Candidate Name: ______________________ (   ) Class: ________

4E
Session 2

KRANJI SECONDARY SCHOOL
Preliminary Examination
Secondary 4 Express

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

Wednesday **25 August 2021** 2 hr 15 min

Question	**Marks**
1	
2	
3	
4	
5	
6	
7	
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10	
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12	
Total	

READ THESE INSTRUCTIONS FIRST:

Do not open this question paper until you are told to do so.

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to three significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Set by: Ms Priscilla Lee

This Question Booklet consists of 20 printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\dfrac{x^4+1}{x^3+2x}$ in partial fractions.
[5]

2 Represent the solution set of $-15 < x^2 - 8x \leq 0$ on a number line and determine if x =
5 satisfies the inequality. [6]

3 The term containing the highest power of x in the polynomial $f(x)$ is $2x^3$. It is given that $x^2 - 2x + 3$ is a quadratic factor of $f(x)$ and one of the roots of the equation $f(x) = 0$ is – 1.

(i) Find an expression for $f(x)$ in descending powers of x. [2]

(ii) Find the number of real roots of the equation $f(x) = 0$, justifying your answer. [2]

(iii) Find the remainder when $f(x)$ is divided by $2x - 3$. [2]

4 **(a)** A circle, *C*, has the equation $x^2 + y^2 + 4x - 6y = 131$.

(i) Find the radius and the coordinates of the centre of *C*. [2]

(ii) The coverage of a Wi-Fi signal is represented by the equation of the circle *C*. Show, with working, why a person standing at the point *A*(10, 7) is not able to make use of the free Wi-Fi. [2]

(b) A rhombus *ABCD* has vertices *A*(1, 5), *B*(*b*, 0), where *b* is a constant, and *C*(–6, 2).

(i) Find the equation of the line *BD*. [3]

(ii)
Hence, or otherwise, find the coordinates of *B* and *D*. [3]

(iii)
Explain why *ABCD* is a square. [2]

5 **(i)** Show that $y = \ln\left(\dfrac{8+3x}{3x-4}\right)$ has no turning point for all values of *x*.
[4]

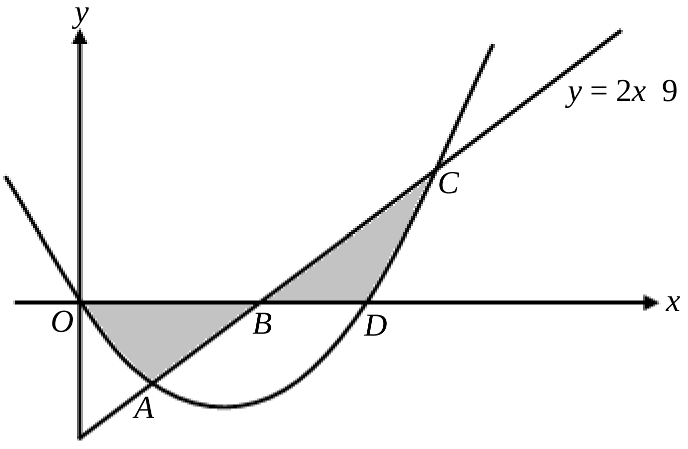
(ii) Find the range of values of x in which the graph of $y = \ln\left(\frac{8+3x}{3x-4}\right)$ is decreasing.

[3]

6 Find, in exact values, the coordinates of the stationary points of the curve $y = \sin x - \sqrt{3}\cos x$ for $0 < x < 2\pi$ and determine the nature of each stationary point.

[7]

7



The diagram shows part of the curve $y = x^2 - 8x$. The straight line $y = 2x - 9$ cuts the curve at the points *A* and *C*. The curve cuts the *x*-axis at the origin, *O* and *D*.

Find

(i) the coordinates of *A*, *B* and *C*,
[4]

(ii) the total area of the shaded regions.
[6]

8 A particle travelling in a straight line passes through a fixed point O with an acceleration of -2 m/s^2. The velocity, v m/s, of the particle, t s after passing through O, is given by $v = ke^{-\frac{1}{5}t} - 2$.

(i)` Show that $k = 10$. [2]

(ii) The particle comes to instantaneous rest at the point *P*. Calculate the distance *OP*. [4]

(ii) Show that the particle is again at *O* at some instant during the twenty-fifth second after first passing through *O*. [2]

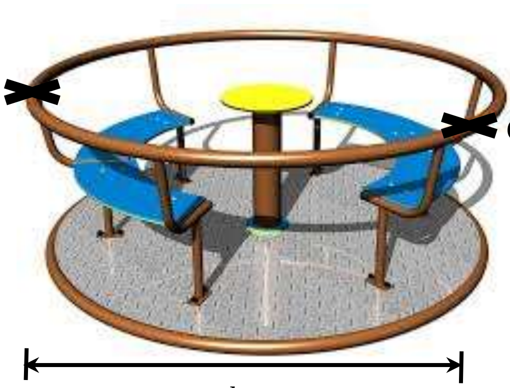
9 **(a)** Prove that $\dfrac{\sin 2A + \cos 2A + 1}{\sin 2A + \cos 2A - 1} = \dfrac{\tan(45° + A)}{\tan A}$.
[4]

(b) It is given that $\sin\theta - \cos\theta = \frac{3}{4}$.

(i) Show that $\sin\theta\cos\theta = \frac{7}{32}$.
[2]

(i) Hence find the value $7\cot\theta + 7\tan\theta$.
[2]

10



The horizontal distance of a child on a carousel, h m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where k is a constant and t is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

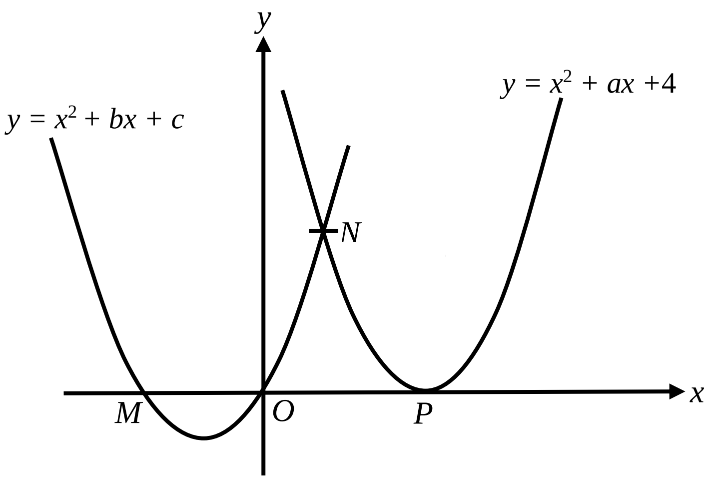
(i) Explain why this model suggests that the diameter of the carousel is 4 m.
[1]

(ii) Show that the value of k is $\frac{\pi}{10}$ radians per second.
[2]

As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

(iii) Find the duration of time for which the child will not be able to view the landmark
during one revolution.
[5]

11 In the diagram, the graph of $y = x^2 + ax + 4$ touches the x-axis at P. The origin O is the mid-point of MP. The graph of $y = x^2 + bx + c$ passes through the points M and O.



(a) Find the values of a, b and c.
[4]

(b) The graph of $y = x^2 + ax + 4$ and the graph of $y = x^2 + bx + c$ intersect at *N*.

(i) Find the coordinates of *N*. [2]

(ii) If another graph of $y = mx^2 + nx + r$ has its turning point at *N* and passes through *P*, find the values of *m*, *n* and *r*, where $r > 0$.
[3]

12 Given the equation $\left(2^{5r}\right)\left(243^{r}\right)\left(\frac{1}{\sqrt{6}}\right)^{r} = 36^{\frac{1}{4r}}$, find r for $r > 0$. [4]

End of paper