2024 AMKSS 4E5N Prelim AM Paper 1 Solutions

Comments: - Minus 1 mark overall for expressing coordinates in improper fraction (Q1 & Q12i) or missing unit such as degree (Q9i, Q10ii)

Qn	Solutions	Marks
1 **[5]**	$\frac{5x}{y}-\frac{12y}{x}=\frac{-7}{xy}$ $\frac{5x^2}{xy}-\frac{12y^2}{xy}=\frac{-7}{xy}$ $5x^2-12y^2=-7 \quad -(1)$ $y=\frac{5x+7}{2} \quad -(2)$ Substitude (2) into (1) $5x^2-12\left(\frac{5x+7}{2}\right)^2=-7$ $5x^2-12\left(\frac{25x^2+70x+49}{4}\right)=-7$ $5x^2-3(25x^2+70x+49)=-7$ $5x^2-75x^2-210x-147=-7$ $70x^2+210x+140=0$ $x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-2$ or $x=-1$ $y=-1.5$ or $y=1$ $A(-2,-1.5),\ B(-1,1)$	M1 (substitution) M1 (expand & simplify to quadratic eqn) M1 (factorise/formula) A2 (coordinates should not be in improper fraction)
2(a) **[2]**	$\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ $=\frac{3+\sqrt{6}}{3-2}$ $=3+\sqrt{6}$	M1 (conjugate) A1

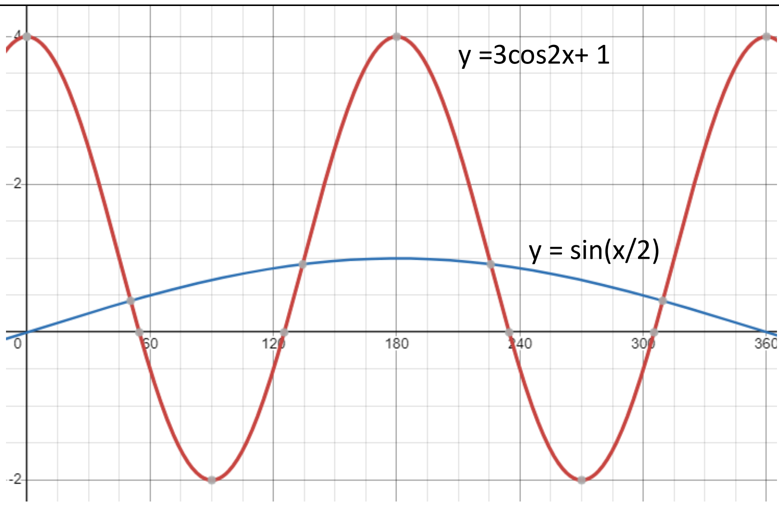
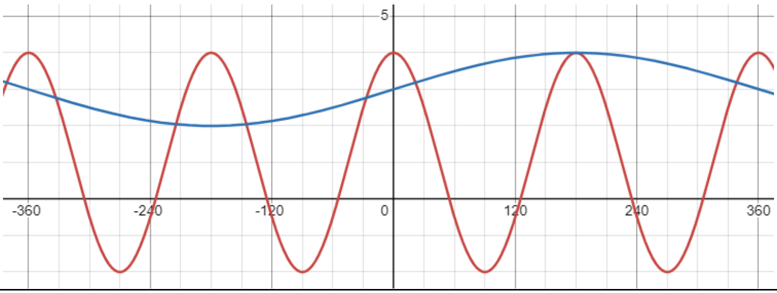
2(b) **[4]**	$\left(\frac{\sqrt{30}}{2}\right)^2+\left(\frac{3+\sqrt{6}}{2}\right)^2=\left(\frac{P}{4}\right)^2$ $\frac{30}{4}+\frac{(3+\sqrt{6})^2}{4}=\frac{P^2}{16}$ $30+(3+\sqrt{6})^2=\frac{P^2}{4}$ $30+9+6\sqrt{6}+6=\frac{P^2}{4}$ $45+6\sqrt{6}=\frac{P^2}{4}$ $P^2=4(45+6\sqrt{6})$ $P^2=180+24\sqrt{6}$	M1 (Pythagoras theorem) M1 (attempt to form an eqn in P) A1, A1
3(i) **[1]**	There is a fixed price of $100 million incurred, even when no submarines were assembled.	B1
3(ii) **[2]**	$y=\frac{5}{2}x^2-20x+100$ $y=\frac{5}{2}\left(x^2-8x+40\right)$ $y=\frac{5}{2}\left(x^2-8x+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2+40\right)$ $y=\frac{5}{2}[(x-4)^2+24]$ $y=\frac{5}{2}(x-4)^2+60$	M1 A1
3(iii) **[1]**	x = 4 occurs at the minimum point of the curve. Hence, the **cost per submarine** will be the lowest when we assemble 4 submarines	B1

4(a) **[4]**	$\log_4 2y - \frac{\log_4(y-3)}{2} = 3\log_4 2$ $\log_4\left(\frac{2y}{\sqrt{y-3}}\right) = \log_4 8$ $\frac{2y}{\sqrt{y-3}} = 8$ $\sqrt{y-3} = \frac{y}{4}$ $y-3 = \frac{y^2}{16}$ $y^2 - 16y + 48 = 0$ $(y-4)(y-12) = 0$ $y = 4$ or $y = 12$	M1 (quotient law of log) A1 (correct expression or equivalent) M1 (factorise/formula) A1
4(b) **[3]**	$\log_{27} z = \log_9 \sqrt{y}$ $\frac{\log_3 z}{\log_3 3^3} = \frac{\log_3 \sqrt{y}}{\log_3 3^2}$ $\frac{\log_3 z}{3} = \frac{\log_3 y^{\frac{1}{2}}}{2}$ $\log_3 z = \frac{3}{2}\log_3 y^{\frac{1}{2}}$ $\log_3 z = \log_3 y^{\frac{1}{2}\times\frac{3}{2}}$ $\log_3 z = \log_3 y^{\frac{3}{4}}$ $z = y^{\frac{3}{4}}$ **NOTE**: If working end up with $z^2 = y^{\frac{3}{2}}$ $z = y^{\frac{3}{4}}$ or $z = -y^{\frac{3}{4}}$ (Rej)	M1 (change of base formula) M1 (power law) A1 Minus 1m if never show reject negative answer

5 **[5]**	$\frac{2x^3+6x^2+1}{(x-1)(x+2)^2}$ $=\frac{2x^3+6x^2+1}{(x-1)(x^2+4x+4)}$ $=\frac{2x^3+6x^2+1}{x^3+3x^2-4}$ Using long division, $2+\frac{9}{(x-1)(x+2)^2}$ $\frac{9}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ $9=A(x+2)^2+B(x-1)(x+2)+C(x-1)$ When $x=1$; $9=9A$ $A=1$ When $x=-2$; $9=-3C$ $C=-3$ When $x=0$; $9=4-2B+3$ $B=-1$ $\therefore 2+\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$	B1 $\left(2+\frac{R}{D(x)}\right)$ M1 (correct case) A2 (minus 1m for each incorrect ans) A1
6(a) **[4]**	$y=\frac{1}{2}x+\frac{7}{2}$ Gradient of $AC=-2$ Equation of $AC: y=-2x+6$ $y=-2x+6 \quad -(1)$ $x+5y=-6 \quad -(2)$ $x+5(-2x+6)=-6$ $x-10x+30=-6$ $-9x=-36$ $x=4$ $y=-2$ $C(4,-2)$	M1 ($\perp$ line: $m_1=-\frac{1}{m_2}$) M1 (solve simultaneous eqns) M1 A1

6b(i) **[3]**	At $x-axis; y=0$ $x=-6$ $D(-6,0)$ Let $B(x,6)$ Area of $ACD = 1.5\ \times$ Area of ABC $\frac{1}{2}\begin{vmatrix}0 & -6 & 4 & 0\\ 6 & 0 & -2 & 6\end{vmatrix} = 1.5\ \times\ \frac{1}{2}\begin{vmatrix}0 & 4 & x & 0\\ 6 & 2 & 6 & 6\end{vmatrix}$ $\frac{1}{2}[36-(-36)] = \frac{3}{4}[24+6x-(-2x+24)]$ $36 = \frac{3}{4}(8x)$ $x=6$ $\therefore B(6,6)$	A1 (find pt D) M1 (must be anticlockwise) A1
6b(ii) **[3]**	$AC = \sqrt{(0-4)^2+(6+2)^2}$ $= \sqrt{80}$ $\frac{1}{2}\times\sqrt{80}\times d = 36$ $d = \frac{72}{\sqrt{80}}$ or 8.05 unit	M1 (length formula) M1 (area of ACD) A1
7(i) **[4]**	$xy+\pi x^2 = 40$ $y = \frac{40-\pi x^2}{x}$ $P = 2\pi x + 2x + 2y$ $P = 2\pi x + 2x + 2\left(\frac{40-\pi x^2}{x}\right)$ $P = 2\pi x + 2x + \frac{80}{x} - 2\pi x$ $P = 2x + \frac{80}{x}$	M1 (make y the subj) M1 (correct P exp) M1 (subt in their y) M1 (show expansion)

7(ii) **[6]**	$\frac{dP}{dx} = 2 - \frac{80}{x^2}$ $2 - \frac{80}{x^2} = 0$ $x^2 = 40$ $x = 6.325 \;\; or \;\; -6.235(\text{NA})$ Stationary value of $P = 2(6.235) + \frac{80}{6.235}$ $= 25.3$ cm $\frac{d^2P}{dx^2} = -2(-2) \times \frac{80}{x^3}$ $= \frac{160}{x^3}$ When $x = 6.325$; $\frac{d^2P}{dx^2} = \frac{160}{(6.235)^3} > 0$ $\therefore P$ is minimum	M1 (-1m for any incorrect term) M1 ($\frac{dP}{dx} = 0$) A1 (must show reject negative x value) A1 M1 (accept 1st or 2nd derivative test) A1 (explanation + conclude min)
8(i) **[4]**	$F(x) = k(x+1)(x-2)(x-5)$ $F(3) = k(4)(1)(-2)$ $30 = -8k$ $k = -\frac{15}{4}$ $F(x) = -\frac{15}{4}(x+1)(x-2)(x-5)$ $F(-3) = 300$ $R = 300$	M1 (do not award if coeff of x^3 assume 1) M1 (remainder theorem) M1 (subt $x = -3$ into their $F(x)$ or correct long division) A1
8(ii) **[2]**	$-\frac{15}{4}(\sqrt{m}+1)(\sqrt{m}-2)(\sqrt{m}-5) = 0$ $\sqrt{m} = -1$ (Rej) or $\sqrt{m} = 2$, $m = 4$ or $\sqrt{m} = 5$, $m = 25$	M1 A1 (must reject one ans)

9(i) **[2]**	Amplitude = 3 Period = 180^0	B1 B1 (must write degree)
9(ii) **[4]**		For $y = 3\cos 2x + 1$ B1- 2 cycles with correct cosine shape through (0°, 4) & (360°, 4) & of amplitude 3 B1- correct turning points & middle values (45°, 135°, 225°, 315°) For $y = \sin(x/2)$ B1- half cycle of sine through (0°, 0) & (360°, 0) & of amplitude 1 B1- correct max point (180°, 1)
9(iii) **[1]**	$k = 3$ 	A1 (given only if both graphs in (ii) are sketched correctly)
10(i) **[1]**	$p - q$	B1
10(ii) **[3]**	$\frac{dV}{dt} = 80$ $\frac{dV}{dh} = \pi h^2$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $80 = \pi(5)^2 \times \frac{dh}{dt}$ $\frac{dh}{dt} = 1.02\text{cm/s}$	 M1 M1 A1 (3sf)

11(a) **[3]**	Let $\angle DBE = x$ $\angle BAD = \angle DBE = x(\angle s$ in alternate segment) $\angle BAE = \angle CAF = x(EA$ is bisector of $\angle BAC)$ $\angle CBE = \angle CAF = x(\angle s$ in same segment) $\therefore \angle CBD = \angle DBE = x$(proven)	M1 M1 M1
11(b) **(i)** **[2]**	$\angle ADB = 90°$(right angle in semi-circle) $\angle AOF = \angle ADB = 90°$ $\angle OAF = \angle DAB$(common $\angle$) $\therefore \Delta AOF$ is similar to ΔADB(AA Similarity test)	M1 M1 (must state AA similarity test)
11(b) **(ii)** **[2]**	Since ΔAOF is similar to ΔADB $\frac{AO}{AD} = \frac{AF}{AB}$ $\frac{AO}{AF+FD} = \frac{AF}{AB}$ $\frac{AO}{AF+FD} = \frac{AF}{2AO}$ (AO is radius and AB is diameter) $2(AO)^2 = AF \times (AF + FD)$	M1 M1
12(i) **[5]**	$\frac{dy}{dx} = \frac{1}{2}(6x-5)^{-\frac{1}{2}}(6)$ $= \frac{3}{\sqrt{6x-5}}$ Gradient of tangent = 1 $\frac{3}{\sqrt{6x-5}} = 1$ $\sqrt{6x-5} = 3$ $6x - 5 = 9$ $x = 2\frac{1}{3}$ $y = 3$ $T\left(2\frac{1}{3}, 3\right)$	M1 (chain rule) A1 M1 (dy/dx =1) M1 A1

12(ii) [6]	$\sqrt{6x-5}=x$ $6x-5=x^2$ $x^2-6x+5=0$ $(x-5)(x-1)=0$ $x=5$ or $x=1$ $\int_1^5 (6x-1)^{\frac{1}{2}}\,dx$ $=\left[\frac{(6x-5)^{\frac{3}{2}}}{\frac{3}{2}\times 6}\right]_1^5$ $=\left[\frac{(6x-5)^{\frac{3}{2}}}{9}\right]_1^5$ $=\frac{125}{9}-\frac{1}{9}$ $=13\frac{7}{9}$ Area of trapezium $=\frac{1}{2}(5+1)\times 4$ $=12$ Shaded Area $=13\frac{7}{9}-12$ $=1\frac{7}{9}$ units^2	M1 (solve simultaneously) A1 M1 (correct limits) M1 (correct integration) M1 A1 (accepts improper fraction for area)

13(i) **[4]**	LHS $= \dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\tan\theta}$ $= \dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\frac{\sin\theta}{\cos\theta}}$ $= \dfrac{\sin\theta}{1-\cos\theta} - \dfrac{\cos\theta}{\sin\theta}$ $= \dfrac{\sin^2\theta - \cos\theta(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$ $= \dfrac{\sin^2\theta - \cos\theta + \cos^2\theta}{\sin\theta(1-\cos\theta)}$ $= \dfrac{1-\cos\theta}{\sin\theta(1-\cos\theta)}$ $= \dfrac{1}{\sin\theta}$ $= \text{cosec}\,\theta$	M1 ($\tan\theta = \dfrac{\sin\theta}{\cos\theta}$) M1 (combined fraction) M1($\sin^2\theta + \cos^2\theta = 1$) M1
13(ii) **[4]**	$\text{cosec}\,2A = 9\sin 2A$ $\dfrac{1}{\sin 2A} = 9\sin 2A$ $9\sin^2 2A = 1$ $\sin^2 2A = \dfrac{1}{9}$ $\sin 2A = \pm\dfrac{1}{3}$ $Acute\angle = 0.33983$ $2A = 0.340, 2.80, 3.48, 5.94$ $A = 0.170, 1.40, 1.74, 2.97$	M1 M1 (2 values) A1, A1