius of the circle.

4) When the peg is shifted nearer to the centre of the turntable, the amplitude of the SHM decreases, but the period or frequency remains unchanged.

5) The shadow reaches maximum speed as it passes O' and is momentarily at rest at the extreme positions.

Consider the following diagram where the OA is the projection of OP along the vertical line OB. P represents the peg and A represents its shadow on the screen OB. As the turntable rotates, P moves in a circle of radius $OP = OB$ with constant angular speed ω. The angle θ it makes with reference to the equilibrium position is thus given by $\theta = \omega t$.



top view of turntable

Considering the right angle triangle OAP, we can easily see that the displacement *x* of the peg's shadow A with respect to O is given by

$$x = x_0 \sin\theta = x_0 \sin\omega t$$

Watch video clip on SHM & uniform CM relationship & phase difference
https://youtu.be/d0p7vDIgqjU

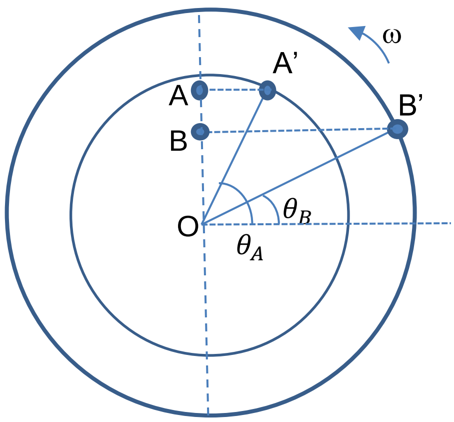
Hence the shadow of the peg does indeed move in SHM.

Phase & Phase Difference

For any object in SHM, we may associate with it the motion of an imaginary peg moving in uniform circular motion such that at any point in time, the angle θ it makes with reference to the equilibrium position is called the **phase (angle)** of oscillation.

Thus the **angular frequency ω** in a simple harmonic motion is also **the rate of change of phase angle** of the imaginary peg.

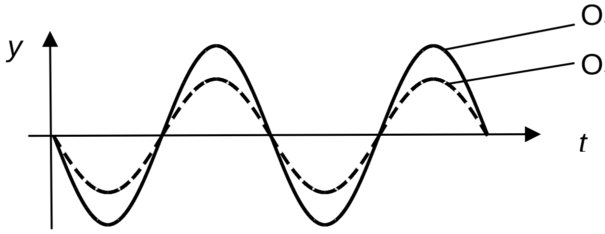
The concept of phase angle is useful for comparing two oscillators of the same frequency (and therefore period), even if they have different amplitudes. We can describe how far ahead one is compared to the other in terms of the difference in their phase angles.



A and B are moving in SHM about O on the vertical dotted line. A is ahead of B by a phase difference of $\Delta\theta = \theta_A - \theta_B$. A and B have different amplitudes but same frequency.

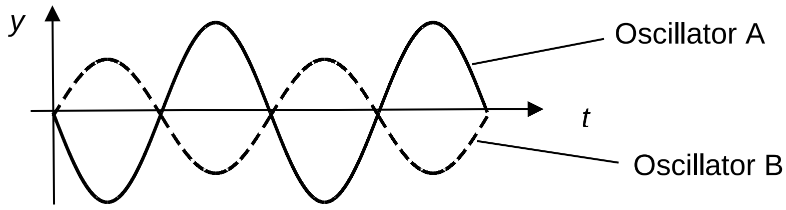
1. The difference in phase angle between two oscillations is called **phase difference Δϕ**.

2. Two oscillations are said to be ***in phase*** when they are in step, i.e., they reach their respective maximum displacements in the same direction at the same time. It does not matter what their respective amplitudes are.



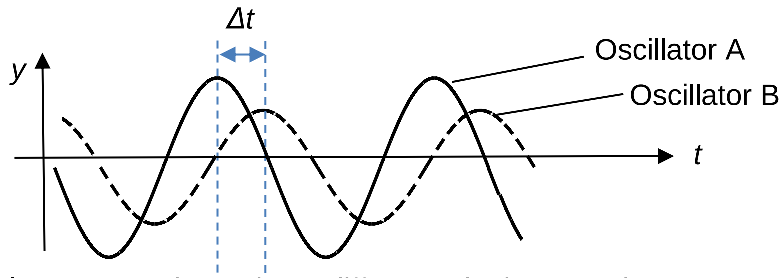
The phase difference $\Delta\phi = 0$ rad or any integral multiple of 2π rad.

3. Two oscillations are said to be ***in anti-phase*** when they are completely out of step, i.e., one oscillation reaches maximum displacement in a given direction at the same instant as the other oscillation reaches maximum displacement in the opposite direction.



The phase difference $\Delta\phi = \pi$ rad or any odd integral multiple of π rad.

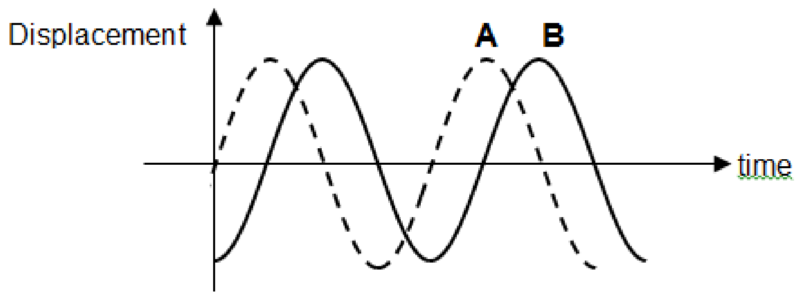
4. Of course the two oscillations may not be in phase or in anti-phase but at some intermediate phase difference. In such a case, the phase difference can be calculated by the following: $\Delta\phi = \frac{\Delta t}{T} \times 2\pi$.



You will learn more about phase difference in the next chapter on Waves.

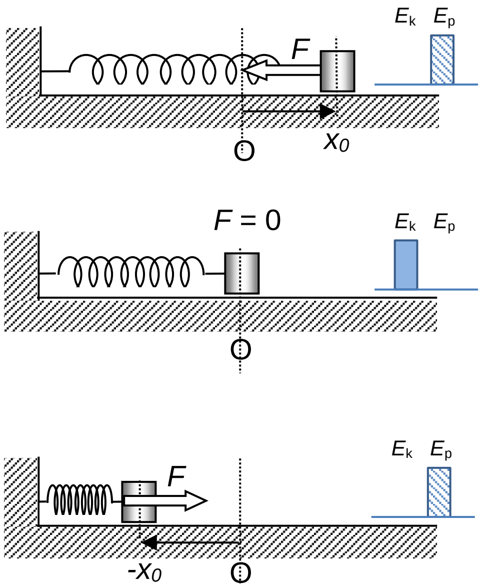
<u>Example 8.8:</u> Phase and Phase Difference

The displacement-time graph of two objects A and B oscillating is shown below. By how much does A lead B in terms of phase angle?



Solution:

8.3 Energy in SHM



Consider a horizontal spring-mass system on a smooth horizontal plane, where the mass is initially at rest. In order for the mass to oscillate in SHM, we need to displace the mass, say to the right from its equilibrium position O. As we do that, we store elastic potential energy, E_p, in the spring.

When we release the mass, the restoring force, F, in the spring will accelerate the mass towards its equilibrium position. As the restoring force is in the direction of motion, it does *positive work*; the initially stored elastic potential energy, E_p, transforms into kinetic energy, E_k. The kinetic energy reaches a maximum value when the restoring force (that is proportional to the displacement) becomes zero at the equilibrium position.

As the mass passes its equilibrium position and moves to the left, the restoring force now acts in the opposite direction to slow it down. The restoring force now does *negative work* on the mass, converting its kinetic energy back to elastic potential energy. The elastic potential energy reaches a maximum value (which is the same as the amount that was stored initially at the start of SHM) when the mass comes to rest momentarily at its extreme end, where the restoring force is maximum.

This repeats throughout SHM where the work done by the restoring force transfers energy to and fro between potential energy stored in the system and kinetic energy of the mass. The total amount of energy involved in this exchange is the mechanical energy, which is also referred to as the total energy associated with the SHM, and is equal to E_T, the amount that was initially invested when we first displaced the mass from its equilibrium position.

8.3.1 Energy-displacement Graphs of SHM

Let us now look at the energy-displacement graphs of the horizontal spring-mass system. We assume that the spring is light (zero mass) so that it may not possess kinetic energy E_k. Hence the total energy E_T associated with the SHM is at any point in time the sum of elastic potential energy stored in the spring and the kinetic energy of the mass.

For SHM, $v = \pm\omega\sqrt{x_0^2 - x^2}$

hence

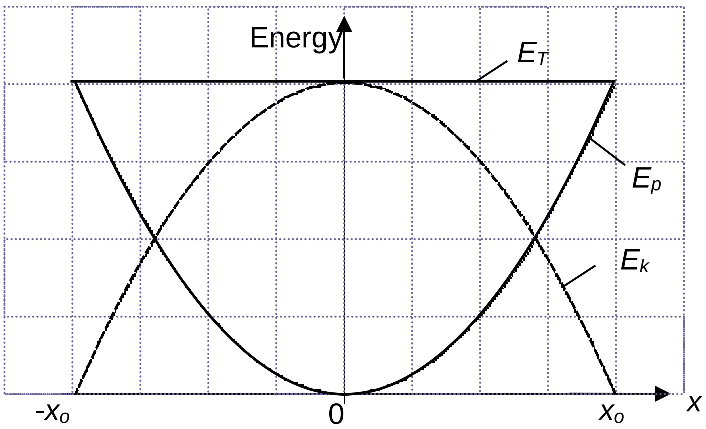
$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

At equilibrium position $(x = 0)$

$$E_T = E_{K,max} = \frac{1}{2}m\omega^2x_0^2$$

As E_T is constant,

$$E_p = E_T - E_K = \frac{1}{2}m\omega^2x^2$$



In some systems there are more than one type of potential energy present in the system. For example, in the vertical spring mass system, the potential energy E_p will then refer to the sum of gravitational potential energy and elastic potential energy. Scan the QR code to watch the energy analysis. (https://youtu.be/FKltat9EHqw)

<u>Example 8.9:</u> Relationship between Energies and Displacement

A mass, m = 0.50 kg, connected to a light spring with spring constant k = 20 N m^{-1} oscillates on a frictionless horizontal surface with an amplitude of 3.0 cm. Given that angular frequency $\omega = \sqrt{\frac{k}{m}}$, calculate the

(a) total energy,

(b) (i) kinetic energy of the system when the displacement is 2.0 cm and

(ii) potential energy of the system when the displacement is 2.0 cm.

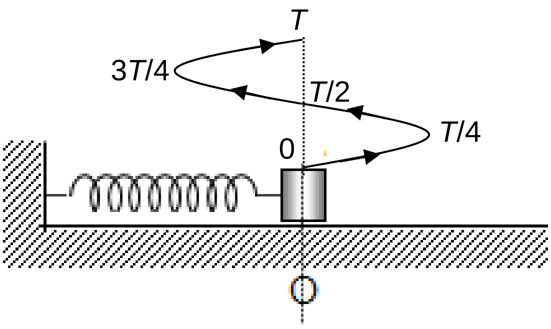
Solution:

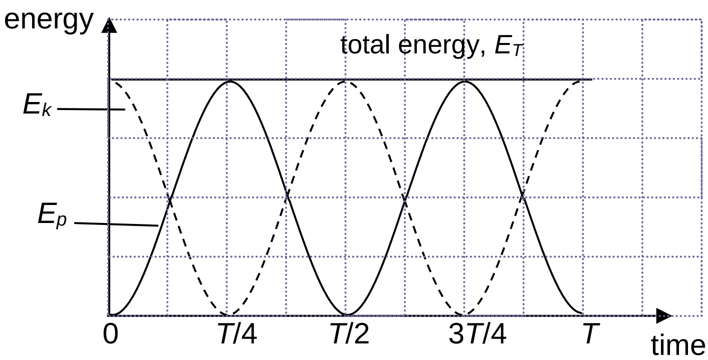
8.3.2 Energy-time Graphs of SHM

The energy-time graphs depend very much on the position and the velocity of the mass at the start of our observation. Suppose the mass is at <u>its equilibrium position when our observation starts</u> ($t = 0, x = 0, v = v_o$).

$\Rightarrow$ $x = x_0 \sin(\omega t)$ and $v = v_0 \cos(\omega t)$.

$\Rightarrow$ $E_k = \frac{1}{2}mv^2 = \frac{1}{2}mx_0^2\omega^2 \cos^2 \omega t$ $\quad$ $E_p = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 x_0^2 \sin^2 \omega t$



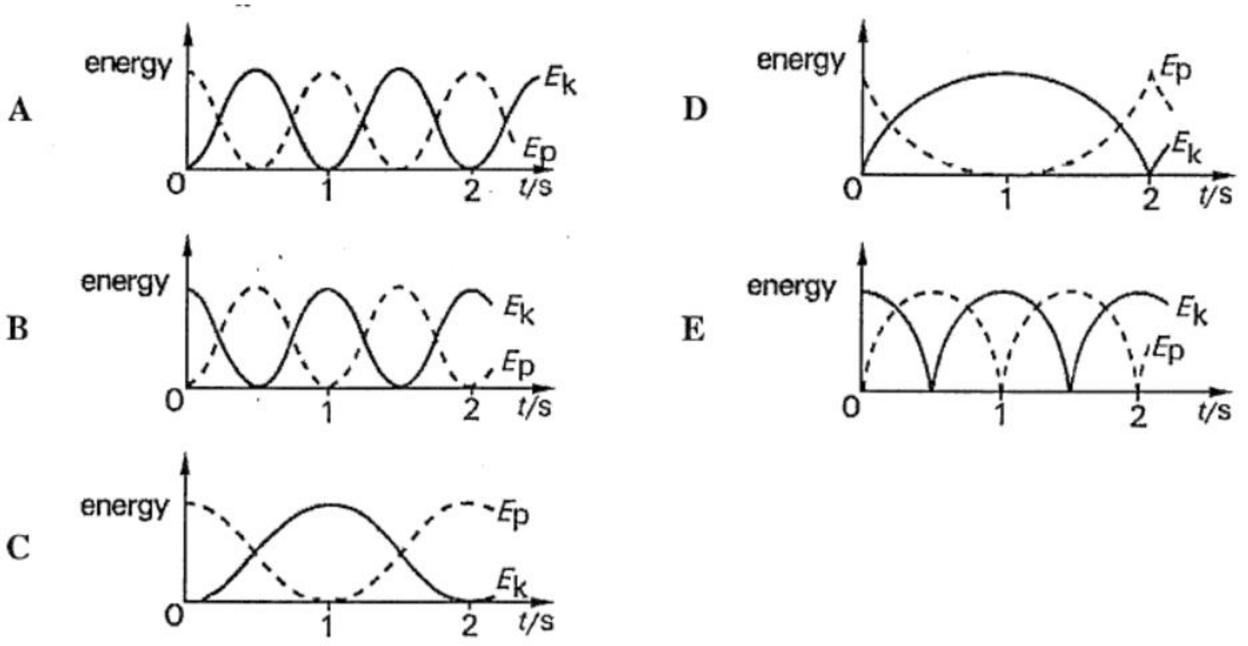


Note that when the oscillatory motion has completed one full cycle in one period, both kinetic and potential energies have gone through two cycles.

<u>Example 8.10:</u> Relationships between Energies and Time

The bob of a simple pendulum of period 2 s is given a small displacement and then released at time t = 0.

Which diagram shows the variation with time of the bob's kinetic energy E_k and its potential energy E_p?



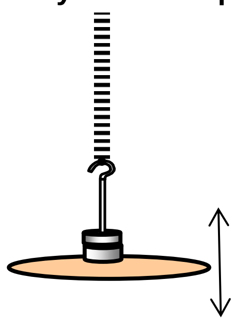
8.4 Damped Oscillations, Forced Oscillations and Resonance

8.4.1 Damped oscillations

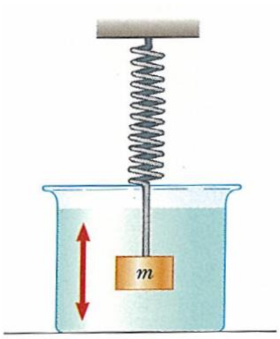
In an ideal oscillating system, there is no friction and the total energy associated with the oscillation is constant. The oscillation continues forever, with no decrease in amplitude.

In real-world systems, there is friction and the total energy associated with the oscillations decreases with time due to the work done against friction. The progressive decrease in amplitude of any oscillatory motion caused by dissipative forces is called ***damping***. Such an oscillatory motion is known as a ***damped oscillation***.

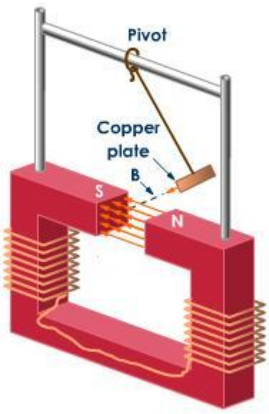
Commonly used damping methods:



A) Attach a piece of cardboard of negligible mass but large surface area to the base of the mass. Air resistance dissipates energy.



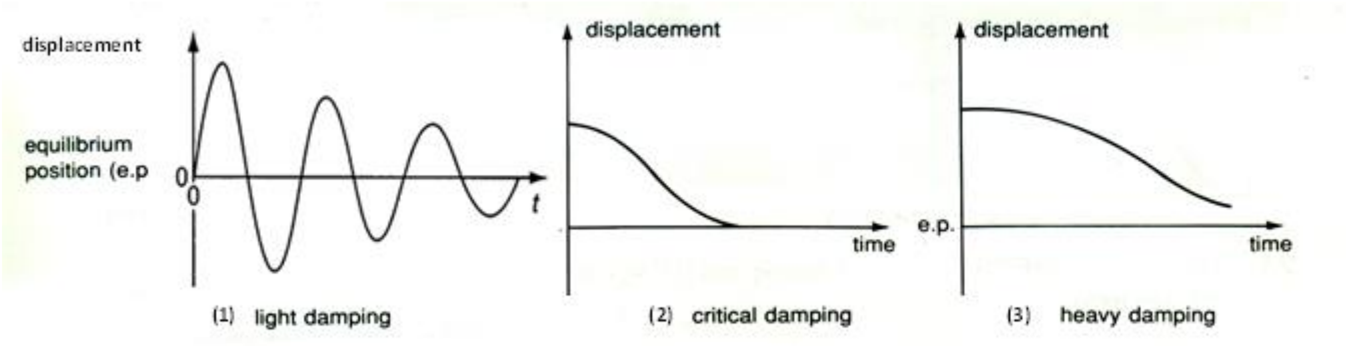
B) Submerge the mass in a viscous fluid. Viscous forces dissipate energy.



C) Swing the copper plate in a magnetic field so that eddy currents are induced in the plate. Eddy currents dissipate energy (will be discussed in Electromagnetic Induction in year 2).

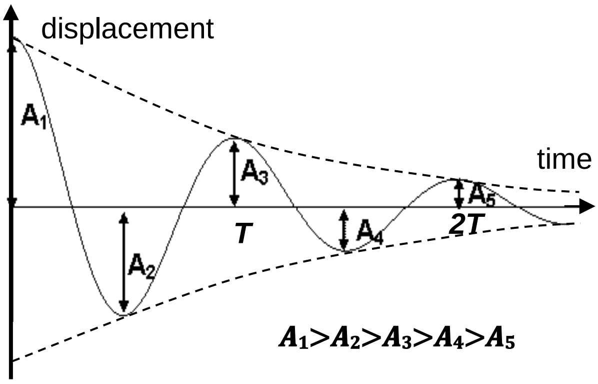
Extent of damping

Damping changes the behaviour of SHM systems. Certain features of the oscillation (amplitude etc.) are dependent on the extent of damping.



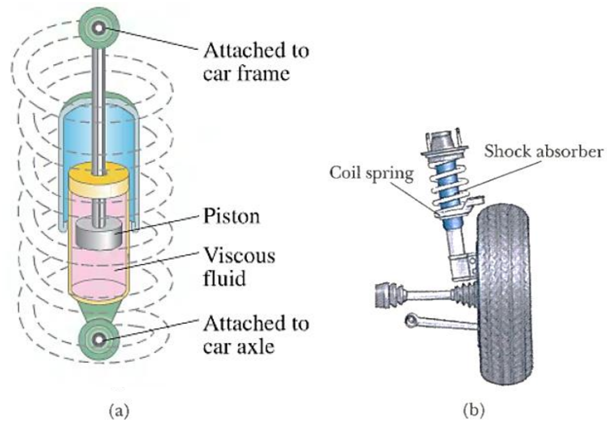
(1) ***Light Damping*** **(*underdamped system*)**

- In a lightly damped system, the total energy of the system decreases with time as its energy is dissipated when the system oscillates against resistive forces.
- The period/frequency remains constant, but the amplitude of the system decreases exponentially with time.



(2) ***Critical Damping (critically damped system)***

- If we are able to gradually increase the resistive force acting on an oscillatory system, damping increases till a point such that when displaced, the system **returns to the equilibrium position in the shortest possible time without oscillating** at all. The system is said to be **critically damped**.
- Critical damping is deliberately introduced into some systems to prevent continuous oscillations. For example, in analogue instruments such as balances, ammeters and voltmeters, the pointer is critically damped so that it stops in the shortest possible time to indicate the reading, instead of oscillating about the reading (underdamped) or taking an unnecessarily long time to crawl to the reading (overdamped).
- Critical damping is also used in **car suspension systems** to ensure a smooth ride when the car moves on a bumpy road. The shock absorbers in the suspension system have hydraulic pistons to prevent the car from bouncing up and down excessively after hitting a bump.



(a) Spring and shock absorber provides damping so that a car will not bounce up and down endlessly.

(b) A spring and shock absorber attached to one of the wheels in a car suspension system.

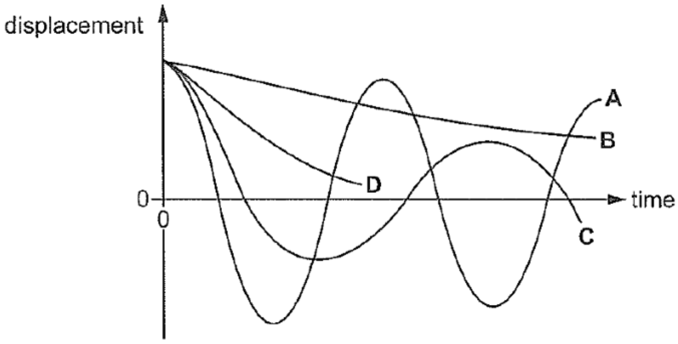
(3) ***Heavy Damping*** **(*overdamped system*)**

- When the damping force is increased beyond the point of critical damping, we say that the system is **overdamped** or that it experiences heavy damping. Once displaced from its equilibrium, the oscillator takes a very long time to return to its equilibrium position.

Example 8.11: **Car Suspension System (N17/I/13)**

The diagram shows four displacement-time curves for oscillations with various degrees of damping.

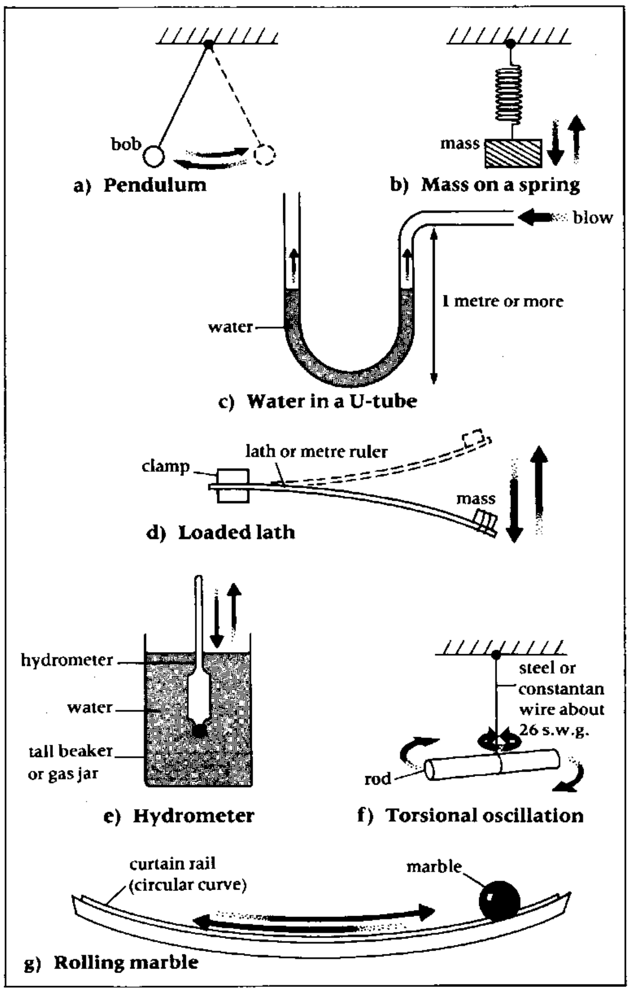
Which curve would ideally characterize a good car suspension system?



8.4.2 Forced oscillations and resonance

Some examples of free oscillations

(the arrows indicate motion, not forces)



Free oscillations

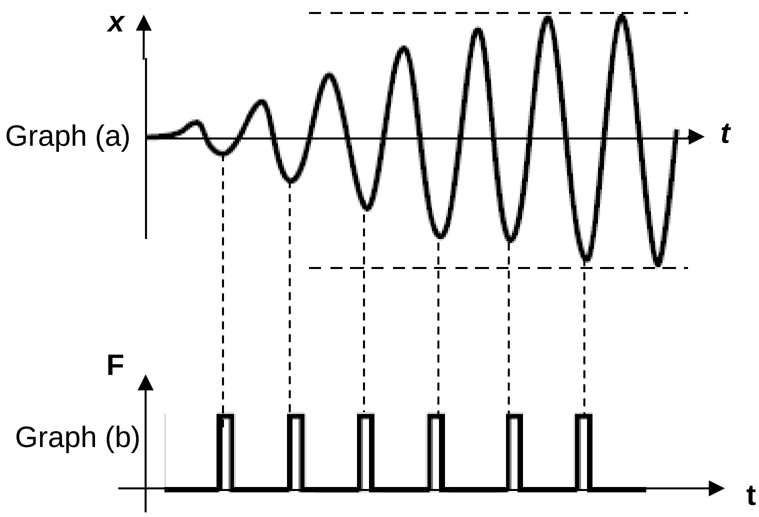
When a system oscillates about its equilibrium position with its own natural frequency and under no external influence, other than the impulse that initiated the motion, the system is said to be in a ***free oscillation***.

Forced oscillations

A real-world system in a free oscillation always experiences damping and will eventually stop oscillating. To keep it oscillating, one needs to periodically put in energy to the system to make up for the energy lost. In other words, one needs to apply a periodic force to the system.

- An oscillation under the influence of an external periodic force is called a ***forced oscillation***.
- The external force that is used to sustain (or drive) the oscillations is known as the **driving force.**
- The frequency with which the periodic force is applied is called the **forced frequency** or **driver frequency**.

One example of forced oscillation is pushing a child on a swing at the playground; one needs to continually give the swing a push at the right moment to sustain its oscillation.



Graph (a) shows the child's *x-t* graph as one periodically transfers a fixed amount of energy into the swinging motion.

Graph (b) shows the variation of driving force with time.

Resonance

Consider the above example of a child on a swing. The child will need to be pushed to keep him swinging.

If the frequency of the driving force matches the natural frequency of the oscillatory motion of the child-swing system, an interesting phenomenon happens - the final amplitude of the oscillation would be the greatest compared to the amplitudes of oscillation for other frequencies of driving force. This phenomenon is known as ***resonance***.

Resonance is a phenomenon in which an oscillatory system responds with maximum amplitude to an external periodic driving force, when the frequency of the driving force (driver frequency) equals the natural frequency of the driven system.

Note:

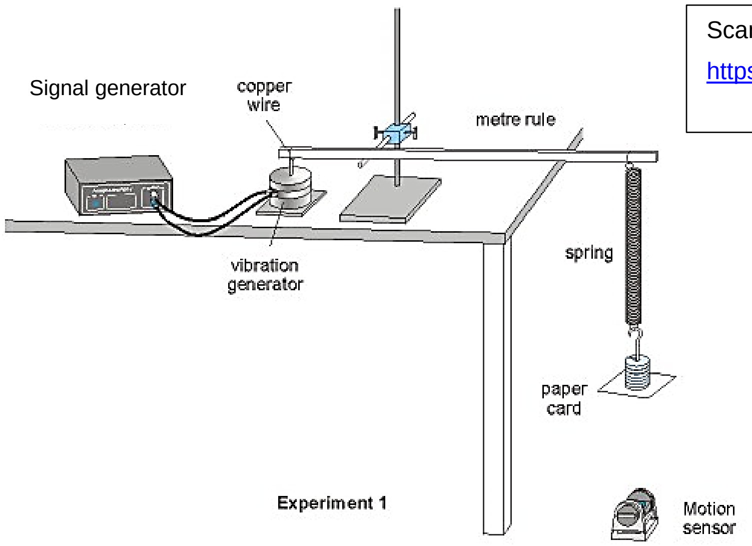
(1) The frequency at which resonance occurs is known as the ***resonant frequency***.

(2) Under ideal conditions, i.e., if no resistive forces are present (there is no damping), the amplitude will build up to an infinitely large value.

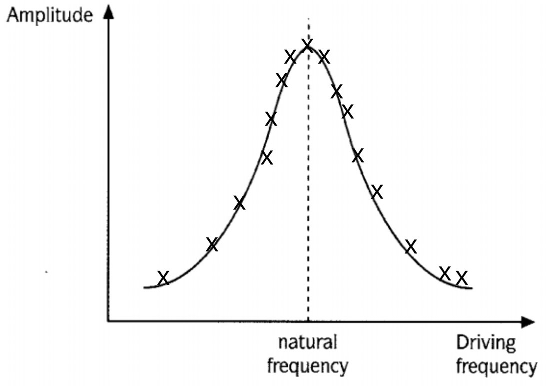
(3) However, in real cases, damping occurs. Whilst putting energy into the system in each cycle, energy is also taken away by damping and the amplitude will stop increasing when the rate of energy supplied is the same as the rate at which it is lost.

(4) Resonance represents the point where (driving) energy is transferred most efficiently into the oscillatory system.

Investigating Resonance in the Lab



Scan the QR code to watch a similar experiment https://youtu.be/jewSVEBkI_s .



The above set-up may be used to investigate the phenomenon of resonance in the lab.

(1) The system under study is a spring-mass system that is driven to oscillate by a metre rule which is in turn driven to vibrate by a vibration generator. To vary the frequency of vibration, the vibration generator is connected to a signal generator.

(2) To vary the amount of damping experienced by the spring-mass system, a light paper card is attached to the mass. By varying the area of the paper card attached, the damping is varied. The larger the paper card, the greater the damping.

(3) To measure the amplitude of vibration, a motion sensor is placed directly below the paper card.

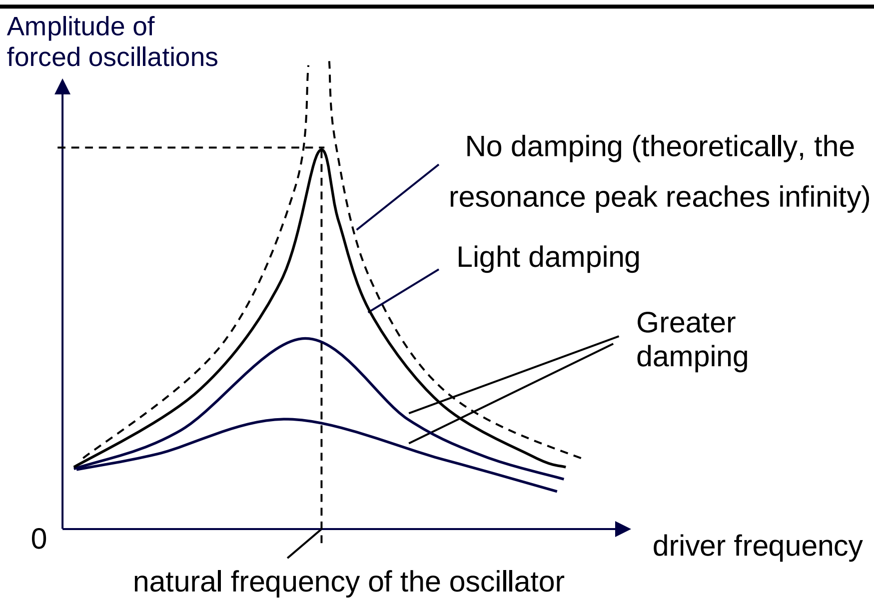
(4) For each frequency of vibration, we can then measure and record the amplitude of vibration of the spring-mass system. We start off with a driving frequency that is much lower than the natural frequency of the spring-mass system and slowly increase this frequency till beyond the natural frequency.

(5) The graph above shows the typical results obtained from the experiment for one area of card.

(6) The experiment is then repeated for different areas of cards in order to produce different amounts of damping.

(7) The data points for each different area of cards / different damping can then be plotted on the same pair of axes for comparison as shown below.

The figure below shows the amplitude vs. driving frequency graphs that will be obtained.



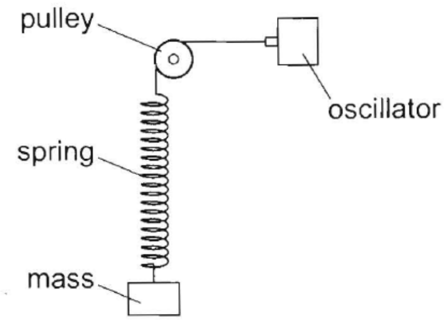
Note:

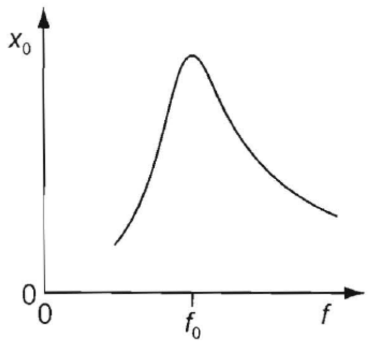
1) The amplitude of a lightly damped system is very large at resonance and the peak is sharp. The amplitude drops off rapidly when the driver frequency differs from the natural frequency of the system.
2) When the driver frequency approaches zero, the amplitude of the forced oscillations will be the same as the amplitude of the driver.
3) As the system experiences greater damping,
    - the amplitude of the oscillation **decreases**.
    - the resonance peak becomes **broader**.
    - the resonance peak shifts slightly to the **left** to a slightly lower value of frequency.

From the graphs, it is evident that damping is a useful way to reduce the effects of resonance. Another way to avoid resonance is to ensure that the frequency of the periodic external force differs from the natural frequency of the driven system.

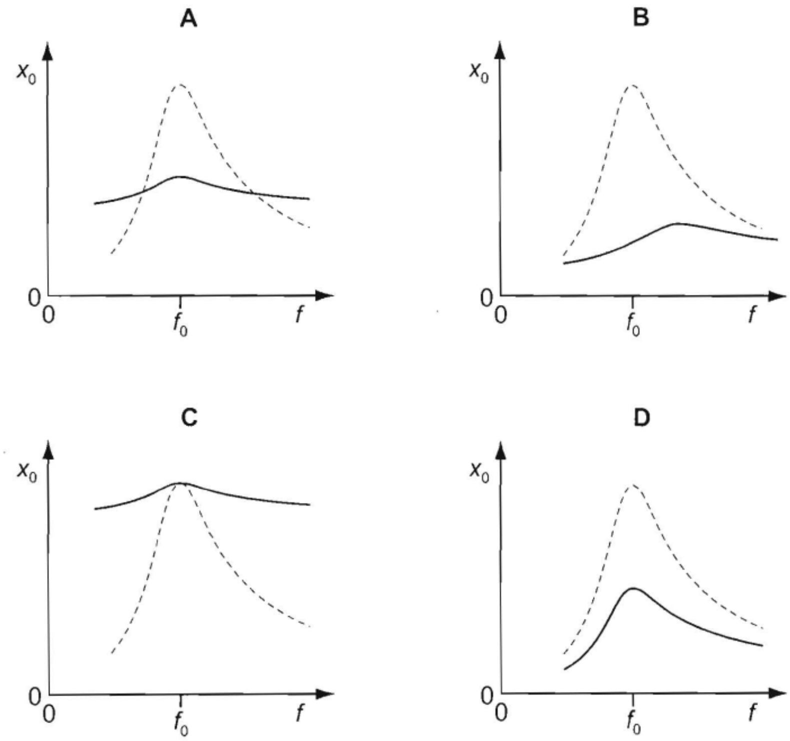
<u>Example 8.12:</u> Damped Oscillation (N10/1/16)

A mass, suspended from a helical spring, is made to oscillate. The graph shows the variation with frequency *f* of the amplitude x_0 of vibration of the mass.





A sheet of cardboard of negligible mass is now fixed to the mass on the spring to cause light damping of the oscillations. Which graph shows how x_0 will vary with *f* over the same frequency range?



8.4.3 Examples of Destructive and Useful Resonance

Examples of destructive resonance

1) Shattering glass with voice

When an opera singer projects a high-pitched note whose frequency matches the natural frequency of a wine glass, resonance may cause the glass to vibrate at a large amplitude that it breaks.

2) Violent vibrations in vehicles and machines

If the panel in a bus rattles violently when the bus is travelling at a certain speed, it is likely that a resonant vibration is occurring. A washing machine with a load which has natural frequency matching the spinning frequency will vibrate violently as resonance occurs.

3) Collapse of some buildings in earthquakes

An earthquake consists of many low-frequency vibrations ranging from 1 to 10 Hz. During an earthquake, when the frequencies of the vibration match the natural frequencies of buildings, resonance may occur and result in serious damage. That also explains why some buildings collapse while others stand almost unaffected. In regions of the world where earthquakes happen regularly, buildings may be built on foundations that absorb the energy of the shock waves. In this way, the vibrations are damped and the amplitude of the oscillations cannot reach a dangerous level.

Examples of useful resonance

1) Radio receptions

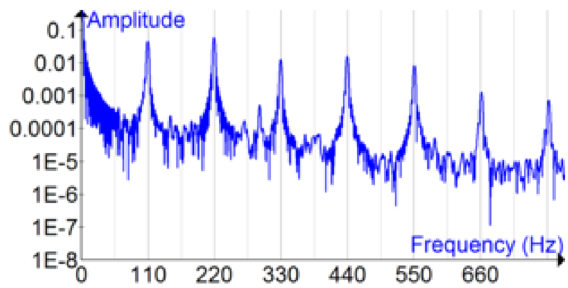
Resonance not only occurs in mechanical oscillations but also in electric circuits. A radio receiver works on the principle of resonance. Our air is filled with radio waves of many different frequencies which the aerial picks up. The tuner can be adjusted so that the frequency of the electrical oscillations in the circuits is the same as that of the radio wave transmitted from a particular station we desire. The effect of resonance amplifies the signals contained in this wave while the radio waves of other frequencies are diminished.

2) Magnetic Resonance Imaging

Magnetic Resonance Imaging (MRI) is increasingly used in medical diagnosis to produce images similar to those produced by X-rays. Strong, electromagnetic fields of varying radio frequencies are used to cause oscillations in atomic nuclei. When resonance occurs, energy is absorbed by the molecules. By analysing the pattern of energy absorption, a computer-generated image can be produced. The advantage of MRI scanners is that no ionising radiation (as in the process of producing X-ray images) is involved.

3) Musical Instruments

Most vibrating objects have more than one resonant frequency. The lowest resonant frequency of a vibrating object is called its fundamental frequency. A harmonic is defined as an integer multiple of the fundamental frequency. (You will learn more about it in the topic Superposition, under Stationary Wave). Strings and air columns used in musical instruments typically vibrate at harmonics of the fundamental frequency. In an acoustic guitar, when you pluck a string, the string initially vibrates at all the frequencies. Those frequencies that are not one of the resonances are quickly filtered out— or attenuated— and all that is left is the harmonic vibrations that are transmitted through air molecules as sound energy into the hollow wooden body of the guitar, making it (and the air inside) resonate and amplifying the musical note (making it considerably louder).



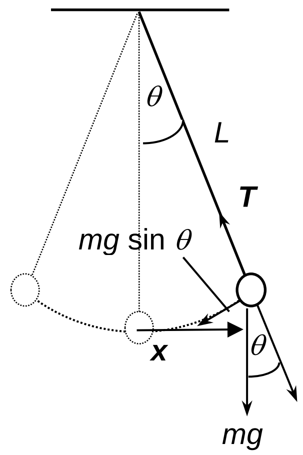
String resonance of a bass guitar A note with fundamental frequency of 110 Hz.

Appendix 1 Case study: Oscillations of A Simple Pendulum

A simple pendulum is in its midst of motion, where its bob is at a horizontal displacement x from its equilibrium position. The string is attached to a bob of mass m and makes a small angle of θ to the vertical as shown.

The restoring force ($mg \sin \theta$) which is pointing towards equilibrium position and is approximately horizontal, is in the opposite direction to the positive direction of the horizontal displacement,

$F_{net} = -mg \sin \theta$



When the angular displacement θ is small,

the horizontal displacement of the bob is approximately equal to

the arc length $x \approx L\theta$

$\Rightarrow \quad \theta \approx x/L$

and $\sin \theta \approx \theta \approx x/L$

$\Rightarrow \quad F_{net} = -mg \sin \theta \approx -mg\dfrac{x}{L}$

From Newton's 2nd law: $ma \approx -mg\dfrac{x}{L}$

$\Rightarrow \quad a \approx -\left(\dfrac{g}{L}\right)x$

i.e., $a = -\omega^2 x$ where $\omega^2 = \dfrac{g}{L}$

Hence, motion of a simple pendulum bob attached to a light inextensible string oscillating with **a small angular displacement of ~ 10° or smaller** is, to a good approximation, a linear SHM with period, T, given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

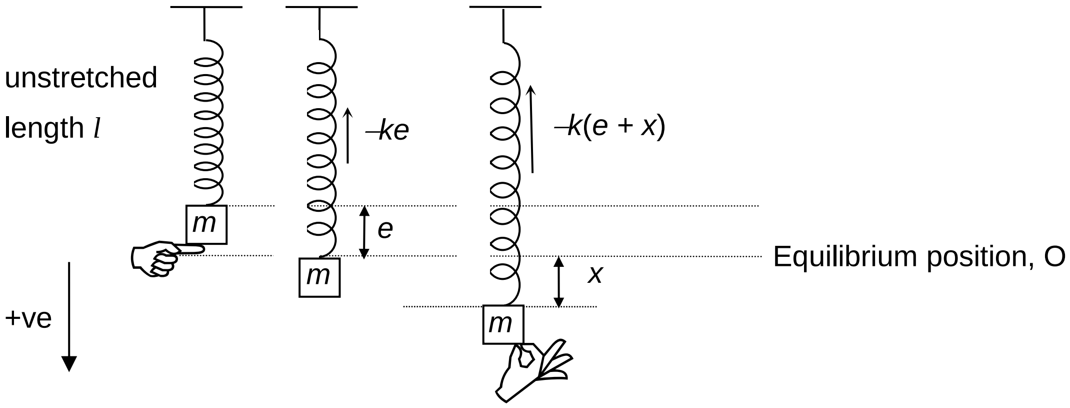
Note:

1) Other assumptions made are that the string is massless, the radius of the bob is small compared with the length of the string and air resistance is negligible.

2) The period T is independent of the amplitude of the oscillation.

Appendix 2 Case study: Oscillations of a Mass on Vertical Spring

A mass m attached to the end of a spiral spring exerts a downward force on it and stretches it by an amount e as shown in the diagram.



If the spring obeys Hooke's law, then the extension e is directly proportional to the extending tension, F_s,

i.e., $F_s = -ke$ where k is a constant.

Since the mass is in equilibrium, the net force on it is zero ($\sum F = 0$).

Thus $mg + (-ke) = 0$

$\Rightarrow mg = ke$

Suppose the mass is now pulled down a further distance x below its equilibrium position, then the tension in the spring is increased to $-k(e + x)$. Thus the resultant force on the mass is given by

$$F = mg - k(e + x)$$
$$= ke - k(e + x) \quad \text{(since } mg = ke\text{)}$$
$$= -kx$$

i.e., the resultant force is acting upwards, with the direction opposite to that of the displacement.

By Newton's 2nd law, $ma = -kx$

$$\Rightarrow a = -\frac{k}{m}x$$

Since k and m are fixed, the oscillation of a mass on spring is simple harmonic.

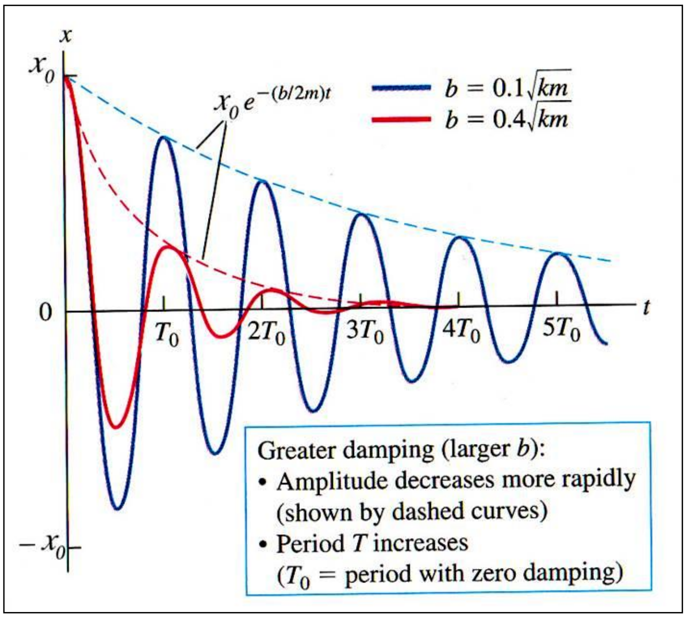
The angular frequency ω is given by $\omega^2 = \dfrac{k}{m}$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Appendix 3 Equations of motion for damped simple harmonic oscillations

In an oscillatory system, the decrease in amplitude caused by dissipative forces is called damping and the corresponding motion is called damped oscillation. The simplest case to analyse is a simple harmonic oscillator under damping, where its dissipative force is directly proportional to its *velocity*. This behaviour occurs in friction involving the flow of viscous fluid, such as in shock absorbers or sliding between oil-lubricated surfaces. The additional force on the body due to viscous drag is $f = -bv$, where *b* is a constant.

With a viscous drag, the net force on the body is $(-kx - bv)$.



Applying Newton's 2nd second law,

$$m\frac{d^2x}{dt^2} = -kx - bv$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

This is a second order linear differential equation with constant coefficients.

If $b < 2\sqrt{km}$, the system is under light damping and the equation of motion is

$$x = \left(x_0 e^{-\left(\frac{b}{2m}t\right)}\right)\cos\left(\left(\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right)t + \phi\right)$$

Its amplitude ($x_0 e^{-\left(\frac{b}{2m}t\right)}$) is not a constant, but decreases with time because of the decaying exponential factor. In addition, the larger the value of *b*, the faster the decrease in the amplitude.

Its angular frequency $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$, is smaller than that without damping, $\omega = \sqrt{\frac{k}{m}}$, and therefore the period is slightly larger.

If $b = 2\sqrt{km}$, then $x = x_0 e^{-\left(\frac{b}{2m}t\right)}$. The system is in critical damping, where it no longer oscillates but returns to its equilibrium position without overshooting the equilibrium position.

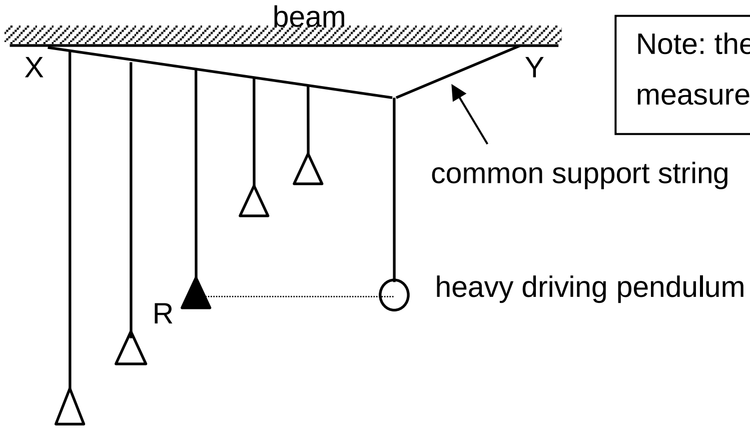
If $b > 2\sqrt{km}$, then the system is heavily damped. Again, there is no oscillation, but the return to its equilibrium takes longer than in the case of critical damping. Its solutions are in the form of $x = C_1 e^{-a_1 t} + C_2 e^{-a_2 t}$, where C_1 and C_2 are constants determined by initial conditions and a_1 and a_2 are constants determined by *m, k* and *b*.

Appendix 4 Demonstration of Forced Oscillation and Resonance - Barton's pendulums

Barton's pendulums consist of several pendulums of different lengths hung from a horizontal string. Each has its own natural frequency of oscillation. The 'driver' pendulum at the end has a large mass and its length is equal to that of pendulum R. When the driver is set swinging, its vibrations are transferred through the common support string to the other pendulums and start the forced oscillations.

Observations show that:

1. All the pendulums vibrate with the same frequency as the driver.
2. Pendulum R whose length matches that of the driver pendulum (therefore has the same natural frequency as the driver) has the largest amplitude. It is said to be resonating with the driver.



Note: the lengths of the pendulums are measured from the beam

Explanation:

All the pendulums are coupled together by the suspension. As the driver swings, it moves the suspension, which in turn moves the other pendulums. The matching pendulum is being pushed slightly once each oscillation, and the amplitude gradually builds up to maximum. The other pendulums are being pushed at a frequency that does not match their natural frequency and they therefore oscillate with smaller amplitudes.

Watch the video clip and further observations on Barton's pendulum

https://bit.ly/3dnr4CQ

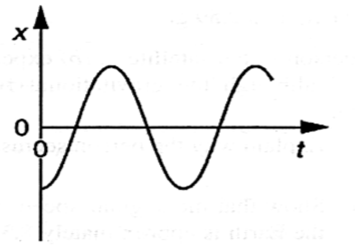
https://bit.ly/3dnk0qb

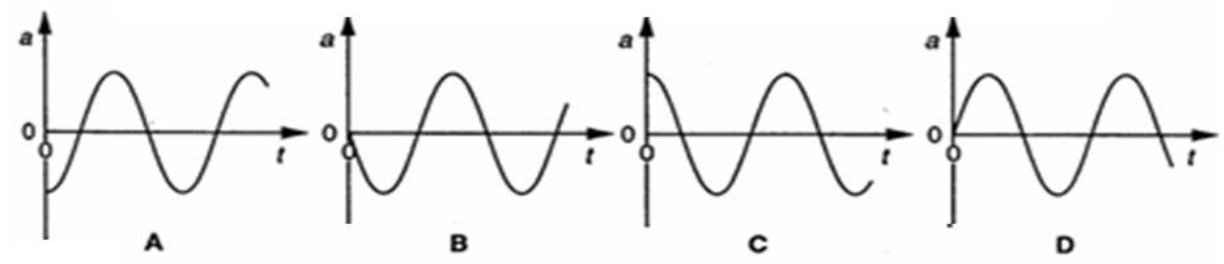
Tutorial 8 Oscillations

Self-Review Questions

Simple Harmonic Motion

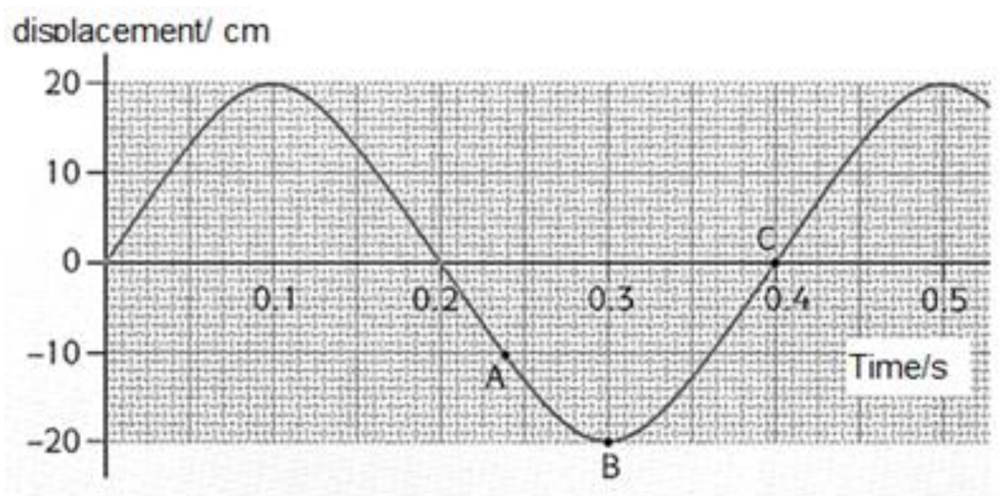
S1 **N03/I/14**: A body moves with simple harmonic motion about point P. The graph shows the variation with time t of its displacement x from P. Which graph shows the variation with time of its acceleration?





S2 **J90/I/10:** A body performing simple harmonic motion has a displacement x given by the equation $x = 30\sin 50t$, where t is in seconds. What is the frequency of oscillation?

S3 The figure below shows the displacement-time graph of a simple harmonic oscillator.



(i) Deduce the following quantities:

(a) amplitude,
(b) period,
(c) frequency,
(d) angular frequency,
(e) displacement at A.

(ii) Using your answers to S3(i)(a) and (d), write down an equation that describes the variation of displacement, *x*, with time, *t*, as shown in the graph.

(iii) Substitute the displacement at A in the figure, into the equation to find the corresponding time. Check that it agrees with the value from the graph.

(iv) At which point (A, B, C) is
(a) the magnitude of the velocity maximum?
(b) the magnitude of the acceleration maximum?
(c) State whether the direction of the velocity in (a) and that of the acceleration in (b) is positive or negative. Explain your answers.

(v) Deduce the following quantities:
(a) velocity at B, (b) velocity at C, (c) acceleration at B, (d) acceleration at C.

(vi) Sketch for this oscillator,
(a) the acceleration-time graph
(b) the velocity-time graph
(c) the velocity-displacement graph
(d) the acceleration-displacement graph.

S4 A 0.50 kg body performs simple harmonic motion with a frequency of 2.0 Hz and amplitude of 8.0 mm. Determine the maximum velocity and the maximum acceleration and the corresponding positions of the body.

Energy in simple harmonic motion

S5 Refer to the displacement-time graph in S3. The mass of the oscillator is 1.0 kg.

(i) At which point (A,B,C) is
(a) the potential energy maximum?
(b) the kinetic energy maximum?

(ii) Find the kinetic energy and the potential energy of the oscillator at C and B.

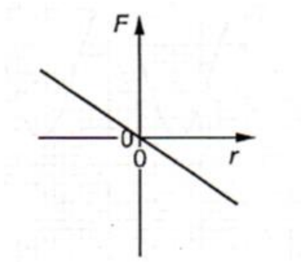
(iii) Sketch for the oscillator, labelled graphs on the same horizontal time axis,
(a) the variation of its kinetic energy,
(b) the variation of its elastic potential energy,
(c) the variation of its total mechanical energy.

(iv) Sketch for the oscillator, labelled graphs on the same horizontal displacement axis,
(a) the variation of its kinetic energy,
(b) the variation of its potential energy,
(c) the variation of its total mechanical energy.

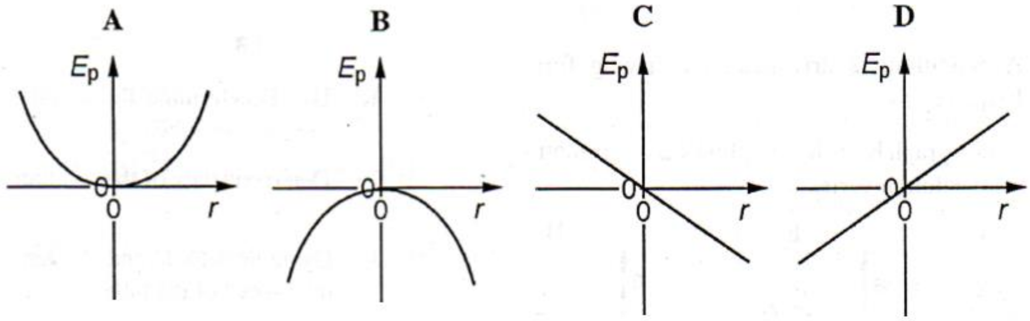
S6 A body oscillates vertically on the end of a light vertical spring with simple harmonic motion of frequency *f* and amplitude *A*. The total energy of the oscillations is proportional to

A $\sqrt{f}$ **B** $\sqrt{A}$ **C** A^2 **D** f

S7 **N04/I/16**: A particle is moving such that the force *F* on it changes with the distance *r* from a fixed point as shown.

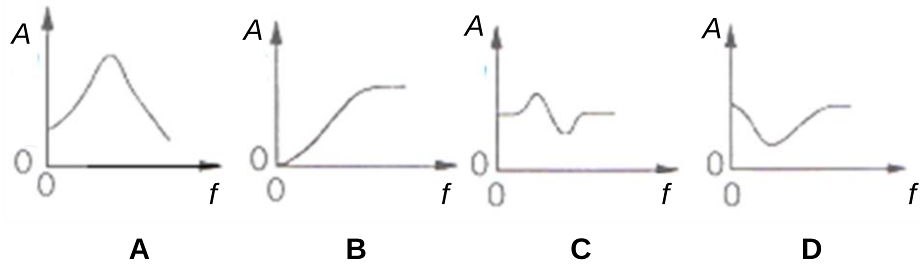


Which graph best shows the relationship between the potential energy *Ep* of the particle and the distance *r*?



Damped Oscillations, Forced Oscillations and Resonance

S8 **N00/I/9:** A pendulum is driven by a sinusoidal driving force of frequency *f*. Which graph best shows how the amplitude *a* of the motion of the pendulum varies with *f* ?



Discussion Questions
Simple Harmonic Motion

D1 J78/II/10

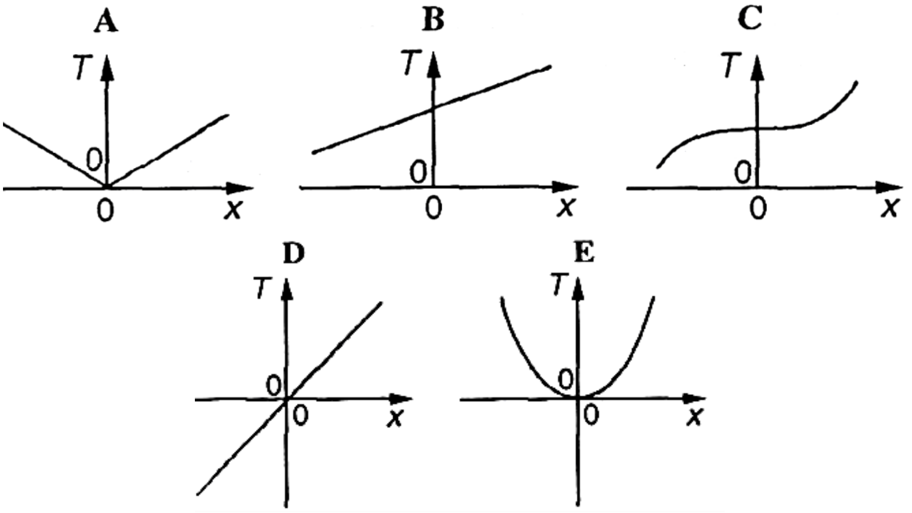
In order to check the speed of a camera shutter, the camera was used to photograph the bob of a simple pendulum moving in front of a horizontal scale.

The extreme positions of the bob were at the 600 mm and 700 mm marks. The photograph showed that while the shutter was open, the bob moved from the 650 mm mark to 675 mm mark.

If the period of the pendulum was 2 s, how long would the shutter remained open?

D2 N88/I/7

A mass is hung from the free end of a light helical spring and then given a small displacement vertically downwards. Which graph best represents how *T*, the tension in the spring, varies with *x*, the displacement of the mass from the equilibrium position during subsequent oscillations?

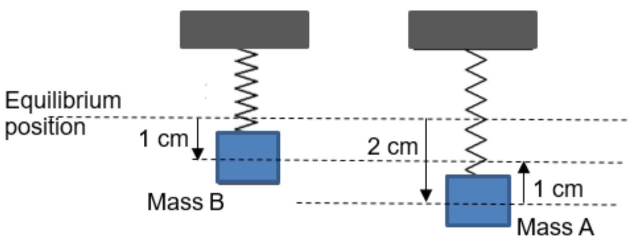


D3 HCIPrelim/15/I/14

Two identical mass-spring systems are set up side by side. Initially, mass B is displaced downwards 1 cm from its equilibrium position and held there.

Mass A is then displaced downwards 2 cm from its equilibrium position and released.

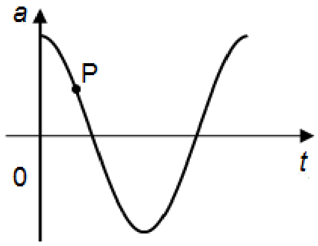
Mass B is released at the instant mass A has moved up a distance of 1 cm.

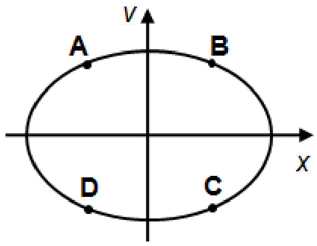


What is the phase difference between mass A and mass B?

A $\frac{\pi}{2}$ rad **B** $\frac{\pi}{3}$ rad **C** $\frac{\pi}{4}$ rad **D** $\frac{\pi}{6}$ rad

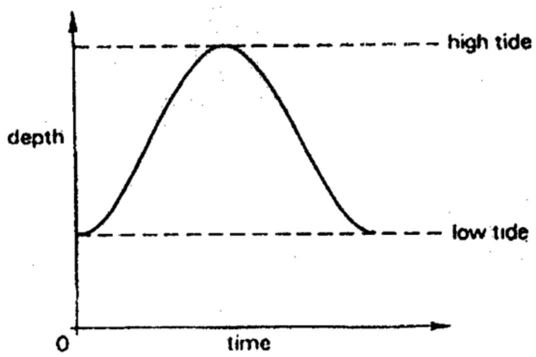
D4 Graph on the left shows how the acceleration of an object undergoing simple harmonic motion varies with time. A graph of velocity against displacement for the same oscillation is shown on the right. Which point in the v-x graph corresponds to point P in the a-t graph?





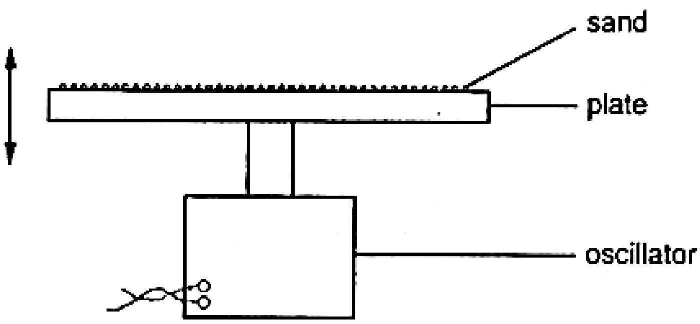
D5 **J91/I/9**

The rise and fall of water in a harbour is simple harmonic. The depth varies between 1.0 m at low tide and 3.0 m at high tide. The time between successive low tides is 12 hours. A boat, which requires a minimum depth of water of 1.5 m, approaches the harbour at low tide. How long will the boat have to wait before entering?



D6 **J99/III/4 (part)**

(b) Some sand is placed on a flat horizontal plate and the plate is made to oscillate with simple harmonic motion in a vertical direction, as shown.



The plate oscillates with a frequency of 13 Hz.

(i) Sketch a graph to show the variation with displacement *x* of the acceleration *a* of the plate.

(ii) The acceleration a is given by the expression

$$a = -\omega^2 x$$

where ω is the angular frequency.

Calculate

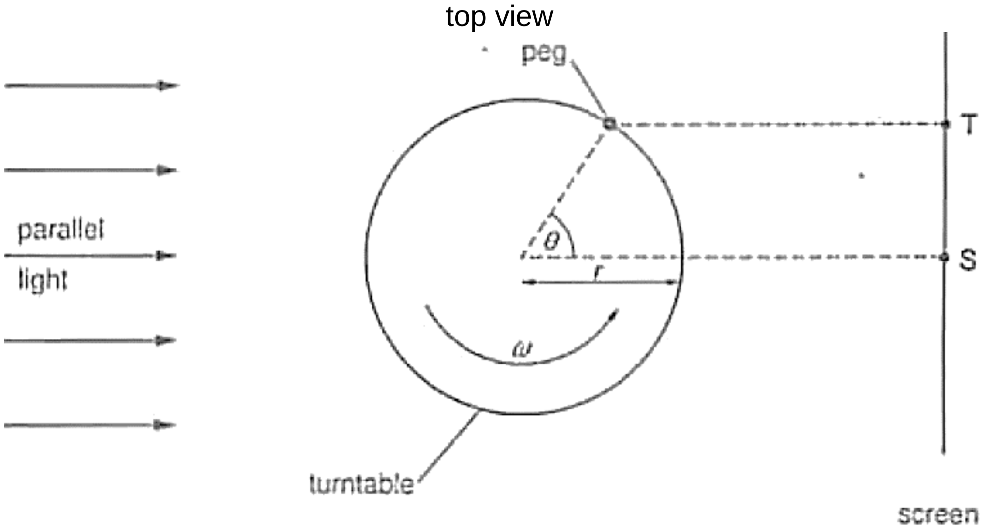
1. the angular frequency

2. the amplitude of oscillation of the plate such that the maximum acceleration is equal to the magnitude of the acceleration of free fall.

(c) Suggest, with a reason, what happens to the sand on the plate in (b) when the amplitude of the oscillations of the plate exceeds the value calculated in (b)(ii)2.

D7 N96/II/2

A vertical peg is fixed to the rim of a horizontal turntable of radius *r*, rotating with a constant angular speed ω, as shown below.



Parallel light is incident on the turntable so that the shadow of the peg is observed on a screen which is normal to the incident light.

At time $t = 0$, $\theta = 0$ and the shadow of the peg is seen at S.
At some later time *t*, the shadow is seen at T.

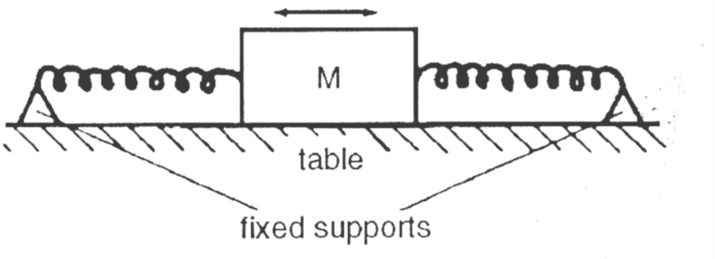
(a) (i) Write down an expression for θ in terms of ω and *t*.
(ii) Derive an expression for the distance ST in terms of *r*, ω and *t*.

(b) By reference to your answer in **(a)(ii)**, describe the motion executed by the shadow on the screen.

(c) The turntable has a radius *r* of 20 cm and an angular speed ω of 3.5 rad s^{-1}. Calculate, for the motion of the shadow on the screen,
(i) the amplitude,
(ii) the period,
(iii) the speed of the shadow as it passes through S,
(iv) the magnitude of the acceleration of the shadow when the shadow is instantaneously at rest.

Energy in Simple Harmonic Motion

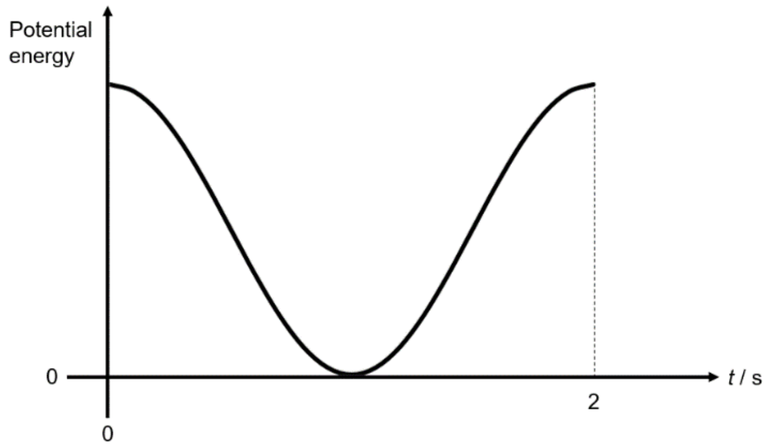
D8 **J81/II/19:** A mass M on a smooth horizontal table is attached by two light springs to two fixed supports as shown below. The mass executes simple harmonic motion of amplitude *a* and period *T*.



Find the energy associated with this simple harmonic motion in terms of *M*, *a* and *T.*

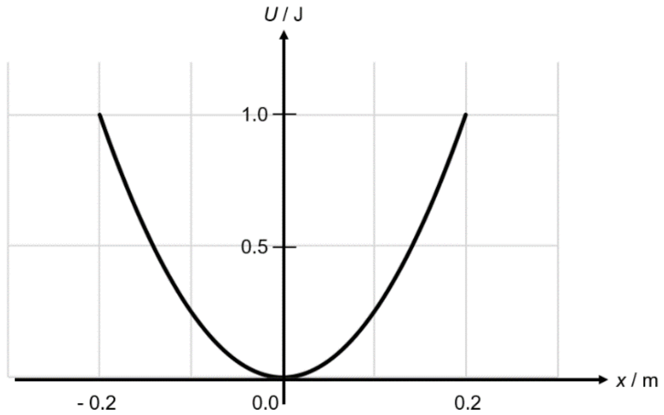
D9 **N21/1/15(modified):** A particle oscillates with simple harmonic motion. The graph shows the variation, with time *t*, of the potential energy of the particle from *t* = 0 to *t* = 2 s.

Sketch the variation of the velocity *v*, with time *t*, of the particle from *t* = 0 to *t* = 2 s.



D10 **N82/II/9:** A particle of mass 4.0 kg moves with simple harmonic motion and its potential energy *U* varies with position *x* as shown below.

What is the period of oscillation of the mass?



D11 One end of a spring is fixed to a support. A mass is attached to the other end of the spring. The arrangement is shown in Fig. 3.1.

This arrangement is used to determine the length l of the spring when mass M is attached to the spring. The procedure is repeated for different values of M. The variation of mass M with length l is shown in Fig. 3.2.

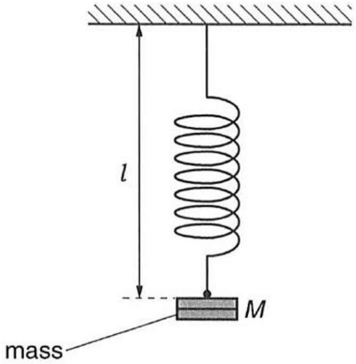


Fig. 3.1

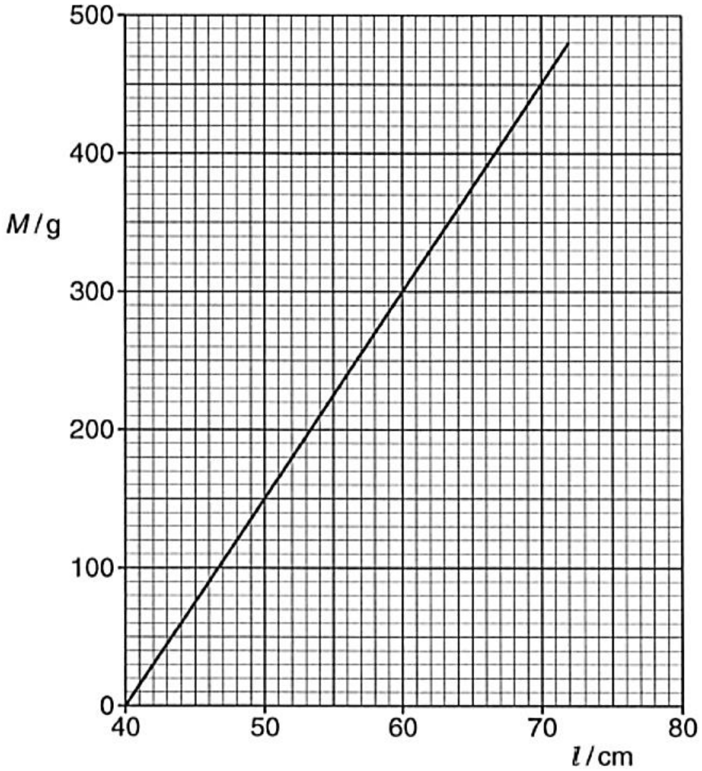


Fig. 3.2

(a) Show that the spring constant k of the spring is 14.7 N m^{-1}.

(b) A mass of 450 g is attached to the spring and is held at rest with length l of 50.0 cm. The mass is then released and the mass oscillates freely. The angular frequency of the spring-mass system is given by the formula $\omega = \sqrt{\frac{k}{m}}$.

Calculate the frequency of the system.

(c) Calculate the speed of the mass during its oscillation when the spring is extended to a length l of 80.0 cm.

(d) The spring is assumed to be light. In practice, the spring will have some mass. Assuming that the spring constant k is unchanged, suggest and explain the effect on the frequency of oscillation of having a spring with mass.

D12 N08/3/6

(a) Distinguish between *frequency* and *angular frequency* for a body undergoing simple harmonic motion.

(b) A spring that has an unstretched length of 0.650 m is attached to a fixed point. A mass of 0.400 kg is attached to the spring and gently lowered until equilibrium is reached. The spring has then stretched elastically by a distance of 0.200 m.

Calculate, for the stretching of the spring,
(i) the loss in gravitational potential energy of the mass,
(ii) the elastic potential energy gained by the spring.

(c) Explain why the two answers to **(b)** are different.

(d) The load on the spring is now set into simple harmonic motion of amplitude 0.200 m.

Calculate
(i) the resultant force on the load at the lowest point of its movement,
(ii) the angular frequency of the oscillation,
(iii) the maximum speed of the mass.

(e) Fig. 6.1 is a table of the energies of the simple harmonic motion. Complete the table.

	gravitational potential energy/J	elastic potential energy/J	kinetic energy/J	total energy/J
lowest point	0			
equilibrium position				
highest point				

Fig. 6.1

(f) On the axes of Fig. 6.2 below, sketch four graphs to show the shape of the variation with position of the four energies. Label each graph.

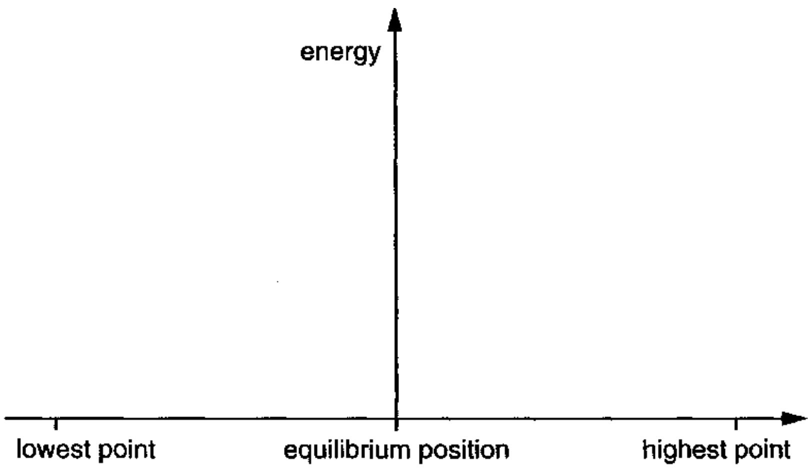


Fig. 6.2

Damped oscillations, forced oscillations and resonance

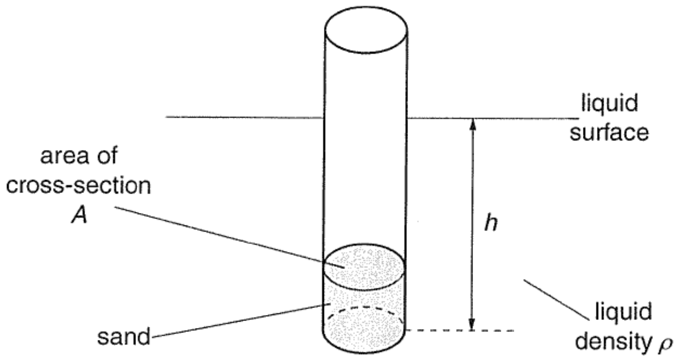
D13 N09/III/1c

In normal use, a loudspeaker produces a range of frequencies of sound. Suggest why it is important that the natural frequency of vibration of the cone of the loudspeaker is not within this range of frequencies.

D14 N13/III/7

(a) State the origin of the upthrust acting on a body in a fluid.

(b) A tube, sealed at one end, has a uniform area of cross-section *A*. Some sand is placed in the tube so that it floats upright in a liquid of density ρ, as shown in the figure.



The total mass of the tube and the sand is *m*. The tube floats with its base a distance *h* below the surface of the liquid. Derive an expression relating *m* to *h, A* and ρ. Explain your working.

(c) The tube in **(b)** is displaced vertically and then released. For a displacement *x*, the acceleration *a* of the tube is given by the expression

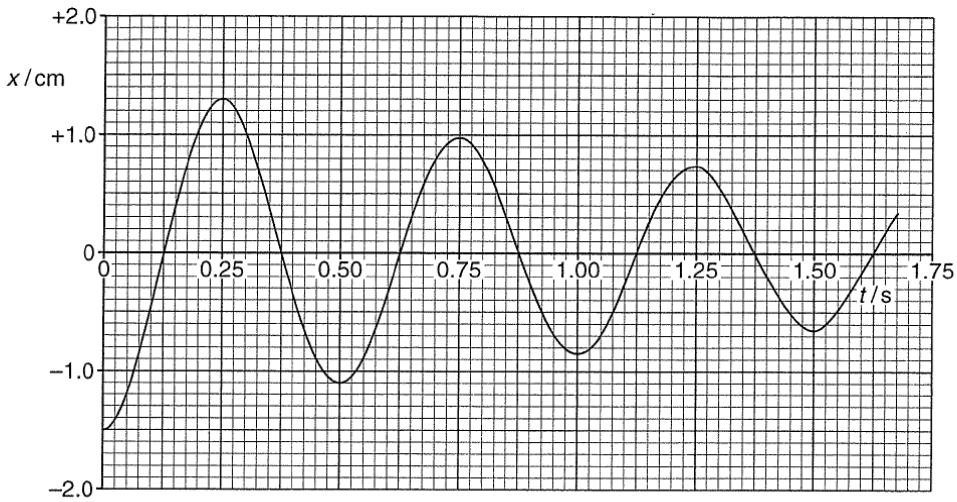
$$a = -\left(\frac{\rho A g}{m}\right)x$$

where *g* is the acceleration of free fall.

(i) Explain why the expression leads to the conclusion that the tube is performing simple harmonic motion.

(ii) The tube has a total mass *m* of 32 g and the area *A* of its cross-section is 4.2 cm^2. It is floating in liquid of density ρ of 1.0×10^3 kg m^{-3}. Show that the frequency of oscillation of the tube is 1.8 Hz.

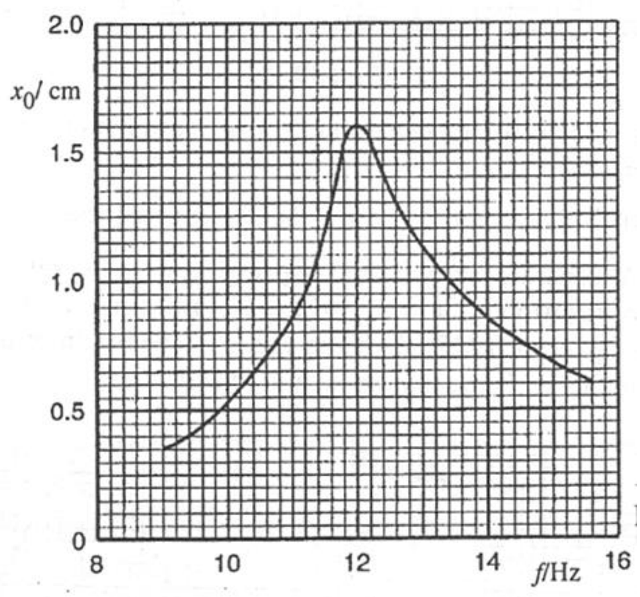
(d) The tube in **(b)** is now placed in a different liquid. The tube oscillates vertically. The variation with time *t* of the vertical displacement *x* of the tube is shown below.



(i) Use the figure to
1. determine the frequency of oscillation of the tube,
2. calculate the density of the liquid.

(ii) 1. Suggest two reasons why the amplitude of the oscillation decreases with time.
2. Calculate the decrease in energy of the oscillation during the first 1.0 s

D15 N05/II/4 (modified)
The figure shows the variation with frequency *f* of amplitude x_0 of the forced oscillations of a machine.



(a) What is meant by a forced oscillation?

(b) At any value of frequency, the oscillations of the machine are simple harmonic.

(i) The machine is vibrating at its maximum amplitude,

1. find the maximum magnitude of the linear speed,

2. find the maximum magnitude of the linear acceleration,

3. state the name of this effect.

(ii) Determine the time interval between maximum linear speed and subsequent maximum linear acceleration.

D16 N22/II/1 (modified)

A mass *m* is suspended from a vertical spring attached to a fixed support. The mass is pulled down and held at a vertical displacement of 0.16 m from its equilibrium position as shown in Fig. 1.1.

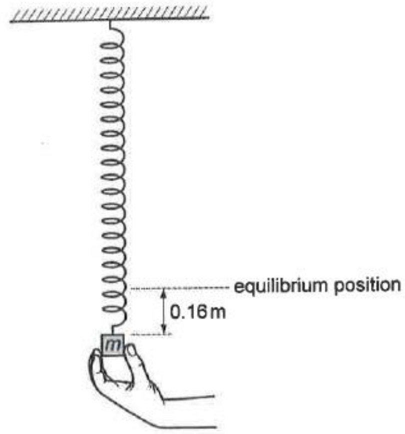


Fig. 1.1

The mass is released.

The variation with vertical displacement *x* of the velocity *v* of the mass on the spring is shown in Fig. 1.2.

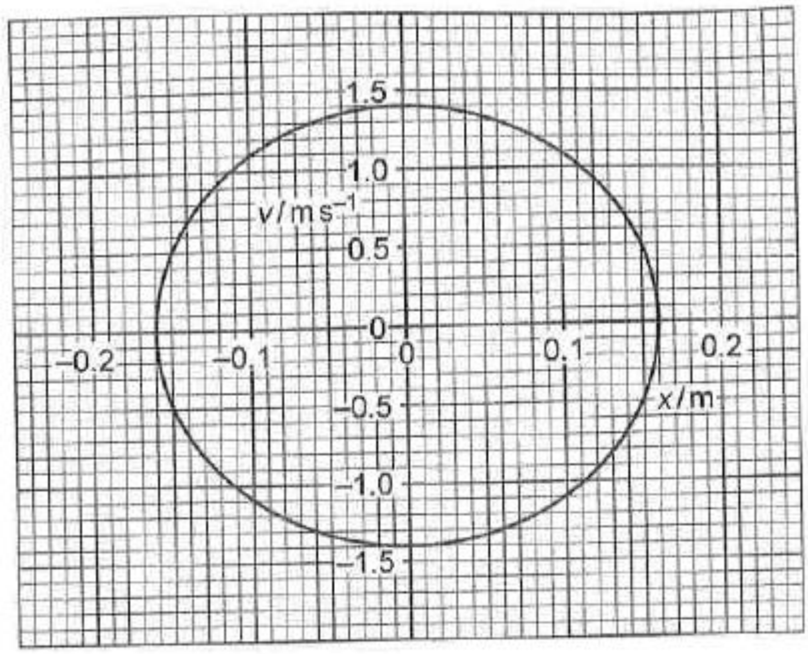


Fig. 1.2

(a) Determine the maximum acceleration a of the mass.

(b) A light piece of card is attached to the bottom of the mass so that the oscillations are lightly damped. The mass is displaced vertically and then released.

On Fig. 1.2, draw the variation of v with x to show the first complete cycle of these damped oscillations.

NUMERICAL ANSWERS

Self-Review Questions

S1 C
S2 8.0 Hz
S3 (i)(a) 20.0 cm; (i)(b) 0.4000 s; (i)(c) 2.500 Hz; (i)(d) 15.71 rad s^{-1}; (i)(e) -10.0 cm
(iii) 0.233 s; (v)(a) 0; (v)(b) 3.14 m s^{-1};
(v)(c) 49.3 m s^{-2}; (v)(d) 0
S4 v_{max} = 0.10 m s^{-1} at equilibrium position; a_{max} = 1.3 m s^{-2} at x = - 8.0 mm
S5 (i)(a) Point B; (i)(b) Point C; (ii) PE_C = 0,
KE_C = 4.93 J, PE_B = 4.93 J, KE_B = 0
S6 C
S7 A
S8 A

Discussion Questions

D1 0.17 s
D5 2.0 h
D6 (b)(ii) 82 rad s^{-1}, 1.47 mm
D7 20 cm, 1.8 s, 70 cm s^{-1}, 2.5 m s^{-2}
D8 $\frac{2m\pi^2a^2}{T^2}$
D10 1.8 s
D11 0.910 Hz; 0.990 m s^{-1};
D12 (b)(i) 0.785 J; (b)(ii) 0.392 J; (d)(i) 3.92 N; (d)(ii) 7.00 rad s^{-1}; (d)(iii) 1.40 m s^{-1}
D14 (d)(i)(1) 2.0 Hz, (2) 1200 kg m^{-3}, (d)(ii)(2) 3.86 x 10^{-4} J
D15 (b)(i)(1) 1.21 m s^{-1}, (2) 91.0 m s^{-2}; (b)(ii) 0.0208 s
D16 (a)12.3 m s^{-2}