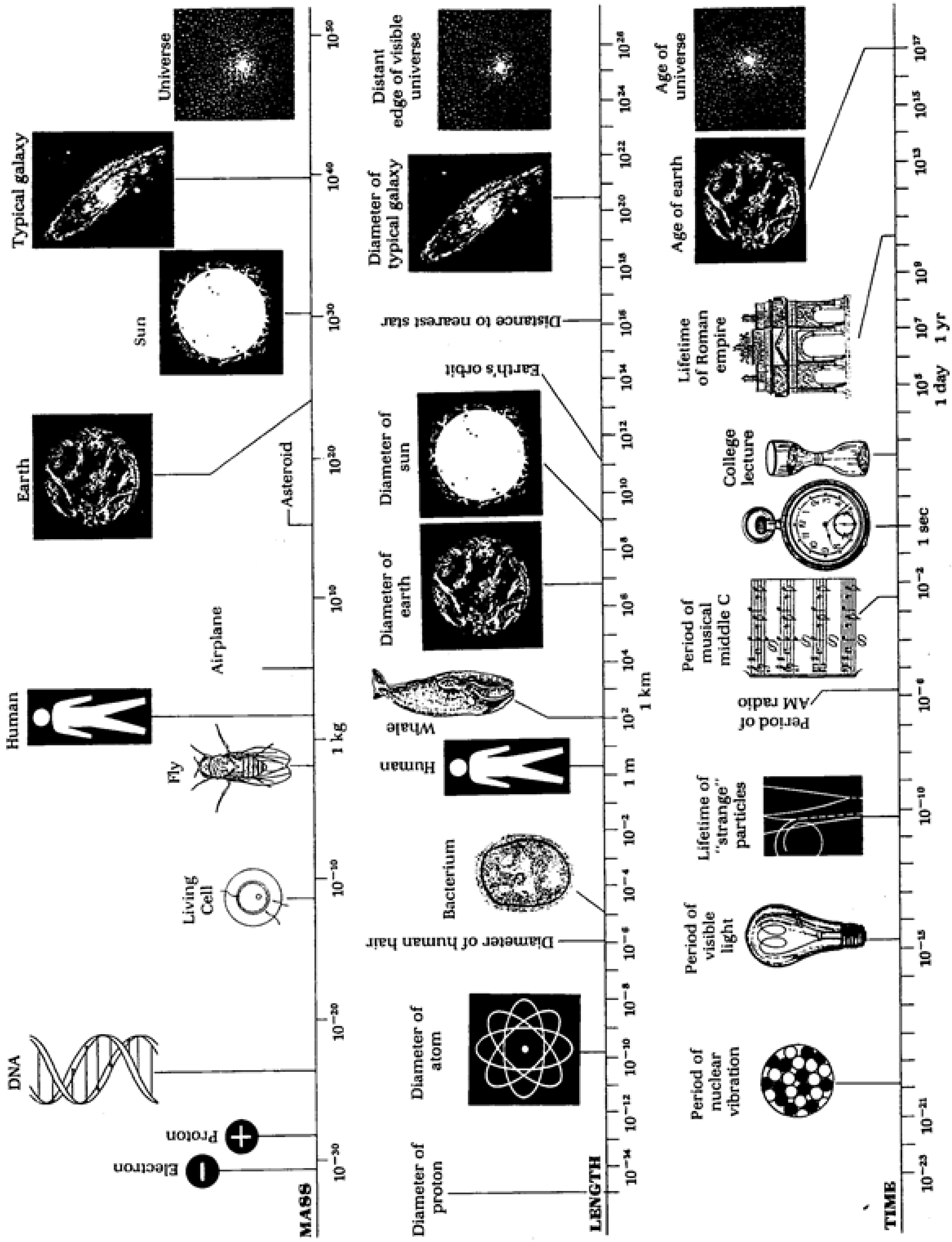
# Chapter 1

# Measurement




An indication of the range over which physical measurements are made.

# TOPIC 1: MEASUREMENT

**H2 Physics Syllabus 9749**

**SECTION I: Measurement**

| **Measurement** | **Learning Outcomes**<br>Students should be able to: |
|---|---|
| Physical quantities and SI units | (a) recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).<br>(b) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.<br>(c) use SI base units to check the homogeneity of physical equations.<br>(d) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics* (*The ASE Companion to 16–19 Science*, 2000).<br>(e) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).<br>(f) make reasonable estimates of physical quantities included within the syllabus. |
| Scalars and vectors | (g) distinguish between scalar and vector quantities, and give examples of each.<br>(h) add and subtract coplanar vectors.<br>(i) represent a vector as two perpendicular components. |
| Errors and uncertainties | (j) show an understanding of the distinction between systematic errors (including zero error) and random errors.<br>(k) show an understanding of the distinction between precision and accuracy.<br>(l) assess the uncertainty in a derived quantity by addition of actual, fractional, percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required). |

**Playlist of teaching videos and lecture examples:**
https://youtube.com/playlist?list=PL_b5cjrUKDlY0aQFdGFfOKuFJQTiXzIeY

## Contents

## 1.1 Introduction

Measuring our observations objectively is an important task that differentiates science from the arts. The scientific method involves making observations that are repeatable by another person. For this reason, the study of physical quantities and their units is the starting point of our learning of science.

## 1.2 Physical Quantities and Units

A physical quantity is a physical property that can be measured and/or calculated and expressed in numbers. For example, "length" is a physical quantity that can be expressed by stating a number and a measurement unit such as metres or inches.

Hence ***the value of a physical quantity*** is expressed as ***a numerical value and a unit of measurement.***

For example,

the length of a football field = 100 m

*Physical Quantity* | *Numerical value* | *Unit*

This has a few implications.

1. Using the unit “metre” helps us to define a particular property of the football field – its length. Using other units could allow us to define other properties (for example, square metres describe the area) of a football field.
2. We can only compare properties of the same unit. Using the “metre” allows us to compare the length of one object with another. We cannot compare a “metre” with a “kilogram”.
3. The numerical value of the physical quantity is dependent on the unit it is expressed in. The length of the same football field above will have a value of 330 when expressed in the unit “foot”.

### 1.2.1 Base Quantities and Units

Through the centuries, scientists in different parts of the world have used different units at different times to measure the same physical quantities. However standardised definitions and units are needed in the modern world.

The current International System of Units[a], universally abbreviated SI (from the French *Le Système International d'Unités*), is the modern metric system of measurement.

Seven well-defined and independent base quantities are chosen. Their corresponding **SI base units** are indicated:

| **Base Quantity** | | **SI Base Unit** | |
|---|---|---|---|
| Name | Symbol | Name | Symbol |
| Mass | $m$ | kilogram | kg |
| Length | $l$ | metre | m |
| Time | $t$ | second | s |
| Electric current | $I$ | ampere | A |
| Thermodynamic temperature | $T$ | kelvin | K |
| Amount of substance | $n$ | mole | mol |
| Luminous intensity | $I_v$ | candela | cd |

Note:

1. The **units**, when **written in full**, are **in small letters**. Ampere and Kelvin are names of scientists.
2. The kilogram is the only SI unit with a prefix as part of its name and symbol.
3. Luminous intensity is not in the Learning Objectives(a) of the Syllabus.
4. Definitions of the units are not required in the Syllabus but are provided in Appendix I at the end of the lecture notes for your reference.

**IMPORTANT TO KNOW**

SI base units were chosen based on their mutual independence. They are the simplest units that cannot be expressed in terms of other SI base units.

### 1.2.2 Derived Quantities and Units

There are a lot of other physical properties that we come across in our daily lives. These quantities and units used scientifically are included in the SI as derived quantities and units.[b] Note that these derived units, such as newton and joule, are also SI units.

**IMPORTANT TO KNOW**

Derived quantities are related to the base quantities through mathematical and scientific equations. Derived units (the units of derived quantities) are defined in terms of the base units using the same equations.

One simple example is the derived quantity of area. It is defined using the mathematical equation area = length × breadth, where both length and breadth are defined in terms of the base quantity, length. Hence the unit for area can be written in terms of base units as $m^2$.

[a] The SI was established in 1960 by the 11th General Conference on Weights and Measures (CGPM, Conférence Générale des Poids et Mesures). (http://www.bipm.org/en/si/).
[b] There are some commonly used units that are not in the SI. Some examples are the minute, hour, day, degree, litre, tonne, electron-volt, unified atomic mass unit, nautical mile, knot, hectare, bar and angstrom.

Some derived units have special names and symbols. An example is the newton (symbol N), a unit for force. We can define this derived quantity - force, using formulae such as $F = ma$, where $F$ is force, $m$ is mass and $a$ is acceleration. You may note that acceleration itself is also a derived quantity. Using such an equation, we can then find the representation of the newton (N) in terms of SI base units (kg m $s^{-2}$).

For more information on the common symbols and units that will be used in A level question papers, refer to Appendix II. As a practice, derive the SI base units for the quantities in the table below.

| Derived quantities, symbols | e.g. of mathematical relationship between quantities | SI base units | SI derived units |
|---|---|---|---|
| Plane Angle | $s = r\theta$<br>$s$ : arc length of circle<br>$r$ : radius of circle<br>$\theta$ : angle subtended by arc | | radian |
| Density, $\rho$ | $\rho = \frac{M}{V}$<br>$M$ : mass of body<br>$V$ : volume of body<br>$\rho$ : density | kg $m^{-3}$ | |
| Force, $F$ | $F = ma$<br>$m$ : mass<br>$a$ : acceleration | kg m $s^{-2}$ | N (newton) |
| Momentum, $p$ | $p = mv$ | kg m $s^{-1}$ | N s (newton-second) |
| Pressure, $p$ | $p = F/A$<br>$A$ : area | kg $m^{-1}$ $s^{-2}$ | Pa (pascal) |
| Energy, $E$ | $KE = ½\ mv^2$<br>$v$ : velocity | kg $m^2$ $s^{-2}$ | J (joule) |
| Power, $P$ | $P = E/t$<br>$E$ : energy<br>$t$ : time | kg $m^2$ $s^{-3}$ | W (watt) |
| Electric charge, $Q$ | $Q = It$<br>$I$ : current<br>$t$ : time | A s | C (coulomb) |
| Electric potential difference, $V$ | $V = W/Q$<br>$W$ : work done<br>$Q$ : electric charge | kg $m^2$ $s^{-3}$ $A^{-1}$ | V (volt) |
| Resistance, $R$ | $R = V/I$<br>$V$ : electric potential difference<br>$I$ : current | kg $m^2$ $s^{-3}$ $A^{-2}$ | Ω (ohm) |
| Frequency, $f$ | $f = 1/T$<br>$f$ : frequency<br>$T$ : period | $s^{-1}$ | Hz (hertz) |

Example 1

According to Newton's Law of Gravitation, the force $F$ between two point masses $M$ and $m$ separated by the distance $r$ is given by the formula $F = \frac{GMm}{r^2}$ where $G$ is the universal gravitational constant.
Obtain the SI base units for $G$. [Ans: $m^3$ $s^{-2}$ $kg^{-1}$]

**Common quantities quoted with <u>no unit of measurement</u>**

1. all pure numbers, e.g. 2, $\frac{1}{2}$, $\pi$, $e$
2. trigonometrical functions, e.g. sine, cosine, tangent
3. all logarithmic functions, e.g. $\log_{10}$, ln
4. powers, e.g. $10^n$, $e^n$, n has no unit.
5. physical constants with no unit of measurement: e.g. refractive index of glass, relative density of a liquid.

### 1.2.3 Homogeneity of equations

For an equation to be physically plausible, the equation must be ***homogenous***. This is to say the terms on both sides of the equations must have the same units when expressed in SI base units.[c] This is because only quantities with the same units may be equated, added or subtracted.

<u>Example 2</u>
Bernoulli's equation, which applies to fluid flow states that

$$P + h\rho g + ½\,\rho v^2 = k$$

where $P$ is pressure, $h$ is height, $\rho$ is density, $g$ is acceleration due to gravity, $v$ is velocity and $k$ is a constant.

Show the LHS of the equation is homogeneous and state the SI unit for $k$. [Ans: kg $m^{-1}$ $s^{-2}$]

[c] An equation where terms on both sides of the equation have the same SI base units also imply that they have the same dimensions. However, knowledge of dimensions is not necessary for 9749 syllabus. See Appendix I (Dimensions of physical quantities) if you wish to understand further.

Checking of homogeneity of an equation (by comparing the SI base units of the left and right hand side terms of the equation) is not only a powerful way of establishing if an equation is reasonable, but it also provides hints for guessing the actual equation.

Example 3

Consider the period *T* of a simple pendulum. The possible factors which may affect it are its length *l*, its mass *m* and the acceleration due to gravity *g*. Use unit analysis to arrive at a plausible relationship between *T* and these quantities.

A physically correct equation must be homogeneous. However, a homogeneous equation need not be physically correct. Ultimately, the validity of an equation can only be verified through experiments.

There are two basic reasons:

(1) The value of the coefficient may be incorrect.

e.g. $E = 3mv^2$ where $E$ = kinetic energy

*The coefficient 3 is incorrect. The value should be* $\frac{1}{2}$ *instead.*

(2) Missing or extra terms that may have the same unit.

e.g. $E = \frac{1}{2}mv^2 + mgh$ where $E$ = kinetic energy

*There is an extra term mgh, which happens to have the same base unit as kinetic energy. This is an extra term.*

### 1.2.4 Prefixes

Prefixes can be used with both base units and derived units. The rationale for prefixes is simple. While it is alright to write your height as 1.65 m, it will be quite cumbersome to write the width of your hair as 0.00005 m, or the size of an atom as 0.0000000001 m. In mathematics, you have learnt the use of standard form. The size of an atom can be rewritten more neatly as $1 \times 10^{-10}$ m but scientists often use prefixes instead of standard form for values between $10^{-12}$ and $10^{12}$.

| Factor | Prefix | Symbol | Name | Decimal equivalent | Order of magnitude[d] |
|---|---|---|---|---:|---:|
| $10^{-12}$ | pico | p | Trillionth | 0.000,000,000,001 | -12 |
| $10^{-9}$ | nano | n | Billionth | 0.000,000,001 | -9 |
| $10^{-6}$ | micro | μ | Millionth | 0.000,001 | -6 |
| $10^{-3}$ | milli | m | Thousandth | 0.001 | -3 |
| $10^{-2}$ | centi | c | Hundredth | 0.01 | -2 |
| $10^{-1}$ | deci | d | Tenth | 0.1 | -1 |
| $10^{0}$ | - | - | One | 1 | 0 |
| $10^{3}$ | kilo | k | Thousand | 1,000 | 3 |
| $10^{6}$ | mega | M | Million | 1,000,000 | 6 |
| $10^{9}$ | giga | G | Billion | 1,000,000,000 | 9 |
| $10^{12}$ | tera | T | Trillion | 1,000,000,000,000 | 12 |

The accepted convention is to use a prefix such that the quantity can be written as a whole number and of least significant figures (s.f.). For example, the wavelength of red light would be written as 650 nm, rather than 0.65 μm. Values larger than the ranges listed in the prefixes above should be written in standard form. (The above list of prefixes are stipulated in LO(e). Other prefixes are listed in Appendix III).

You may notice that the symbol for the prefix milli (m) is the same as that for the base unit metre (m), leading to a confusion when we see, for example, "$ms^{-1}$". It could mean "metre per second", or "per millisecond". To distinguish the units especially in print, the A-level standard is to **use "$ms^{-1}$" for "per millisecond" and leave a space "$m\ s^{-1}$" for the derived unit "metre per second".**

**Further reference:**

Powers of Ten video
https://www.youtube.com/watch?v=0fKBhvDjuy0

The Scale of the Universe 2 video
https://www.youtube.com/watch?v=uaGEjrADGPA

[d] http://en.wikipedia.org/wiki/Order_of_magnitude

## 1.3 Estimation

When was the last time you estimated something?

Physicists frequently use "back-of-the-envelope" calculations or "Fermi" problems, named after Physicist Enrico Fermi who worked on the Manhattan Project during World War II. Fermi was known for making approximate calculations with little or no actual data. One well-documented example was his estimate of the strength of the atomic bomb based on the distance travelled by bits of paper dropped from his hand at the test blast.[e]

Estimation in Physics uses simple numbers (1, 2 or 5) with the correct order of magnitude ($10^3$ or $10^{-4}$). It is important to know whether a lecture theatre can sit a few students, tens, hundreds or thousands of students, but not so important to know that it has a capacity of 327. When the exact answer or values are not known, assumptions and estimation are used to find a rough answer.

Estimation is also very useful as a check to the answer that we would get from complex calculations.

Examples of "Fermi" problems:

- Estimate the total number of hairs on your head.
- How many bricks are needed to build a home?
- Estimate the efficiency of an electric kettle.
- How much paper is used by Hwa Chong Institution for lecture notes in 1 year?

Some important estimates that you should memorize:

- Typical wavelength of visible light: $10^{-7}$ m (range: 400 to 700 nm)
- Range of wavelengths of other electromagnetic regions
- Size of an atom: $10^{-10}$ m or 1 angstrom (Å)
- Size of a nucleus: $10^{-15}$ m

Strategy for estimation:

1. Identify the unknown: Define specifically what you need to estimate
2. Identify the known: Find your experience that may help you relate to the unknown
3. Find a relationship between the known and the unknown: make a connection between what you know and what needs to be estimated.

[e] http://en.wikipedia.org/wiki/Fermi_problem

Example 4 [N09/I/2]

Which estimate is realistic?

A The kinetic energy of a bus travelling on an expressway is 30 000 J.
B The power of a domestic light is 300 W.
C The temperature of a hot oven is 300 K.
D The volume of air in a tyre is 0.03 $m^3$.

**Further reference:**

MIT OpenCourseWare Video – the Art of Approximation
https://youtu.be/X8DlaW83HJc

## 1.4 Errors and Uncertainties

Whenever we attempt to make a measurement of a physical quantity, we are prone to all sorts of errors. As such, it is actually quite difficult, maybe impossible, to obtain a true value of the physical quantity. This can be a serious problem. It is essential in any experiment, in order to check the validity of a hypothesis, to be confident of our measurements, so that if the value of a physical quantity obtained in the experiment differs from the value predicted by the hypothesis, we can reject the hypothesis.

A measurement of a physical quantity $X$ is reported in the form, $X = (x \pm \Delta x)$ where $\Delta x$ refers to the uncertainty associated with the measurement of $X$.

It is to be interpreted as we are pretty confident that the true value of $X$ lies in the interval $(x - \Delta x, x + \Delta x)$. If the predicted value falls within this interval, then we have no evidence to reject the hypothesis. However, if the predicted value falls outside this interval, we may claim that we have evidence to reject the hypothesis.

Two scientists who performed their experiments independently in an attempt to measure the quantity $X$ will also say that their values of $X$ are consistent with each other if the two intervals overlap. If the intervals established by each scientist do not overlap, then their values of $X$ are inconsistent.

In attempting to measure $X$, we will naturally strive to do our best and make several measurements. But let's first suppose that we only make one measurement. In such a situation, the precision of the instrument we used limits our ability to obtain the true value. For example, when using a metre rule to measure the length of a pencil, we may at most be certain that our measured value is right up to the nearest mm, as the smallest division on the metre rule is 1 mm. We will hence report our measurement as $X = (14.6 \pm 0.1)$ cm.

$$x \pm \Delta x = (14.6 \pm 0.1)\ \text{cm}$$

Physical Quantity (length of a pencil)

Actual uncertainty

Measured physical quantity

Estimated uncertainty (limited by instrumental precision)

However, there could be other sources of error that we did not realise. With only a single measurement, we will not be able to tell whether we might get a different reading the next time we tried and how far off that reading will be. As such, even though we have established an interval based on just one reading, it is entirely possible that the true value could fall outside this interval.

If we are aware of the presence of experimental errors other than instrumental errors, we could provide for it. For example, when we attempt to measure the length of the pencil, the butt of the pencil was broken with jagged edges, and it is difficult to ascertain the edge very precisely. As such, we could take that into consideration and report our measurement as $X = (14.6 \pm 0.3)$ cm with a reasonable subjective estimated uncertainty of 0.3 cm.

$$x \pm \Delta x = (14.6 \pm 0.3)\ \text{cm}$$

Physical Quantity (length of a pencil)

Actual uncertainty

Measured physical quantity

Estimated uncertainty (considering errors that exceed the instrumental precision)

Ideally, we should make more attempts at measuring $X$. If we make multiple attempts, the spread of the data could give us a better idea of the impact of errors that could be present and help us to make a more realistic guess at the interval which could contain the true value. As a rule of thumb, a reasonable estimate would be to determine the uncertainty as $\Delta x = \frac{1}{2}(x_{max} - x_{min})$. We would also use the mean $\langle x \rangle$ of our readings as the true value of $X$. Hence we will report our measurement of $X$ as

$$X = \langle x \rangle \pm \frac{x_{max} - x_{min}}{2}$$

| | 1st attempt | 2nd attempt | 3rd attempt |
|---|---|---|---|
| Length of pencil / cm | 14.6 | 14.3 | 14.4 |

$$X = \left(\frac{14.6 + 14.3 + 14.4}{3}\right) \pm \left(\frac{14.6 - 14.3}{2}\right)$$
$$= (14.4 \pm 0.2)\text{ cm}$$

$$x \pm \Delta x = (14.4 \pm 0.2)\text{ cm}$$

Physical Quantity (length of a pencil)

Actual uncertainty

Mean value of multiple readings

Estimated uncertainty (considering max-min values)

You would realise by now that there is some level of arbitrariness in declaring the uncertainty associated with our measurement. Nevertheless, while declaring a larger uncertainty will result in a larger interval that is more likely to contain the true value, remember that too large an interval will also cause any subsequent conclusions derived from the experiment to be meaningless.

In practice, many attempts are made and we have a very large set of data and a statistical approach is used to establish the uncertainty but we will not go into that as it is not in our syllabus.

**Fractional uncertainty**

The **fractional uncertainty** is the ratio of the actual uncertainty to the measured value.

The fractional uncertainty in a quantity $x$ is $\dfrac{\Delta x}{x}$.

Example: The fractional uncertainty of $x$, $\dfrac{\Delta x}{x} = \dfrac{0.2}{14.4} = 0.014$.

**Percentage uncertainty**

The **percentage uncertainty** is obtained by converting the fractional uncertainty into percentage form by multiplying by 100%.

The percentage uncertainty in a quantity $x$ is $\dfrac{\Delta x}{x} \times 100\%$.

Example: The percentage uncertainty of $x$, $\dfrac{\Delta x}{x} \times 100\% = \dfrac{0.2}{14.4} \times 100\% = 1.4\%$.

### 1.4.1 Significant Figures and Decimal Places

The general rules are:

1. **Express uncertainties to 1 s.f.**
   If the uncertainty in a measurement is estimated to be 0.025 for example, it should be rounded to 0.03.

2. **Express the quantity to the same place value as its uncertainty.**
   Using the same example, 0.03 has its most significant digit in the second decimal place. Hence the quantity should be written to exactly two decimal places:

   $12.10 \pm 0.03$

   and NOT $12.1 \pm 0.03$ or $12.102 \pm 0.03$.

Example 5

A student makes measurements from which he calculates the speed of sound to be 327.66 m s$^{-1}$. He estimates that the percentage uncertainty is 3%. Round off the speed to an appropriate number of significant figures. [Ans: 330 m s$^{-1}$]

### 1.4.2 Combining Uncertainties

There are established statistical rules for propagation of uncertainty from individual pieces of information. The A-level course only requires a simplified version of the statistical treatment.

#### (i) Adding or Subtracting Measured Quantities

When two measured quantities are added together or one subtracted from another, the actual uncertainty in the result is equal to the sum of the actual uncertainty of the two quantities.

We always add up the uncertainties even if the equation involves subtraction because we do not know the sign of the actual uncertainty and thus need to estimate the *largest* possible uncertainty in our quantities.

**Rule 1**: For $c = a + b$, the actual uncertainty in $c$, $\Delta c = \Delta a + \Delta b$
For $d = a - b$, the actual uncertainty in $d$, $\Delta d = \Delta a + \Delta b$

#### (ii) Multiplying or Dividing Measured Quantities

When two measured quantities are multiplied together or one divided by the other, the fractional uncertainty in the result is equal to the sum of the fractional uncertainties of the two quantities.

**Rule 2**: For $p = ab$, the fractional uncertainty in $p$, $\dfrac{\Delta p}{p} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}$

For $q = \dfrac{a}{b}$, the fractional uncertainty in $q$, $\dfrac{\Delta q}{q} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}$

Here, percentage uncertainty can be used interchangeably with fractional uncertainty as they differ only by a factor of 100 which is multiplied equally to every term in the equation. The percentage uncertainty in $p$ is equal to the sum of the percentage uncertainties in $a$ and $b$,

$$\frac{\Delta p}{p} \times 100\% = \frac{\Delta a}{a} \times 100\% + \frac{\Delta b}{b} \times 100\% .$$

Let us use a simple equation to illustrate the analysis of most mathematical or scientific equations we will encounter. Consider the linear equation $y = mx + c$, where $m$, $x$ and $c$ are all measurements with their associated uncertainties. Our experimental result, $y$, will also have an uncertainty that depends on that of $m$, $x$ and $c$.

The addition is simple: $\Delta y = \Delta(mx) + \Delta c$ but for the product $mx$, $\dfrac{\Delta(mx)}{(mx)} = \dfrac{\Delta m}{m} + \dfrac{\Delta x}{x}$. Rearranging,

$\Delta(mx) = \left(\dfrac{\Delta m}{m} + \dfrac{\Delta x}{x}\right)(mx)$ and hence $\Delta y = \left(\dfrac{\Delta m}{m} + \dfrac{\Delta x}{x}\right)(mx) + \Delta c$ .

Example 6

The measurements of the dimensions of a particular piece of rectangular cardboard are (18.5 ± 0.5) mm and (12.5 ± 0.5) mm. Determine the area of the cardboard with its associated uncertainty.
[Ans: $(2.3 \pm 0.2) \times 10^2$ mm$^2$]

**(iii) Scaling**

Actual uncertainty in a measured quantity is *scaled together* with the measured quantity.

| | |
|---|---|
| **Rule 3**: | If $r = \mathrm{k}a$, then $\Delta r = \mathrm{k}(\Delta a)$ where k is a constant |
| E.g. | if $r = \frac{1}{4}b$ then $\Delta r = \frac{1}{4}(\Delta b)$ |

Think of the above as a special case of Rule 1. For instance, $r = 3a = a + a + a$
For the case of dividing measurement by a numerical constant *k*, think of it as multiplying the inverse, 1/*k*.

Note: the fractional uncertainty $\frac{\Delta r}{r}$ is equal to $\frac{\Delta a}{a}$, as the constant *k* is cancelled out in the ratios.

Example 7

The radius of a circle is $r$ = (3.0 ± 0.2) cm. Find the circumference with its uncertainty. [(19 ± 1) cm]

### (iv) Powers

This applies to all exponents, $n$, both larger and smaller than 1, whether integer or fraction. However, if $n$ is negative, the error is still considered as positive.

**Rule 4**: If $s = a^n$, then $\frac{\Delta s}{s} = |n| \frac{\Delta a}{a}$

E.g. if $s = b^{-½}$, then $\frac{\Delta s}{s} = \frac{1}{2}\frac{\Delta b}{b}$

<u>Example 8</u>

Given a sphere of radius $r$ = (18.5 ± 0.5) mm, find the volume of the sphere with its associated uncertainty. [Ans: (2.7 ± 0.2) × $10^4$ $mm^3$]

<u>Example 9</u>

In an experiment, the external diameter $d_1$ and internal diameter $d_2$ of a hollow tube are found to be (64 ± 2) mm and (47 ± 1) mm respectively. Calculate the thickness of the tube and the associated uncertainty. What is the corresponding percentage uncertainty? [Ans: (9 ± 2) mm; 22 %]

Example 10

The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ . A student conducts an experiment to find the acceleration of free fall, *g*. He measures the length of the pendulum, $l = 0.23 \pm 0.01$ m, and the time for 20 oscillations, $t = 19.24 \pm 0.01$ s. Find *g* and its associated uncertainty. [Ans: (9.8 ± 0.4) m s$^{-2}$]

**(v) Special functions or complicated functions, X**

If measurements are put together with trigonometric functions, exponential functions or other complicated formulae, use simple numerical substitutions to evaluate the uncertainty directly. The following rule may be applied.

**Rule 5:**
Actual uncertainty of X = ½ (maximum possible X – minimum possible X)
$\Delta X = ½ (X_{max} – X_{min})$

Example 11

Consider S = $x \cos \theta$ for $x$ = (2.0 ± 0.2) cm, $\theta$ = (53 ± 2)°. Find S with its uncertainty.

**Summary of Rules** for propagation of uncertainties derived from measured values A ± ΔA and B ± ΔB

1. If $R = mA + nB$, then $\Delta R = |m|\Delta A + |n|\Delta B$
2. If $R = kA^m \times B^n$, where k is a numerical constant, then $\frac{\Delta R}{R} = |m|\frac{\Delta A}{A} + |n|\frac{\Delta B}{B}$
3. Other functions: e.g. $R = \sin A$, $R = \ln B$

Use the general approach: $\Delta R = \frac{1}{2}(R_{max} - R_{min})$

### 1.4.3 Precision and Accuracy

These are two terms which we need to understand in our approach to measurements.

***Precision*** is a measure of how well a result can be determined (without reference to a theoretical or true value). It is the degree of consistency and agreement among independent measurements of the same quantity.

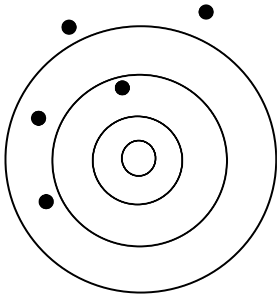
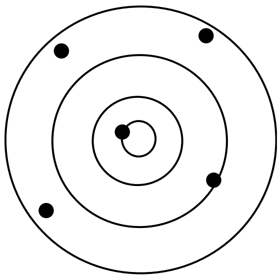
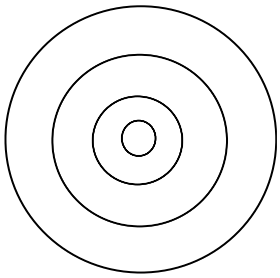
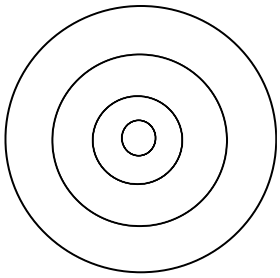
Measurements are said to be precise when *repeated* measurements remain very close to one another. If a certain measurement which is done, repeated several times, produces widely varying readings, the precision is poor.
For example, three measurements of my height: 1.75 m, 1.76 m, 1.75 m are precise but 1.72 m, 1.78 m, 1.76 m are not precise.

***Accuracy*** is the closeness of agreement between a measured value and a true or accepted value.

For example, if we are told that an object has a true mass 500 g, a measured value of 400 g is inaccurate. Similarly, the measurement of the acceleration of free fall, $g$ = 9.40 m s$^{-2}$ is also inaccurate, compared to $g$ = 9.70 m s$^{-2}$.

Example 12: Check your understanding

Complete the diagram below with appropriate ticks and crosses.

| Target | Precise | Accurate |
|---|---|---|
|  | | |
|  | | |
|  | ✓ | ✗ |
|  | ✓ | ✓ |

**Further reference:**

Rules for calculating uncertainties ~xmphysics0 (summary and examples)
https://www.youtube.com/watch?v=Ddfgq8bjb7E

Proof of product rule (beyond the syllabus) ~xmphysics0
https://www.youtube.com/watch?v=GP8r3E529kI

### 1.4.4 Systematic and Random Errors

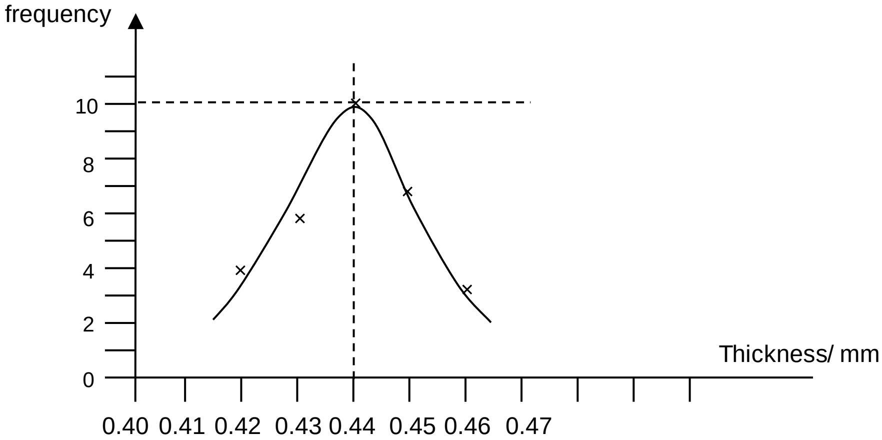
We have asserted that the true and exact value of a physical quantity can never be known. This is because our measurement of a physical quantity will always be limited by experimental errors. The causes of these experimental errors can be split into two broad categories – systematic errors and random errors.

#### 1.4.4.1 Random Error

Suppose we take many measurements to determine the diameter of a thin piece of wire using a micrometer screw gauge and tabulate our measurements in a table.

| Readings / mm | Frequency (no. of times of occurrence) |
|---|---|
| 0.42 | 4 |
| 0.43 | 6 |
| 0.44 | 10 |
| 0.45 | 7 |
| 0.46 | 3 |
| Total no. of readings | 30 |

The distribution of the measurements can be represented in the plot as shown.

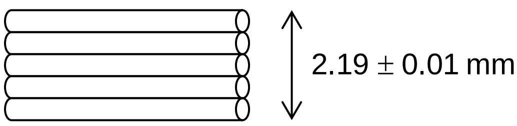



A *scattering* of readings about the **mean** value of the measurements suggests the presence of random errors (deviations in varying magnitude & direction). Random errors can occur despite repeating the experiments under the same conditions. Random errors are caused by environmental conditions, irregularity of quantity being measured, and limitations of the measurement equipment or the observer.

There are two ways to manage random errors.

**1. Improving the procedure to minimize uncertainty due to random errors.**

E.g., instead of measuring the diameter of only one thin wire, an improvement in the experimental procedure can be done by lining up a few thin wires and stick them together side-by-side using scotch tape. We will then measure the combined diameter instead.




Now if we use the same micrometer screw gauge, the actual uncertainty of our measurement is still the same as ± 0.01 mm. What about the diameter of one thin wire?

Since we measured the combined diameter of five thin wires, we can write $5d = (2.19 \pm 0.01)$ mm. Hence $d = \frac{1}{5}(2.19 \pm 0.01) = (0.438 \pm 0.002)$ mm. The actual uncertainty in our result has decreased tremendously and the diameter can now be expressed to one more significant figure than before.

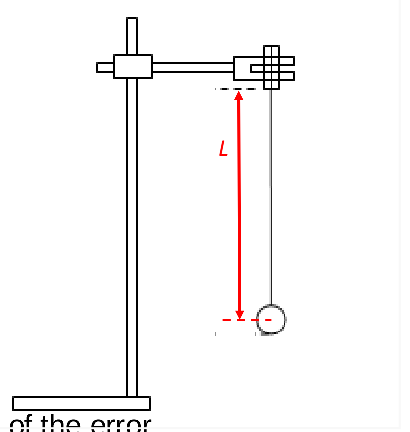
2. **A need for a large sample size in order for the mean value to be a good estimate of the true value**

When there are random errors present during measurement, due to the random nature of the errors, taking the mean of only a small number of repeated readings can result in a value that is still quite different from the true value. For the mean value to be a good estimate of the true value, a large number of repetitions is necessary. The more the better!

Example 13: Oscillations of a pendulum

The period of one oscillation of the pendulum $T$ is related to the length of the pendulum $L$ according to the equation $T = 2\pi\sqrt{\frac{L}{g}}$ , where g = 9.81 m s$^{-2}$. A diagram of the experiment is shown on the right. $T$ is measured for one oscillation with a stop watch and $L$ is measured with a metre ruler held in hand.

| *L*/cm | *T*/s (1st reading) | *T*/s (2nd reading) |
|---|---|---|
| 22.0 | 1.16 | 0.98 |
| 23.5 | 1.07 | 1.23 |
| 25.1 | 1.15 | 1.19 |
| 25.6 | 1.30 | 1.10 |
| 27.3 | 1.42 | 1.20 |
| 28.1 | 1.38 | 1.25 |




Using the given information,

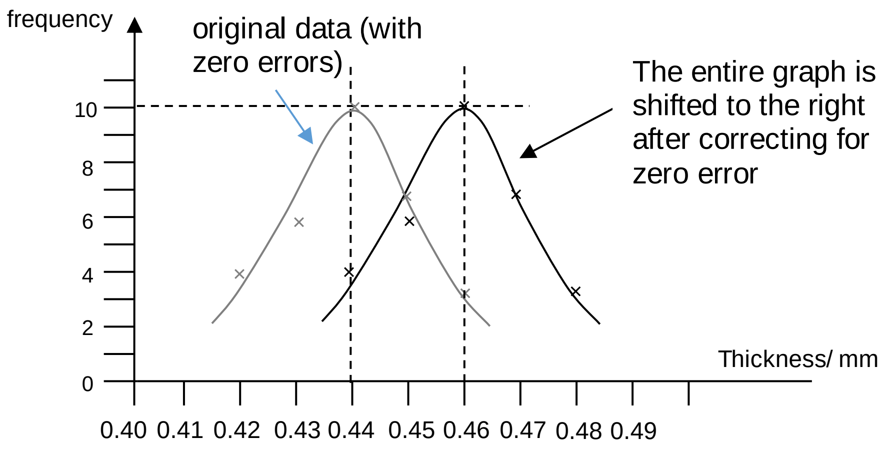
a) identify a random error in this experiment and explain the source of the error

b) suggest an appropriate method to reduce the random error you have identified

### 1.4.4.2 Systematic Error

Suppose now we discover that the micrometer screw gauge used has a *zero error* of - 0.02 mm. Every single measurement is off by 0.02 mm.

The distribution of the readings would look like this:




This shift of values from the real values is the result of a *systematic error*. Systematic errors are reproducible errors which cause a set of readings to deviate in a fixed direction from the true value. Systematic errors are caused by instrumental errors, environmental conditions, and poor experimental techniques. The source of systematic errors can be determined and eliminated by corrective action.

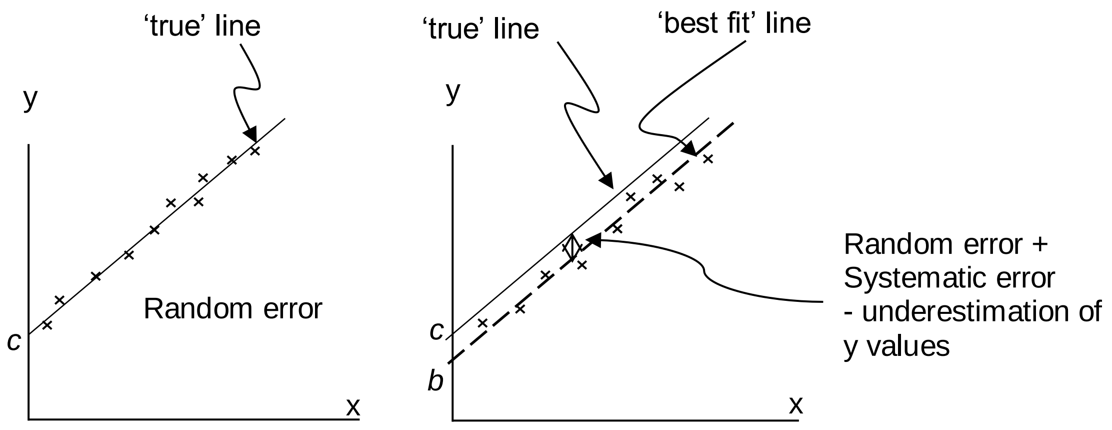
**Question: Distinguish random errors from systematic errors.**

**Answer: Random errors** are deviations of the measured value from the mean value, with varying signs and magnitudes. **Systematic errors** are deviations of the mean value from the true value, with same sign and similar magnitude.

Eliminating Systematic Error

In some cases, systematic error can be accounted for and corrected after it has been detected. The easiest way to detect systematic error is to note the vertical-intercept of the graph that is plotted. For example the graph of $y = mx + c$ when plotted gives a y-intercept of $b$ instead of $c$ as shown in the diagram below. The systematic error of $(c – b)$ results in an underestimation of the y- values.

The effects of random errors and of systematic errors appear in graphs as illustrated below.



Proper usage of measuring instruments is essential in eliminating systematic error. Measuring instruments need to be checked before usage. Always ensure balances, callipers, scale pointers, digital devices all read zero before being used. Calibration of instruments should be cross-checked if possible by comparing a measurement using two measuring devices.

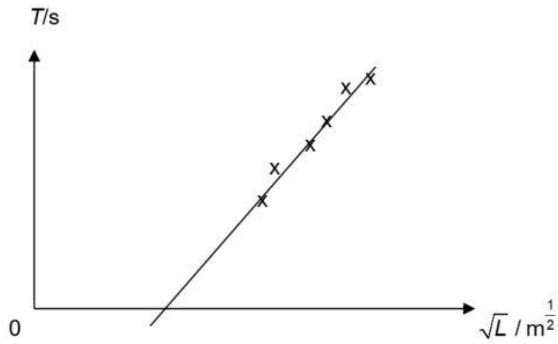
In many experiments, the most glaring error is due to the experimenter himself. Carelessness while taking measurements can result in severe systematic errors and random errors. However, blunders such as misreading a ruler or calculation mistakes should NOT be quoted as examples of errors in practical reports or exams!

**Examples of Systematic Errors**

| **Descriptions of errors** | **Error Sources** | **Corrections** |
|---|---|---|
| • zero errors on the scales of instruments<br>• poor calibration of instruments | Due to apparatus | • correct all measured readings by negating the error accordingly<br>• calibrate the instrument properly before experiment |
| • consistent parallax error which affects all the readings in the same way, for instance, taking readings off a scale from a fixed angle | Due to poor experimental technique | • adopt the correct way to take reading: ensure that the *line of sight* is *perpendicular* to the measuring scale |
| • *background radiation* causes the count rate of your radioactive sample to be consistently higher than the true reading | Due to external factors | Take the external factor(s) into account and adjust all readings appropriately. For instance,<br>• Measure the average background count rate and subtract it from the measured count rate. |

Example 14: Oscillation of a pendulum

Refer to the Example 13 for the same setup. The graph of $T$ against $\sqrt{L}$ is shown:




Using the given information,

(a) identify a systematic error in this experiment and explain the source of the error.

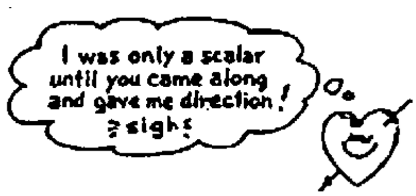
(b) suggest an appropriate method to eliminate the systematic error you have identified.

### 1.4.4.3 Summary Table

| **Type of error** | **Characteristics** | **If this error exists, will *precision* be affected?** | **If this error exists, will *accuracy* be affected?** |
|---|---|---|---|
| random | varying in both magnitude and direction about a mean value, can be reduced by taking average of repeated readings, but not eliminated. | Yes | No |
| systematic | varying in fixed direction about a true value, CANNOT be reduced by taking average of repeated readings, but may be eliminated by good experimental techniques. | No | Yes |

## 1.5 Scalars and Vectors

Physical quantities that have *no direction* associated with them are known as *scalars*. Scalars are specified completely by numerical values and units. Examples of scalars include distance, speed, mass, time, temperature, energy, gravitational and electric potentials.




Physical quantities that have both a magnitude and a *direction* are known as *vectors*. Examples of vectors include displacement, velocity, acceleration, force, gravitational and electric field strengths.

In books, a vector is often denoted in one of the following ways: ***F***, $\overrightarrow{F}$ or $\underset{\sim}{f}$ . The magnitude is indicated as |***F***| or simply *F*.

The direction of vectors should always be clearly presented. In written form, we can express, for example, velocity as 5 m $s^{-1}$ towards the east, or acceleration of free fall as 9.81 m $s^{-2}$ downwards. This is somewhat troublesome to write when we come to diagonal motions.

Diagrams are always helpful in Physics when dealing with vector quantities. Each vector is represented by an arrow. The arrow is always drawn pointing in the direction of the vector quantity and the length of the arrow is proportional to the magnitude of the vector quantity. If more than one vector is drawn on a diagram, the lengths of the different arrows should be representative of the relative magnitudes of the vector quantities.

For example, a boat's engine can propel the boat to move at 6.0 m $s^{-1}$ to the east and the current in the river is flowing at 3.0 m $s^{-1}$ also to the east. The two vectors can be drawn as shown.


Velocity of boat
= 6.0 m $s^{-1}$

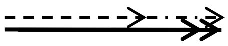

Velocity of current
= 3.0 m $s^{-1}$

Note the relative lengths of the arrows: the velocity of the boat is twice that of the current, hence its arrow is also twice as long.

Multiplication of a vector by a scalar simply scales the length of the arrow. Multiplication of a vector by –1 (or a negative sign, "–") reverses the direction in which the vector points.

### 1.5.1 Adding and Subtracting Coplanar Vectors

It is obvious that the boat moves down the river faster with the presence of a current than without. In addition the boat will in fact move slower if it is moving upstream or against the flow of current. Hence there must be an additive and subtractive effect when there are two of more vector quantities present in one situation.


Velocity of boat flowing
with current = 9.0 m $s^{-1}$


Velocity of boat flowing
against current = 3.0 m $s^{-1}$

The sum of two or more vectors is called the *resultant* vector. The resultant velocity vector of the boat is 9.0 m $s^{-1}$ when flowing with the current and 3.0 m $s^{-1}$ when flowing against the current.
As demonstrated in the diagram above, resultant vectors are always found by connecting one arrow-head to the tail of the next arrow. The resultant vector is then drawn from the tail of the very first arrow to the arrow-head of the last arrow. This summation process can be repeated step-by-step for any number of vectors.

### 1.5.1.1 Vectors in 1D

Vectors in 1D can be added or subtracted via two methods.

The first method is via vector diagrams as we have described earlier. However, because in 1D, the directions of the vectors are limited to either forward or backward, we can actually add or subtract them just like how we add or subtract scalar quantities with the help of a sign convention.

By adopting a sign convention where the direction along the flow of river is taken as positive,
the velocity of the river current is then $v_{current}$ = +3 m s$^{-1}$.
The velocity of the boat traveling along the river is $v_{boat}$ = +6 m s$^{-1}$.
Hence the velocity of the boat as seen by someone on the shore is given by
$v_{shore} = (+3) + (+6) = +9\ ms^{-1}$.

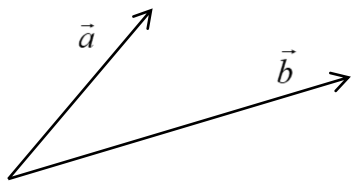
If the boat is traveling against the flow of river, using the same sign convention,
the velocity of the river current is then $v_{current}$ = +3 m s$^{-1}$.
the velocity of the boat traveling along the river is $v_{boat}$= - 6 m s$^{-1}$.
Hence the velocity of the boat as seen by someone on the shore is given by $v_{shore}$=(+3)+(-6) = -3 m s$^{-1}$.

Notice that we have managed to resolve the vector equation $\vec{v}_{shore} = \vec{v}_{boat} + \vec{v}_{current}$ without having to draw arrows.

### 1.5.1.2 Vectors in 2D

Two dimensional vectors lie on the same plane (coplanar) but point in different directions that are not along a single straight line.

In the diagram below, $\vec{a}$ and $\vec{b}$ are two coplanar vectors lying on the plane of the paper but not along a single straight line.




The tail of each arrow indicates the position where a vector quantity is acting upon. In the diagram above, if both $\vec{a}$ and $\vec{b}$ are forces, they would be acting on the same point, or the same object.

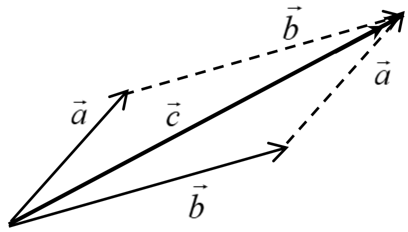
Imagine if you were pulled by two friends, one at each arm, in different directions. In which direction would you move? Can you possibly move in both directions you are pulled? There only exists one resultant force that will determine the direction you will move.

Similarly, the resultant vector $\vec{c}$ is obtained by adding vectors $\vec{a}$ and $\vec{b}$, or $\vec{c} = \vec{a} + \vec{b}$.

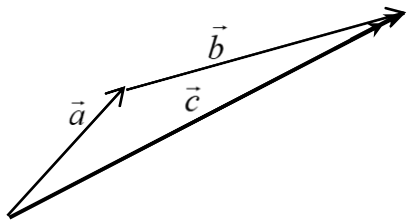
**Addition of vectors:** $\vec{c} = \vec{a} + \vec{b}$

Methods of Determining the Resultant Vector

a) Parallelogram Method: By putting the vectors to be added ‘tail to tail’. Complete the parallelogram. The resultant vector is the diagonal from the tail of the two vectors to the other vertex of the parallelogram.




b) Triangle Method: By joining the end of the next arrow onto the tip of the previous one to form a chain of arrows, the resultant vector is a straight arrow that goes from the tail of the chain directly to the head. The order of adding the vectors does not affect the resultant.

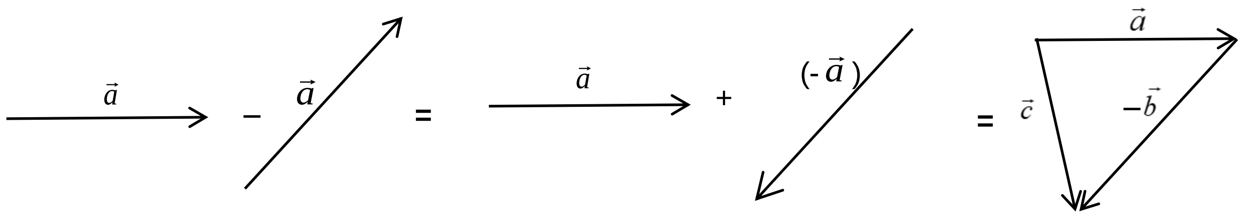



c) Component Method: The vectors are resolved into two perpendicular directions and then added. **This method is extremely important because it is the most convenient method to add three or more vectors.** (This method is described in section 1.5.2.)

**Subtraction of vectors:** $\vec{c} = \vec{a} - \vec{b}$

Subtraction of vectors can be evaluated by the same procedure as addition of two vectors since we can view subtraction of $\vec{b}$ from $\vec{a}$ as the summation of $-\vec{b}$ and $\vec{a}$.

$$\vec{c} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$




In Physics, vector subtraction can come about when we want to determine the ***change*** in a certain physical vector quantity. In general,

**A change in a physical quantity = final value – initial value**

$$\Delta\vec{x} = \vec{x}_{final} - \vec{x}_{initial}$$

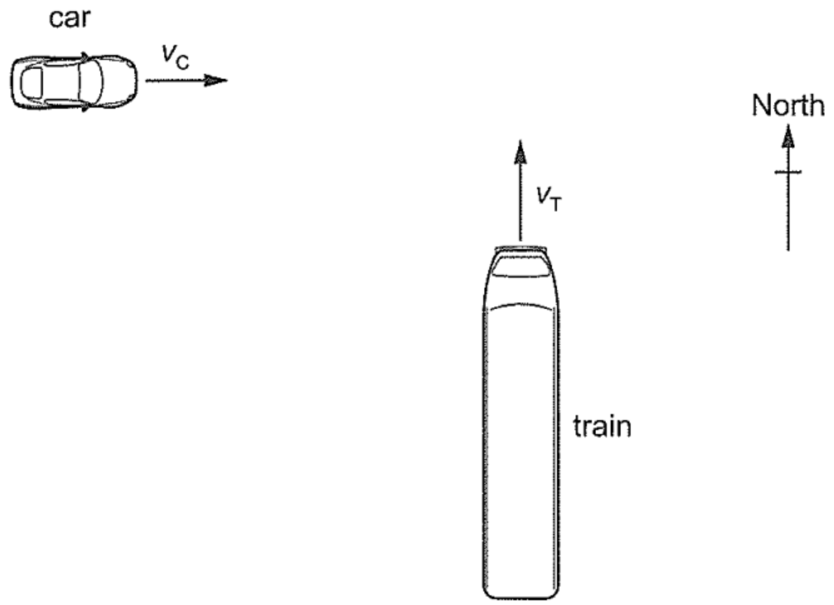
which is evaluated as: $\Delta\vec{x} = \vec{x}_{final} + (-\vec{x}_{initial})$

**Further reference:**

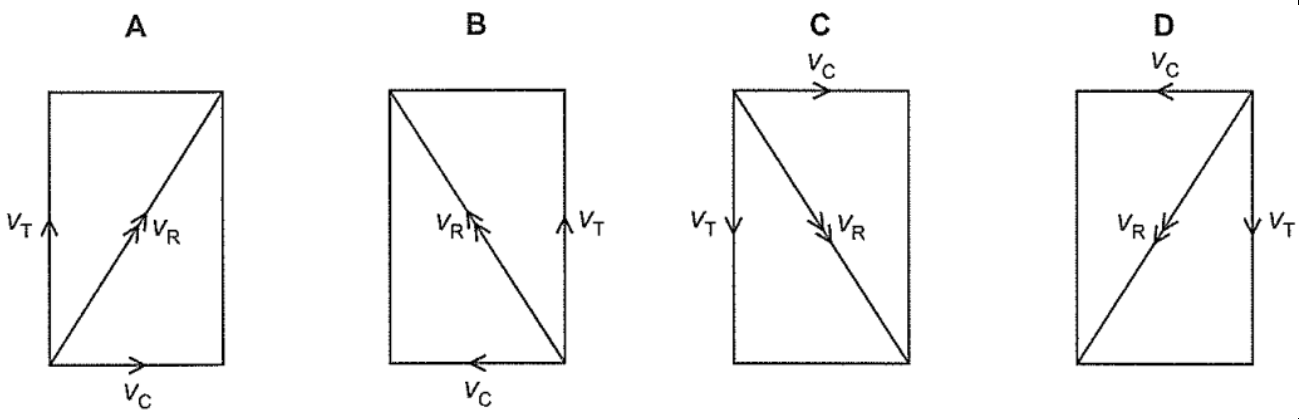
Vector addition and subtraction ~xmphysics0 (summary and examples)
https://youtu.be/FhoiORrgPFw

Example 15: H1/2016/P1/Q5

A passenger in a train travelling due north at speed $v_T$ sees a car travelling due east at speed $v_C$.

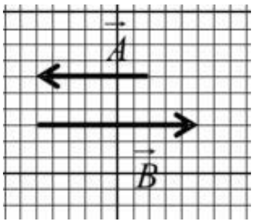


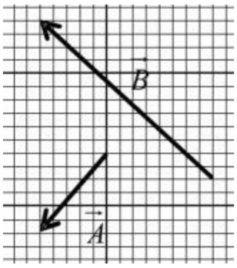

Which diagram shows the velocity $v_R$ of the car relative to the passenger in the train?



Example 16 (a)

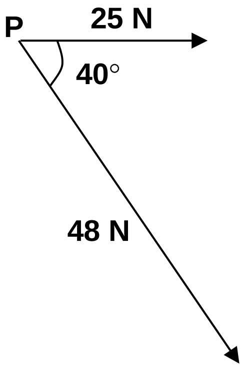
In the figure below, for each pair of vectors $\vec{A}$ and $\vec{B}$, draw the resultant vector $\vec{R}$ where $\vec{R} = \vec{A} + \vec{B}$.




Example 16 (b)

Two forces act at a point P as shown below. Determine (magnitude and direction of) the resultant, ***R***, of these two forces.

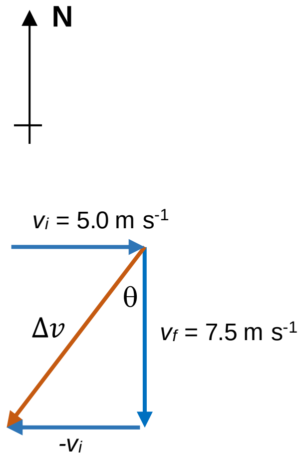


Example 16(c)

An object is moving at 5.0 m s$^{-1}$ due east. Its direction changes to due south with a speed of 7.5 m s$^{-1}$. Determine (i) the change in speed) and (ii) the change in velocity.

Suggested solution

(i) change in speed $= 7.5 - 5.0 = 2.5\ \mathrm{m\,s^{-1}}$




(ii) change in velocity

Component method

The x and y components of the initial and final velocities are
Initial velocity x-component: $v_{ix} = 5.0\ \mathrm{m\,s^{-1}}$
Initial velocity y-component: $v_{iy} = 0.0\ \mathrm{m\,s^{-1}}$

Final velocity x-component: $v_{fx} = 0.0\ \mathrm{m\,s^{-1}}$
Final velocity y-component: $v_{fy} = -7.5\ \mathrm{m\,s^{-1}}$

Change in velocity, $\Delta v = v_f - v_i$:
x-component: $\Delta v_x = v_{fx} - v_{ix} = 0.0 - 5.0 = -5.0\ \mathrm{m\,s^{-1}}$
y-component: $\Delta v_y = v_{fy} - v_{iy} = -7.5 - 0.0 = -7.5\ \mathrm{m\,s^{-1}}$

Magnitude of $\Delta v = \sqrt{\left(\Delta v_x^2 + \Delta v_y^2\right)} = \sqrt{\left((-5.0)^2 + (-7.5)^2\right)} = 9.0\ \mathrm{m\,s^{-1}}$

The angle as show in in the diagram, $\theta = \tan^{-1}\left(\frac{\Delta v_x}{\Delta v_y}\right) = \tan^{-1}\left(\frac{5.0}{7.5}\right) = 33.7^o$

Trignometry method

From the diagram, we can find magnitude using pythagoras theorem
$\Delta v = \sqrt{\left((5.0)^2 + (7.5)^2\right)} = 9.0\ \mathrm{m\,s^{-1}}$

The angle as shown in the diagram, $\theta = \tan^{-1}\left(\frac{5.0}{7.5}\right) = 33.7^o$

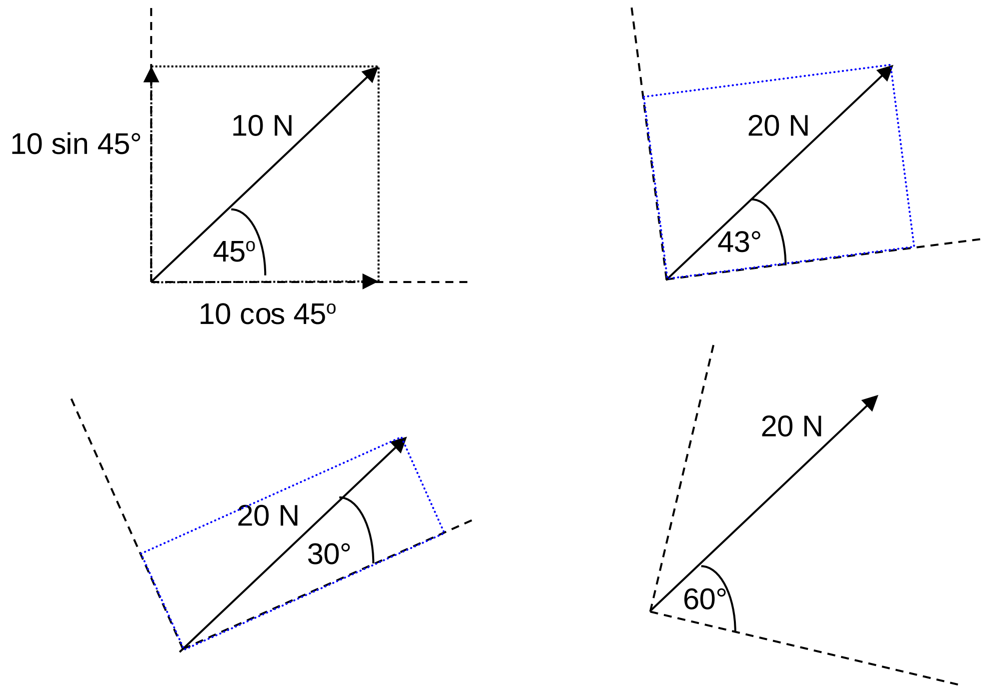
### 1.5.2 Resolution of Vectors

Since two vectors can be added to give a resultant vector, any vector can be broken up (or resolved) into two vectors or components. We will usually resolve a vector into two mutually-perpendicular components through the use of trigonometry and Pythagoras' theorem. Mutually-perpendicular vectors are independent of each other.

**Practice**

For each vector below, draw the two perpendicular components in the direction given by the dotted lines and state their magnitudes in terms of the respective values and angles.

The first example below has been done for you. Note that the dotted rectangle is an important working to account for the exact magnitudes of component vectors.



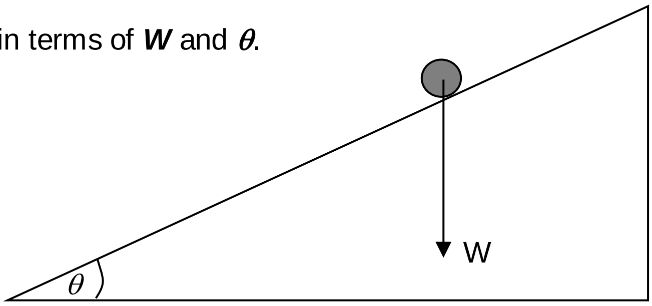

Each of the vectors above is resolved into two perpendicular components. A vector can be resolved into infinite pairs of perpendicular components. The choice of directions depends on the problem at hand.

Example 17

(a) An object rests on the plane of an inclined slope as shown. The weight ***W*** acts vertically down. Draw components of the weight

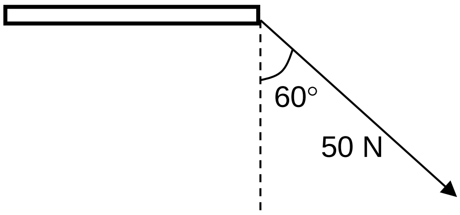
(i) parallel to the slope, $W_P$
(ii) perpendicular (normal), $W_N$, to the slope.

Label the magnitude of the two components in terms of ***W*** and ***θ***.



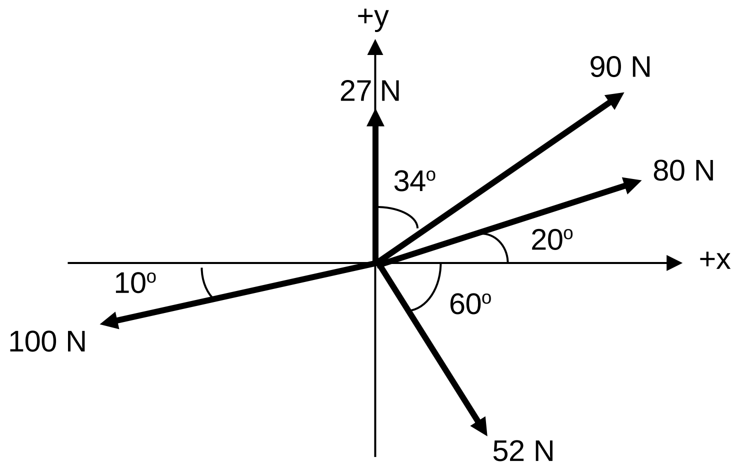

(b) A force of 50 N acts on a horizontal plank at angle of 60° to the vertical as shown. Draw components of this force (i) parallel to the plank, (ii) perpendicular to the plank.

Determine the magnitude of these components.



Example 18

Five forces shown act on an object. Find the resultant force.




- Resolve the vectors into two mutually perpendicular components:

| Vector/N | x-component /N (+ →) | y-component /N (+↑) |
|---|---|---|
| 80 | | |
| 90 | | |
| 27 | | |
| 100 | | |
| 52 | | |
| Resultant | | |

The magnitude of the resultant vector, $R =$

The direction of the resultant vector anti-clockwise from the positive x-direction,

$\theta =$

**Further reference:**

MIT OpenCourseWare Video – Vectors
https://youtu.be/mVQOmLTXLbQ

Further Reading and References

## Appendix I International System of Units (SI)

The current International System of Units, universally abbreviated SI (from the French *Le Système International d'Unités*), is the modern metric system of measurement. This collection of units consists of seven defining constants [f], seven base units (from the seven defining constants), derived units (combinations of these seven base units) and a set of decimal-based multipliers used as prefixes. While there are units not included in the SI, the units in the SI are actually sufficient for use for all physical quantities known to us.

| Base Quantity | SI Base Unit | | |
|---|---|---|---|
| | Name | Symbol | Definition |
| length | metre | m | The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second. |
| mass | kilogram | kg | The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.626 070 15 × $10^{-34}$ when expressed in the unit J s, which is equal to kg $m^2$ $s^{-1}$, where the metre and the second are defined in terms of speed of light and $\Delta v_{cs}$. |
| time | second | s | The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom. ($\Delta v_{cs}$ = 9 192 631 770 hertz) |
| electric current | ampere | A | The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be 1.602 176 634 x $10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta v_{cs}$. |
| thermodynamic temperature | kelvin | K | The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant *k* to be 1.380 649 x $10^{-23}$ when expressed in the unit J $K^{-1}$, which is equal to kg $m^2$ $s^{-2}$ $K^{-1}$, where the kilogram, meter and second are defined in terms of *h*, *c* and $\Delta v_{cs}$. |
| amount of substance | mole | mol | The mole is the amount of substance of a system which contains exactly 6.022 140 76 x $10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, $N_A$, when expressed in the unit $mol^{-1}$ and is called the Avogadro number. The amount of substance, symbol *n*, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles. |
| luminous intensity | candela | cd | The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x $10^{12}$ hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. |

Adapted from the National Institute of Standards and Technology (NIST) Reference on Constants, Units and Uncertainty.

[f] CGPM announced the new definition of SI units from 7 defining constants on 16 Nov 2018. Details of the definitions are found here: https://www.bipm.org/en/measurement-units/base-units.html

**Dimensions of physical quantities**

The dimension of a quantity denotes the physical nature of the quantity. It only makes sense to add and subtract two quantities from each other when they are of the same nature.

***In order for a mathematical equation that relates different physical quantities to be valid, the terms on both sides of an equation must have the same dimensions.***

***An equation where terms on both sides of the equation have the same dimensions is said to be homogeneous.***

***Dimensions of physical quantities can be treated as algebraic quantities. The dimensions of derived quantities can be determined using the same mathematical equations that relate the quantities.***

In the SI, we have identified seven base quantities, each of them is of a different dimension (there are seven dimensions). All physical quantities of the same dimension have the same SI base unit. E.g., the length of a football field, a person's height, the thickness of a piece of paper, these physical quantities all have the same dimension of length. They all have the same SI base unit of metre.

To every dimension, we can associate an SI base unit. In this way, when we check the homogeneity of an equation by comparing the units of every term in the equation in terms of SI base units, we are also checking the dimensions of each term. If all the terms of the equation have the same unit in terms of SI base units, then the terms will have the same dimensions as well and hence the equation will be homogeneous.

| **Base Quantity** | | **SI Base Unit** | | **Dimension** |
|---|---|---|---|---|
| Name | Symbol | Name | Symbol | Symbol |
| Mass | $m$ | kilogram | kg | M |
| Length | $L$ | metre | m | L |
| Time | $t$ | second | s | T |
| Electric current | $I$ | ampere | A | I |
| Thermodynamic temperature | $T$ | kelvin | K | Θ |
| Amount of substance | $n$ | mole | mol | N |
| Luminous intensity | $I_v$ | candela | cd | J |

## Appendix II  Summary of Key Quantity, Symbols and Units
(from GCE A-level syllabus)

# SUMMARY OF KEY QUANTITIES, SYMBOLS AND UNITS

The following list illustrates the symbols and units that will be used in question papers.

| *Quantity* | *Usual symbols* | *Usual unit* |
|---|---|---|
| *Base Quantities* | | |
| mass | $m$ | kg |
| length | $l$ | m |
| time | $t$ | s |
| electric current | $I$ | A |
| thermodynamic temperature | $T$ | K |
| amount of substance | $n$ | mol |
| *Other Quantities* | | |
| distance | $d$ | m |
| displacement | $s$, $x$ | m |
| area | $A$ | $m^2$ |
| volume | $V$, $v$ | $m^3$ |
| density | $\rho$ | kg $m^{-3}$ |
| speed | $u$, $v$, $w$, $c$ | m $s^{-1}$ |
| velocity | $u$, $v$, $w$, $c$ | m $s^{-1}$ |
| acceleration | $a$ | m $s^{-2}$ |
| acceleration of free fall | $g$ | m $s^{-2}$ |
| force | $F$ | N |
| weight | $W$ | N |
| momentum | $p$ | N s |
| work | $w$, $W$ | J |
| energy | $E$,$U$,$W$ | J |
| potential energy | $E_p$ | J |
| kinetic energy | $E_k$ | J |
| heating | $Q$ | J |
| change of internal energy | $\Delta U$ | J |
| power | $P$ | W |
| pressure | $p$ | Pa |
| torque | $T$ | N m |
| gravitational constant | $G$ | N $kg^{-2}$ $m^2$ |
| gravitational field strength | $g$ | N $kg^{-1}$ |
| gravitational potential | $\phi$ | J $kg^{-1}$ |
| angle | $\theta$ | °, rad |
| angular displacement | $\theta$ | °, rad |
| angular speed | $\omega$ | rad $s^{-1}$ |
| angular velocity | $\omega$ | rad $s^{-1}$ |
| period | $T$ | s |
| frequency | $f$ | Hz |
| angular frequency | $\omega$ | rad $s^{-1}$ |
| wavelength | $\lambda$ | m |
| speed of electromagnetic waves | $c$ | m $s^{-1}$ |
| electric charge | $Q$ | C |
| elementary charge | $e$ | C |
| electric potential | $V$ | V |
| electric potential difference | $V$ | V |
| electromotive force | $E$ | V |
| resistance | $R$ | Ω |
| resistivity | $\rho$ | Ω m |
| electric field strength | $E$ | N $C^{-1}$, V $m^{-1}$ |
| permittivity of free space | $\varepsilon_o$ | F $m^{-1}$ |
| magnetic flux | $\Phi$ | Wb |

| ***Quantity*** | ***Usual symbols*** | ***Usual unit*** |
|---|---|---|
| magnetic flux density | $B$ | T |
| permeability of free space | $\mu_o$ | H $m^{-1}$ |
| force constant | $k$ | N $m^{-1}$ |
| Celsius temperature | $\theta$ | °C |
| specific heat capacity | $c$ | J $K^{-1}$ $kg^{-1}$ |
| molar gas constant | $R$ | J $K^{-1}$ $mol^{-1}$ |
| Boltzmann constant | $k$ | J $K^{-1}$ |
| Avogadro constant | $N_A$ | $mol^{-1}$ |
| number | $N$, $n$, $m$ | |
| number density (number per unit volume) | $n$ | $m^{-3}$ |
| Planck constant | $h$ | J s |
| work function energy | $\Phi$ | J |
| activity of radioactive source | $A$ | Bq |
| decay constant | $\lambda$ | $s^{-1}$ |
| half-life | $t_{1/2}$ | s |
| relative atomic mass | $A_r$ | |
| relative molecular mass | $M_r$ | |
| atomic mass | $m_a$ | kg, u |
| electron mass | $m_e$ | kg, u |
| neutron mass | $m_n$ | kg, u |
| proton mass | $m_p$ | kg, u |
| molar mass | $M$ | kg |
| proton number | $Z$ | |
| nucleon number | $A$ | |
| neutron number | $N$ | |

## Appendix III SI Prefixes

| Prefix | Symbol | Decimal | Power of ten | Order of magnitude |
|---|---|---|---|---|
| quecto- | Q | 0.000000000000000000000000000001 | $10^{-30}$ | −30 |
| ronto- | r | 0.000000000000000000000000001 | $10^{-27}$ | −27 |
| yocto- | y | 0.000000000000000000000001 | $10^{-24}$ | −24 |
| zepto- | z | 0.000000000000000000001 | $10^{-21}$ | −21 |
| atto- | a | 0.000000000000000001 | $10^{-18}$ | −18 |
| femto- | f | 0.000000000000001 | $10^{-15}$ | −15 |
| pico- | p | 0.000000000001 | $10^{-12}$ | −12 |
| nano- | n | 0.000000001 | $10^{-9}$ | −9 |
| micro- | μ | 0.000001 | $10^{-6}$ | −6 |
| milli- | m | 0.001 | $10^{-3}$ | −3 |
| centi- | c | 0.01 | $10^{-2}$ | −2 |
| deci- | d | 0.1 | $10^{-1}$ | −1 |
| - | - | 1 | $10^{0}$ | 0 |
| deca- | da | 10 | $10^{1}$ | 1 |
| hecto- | h | 100 | $10^{2}$ | 2 |
| kilo- | k | 1,000 | $10^{3}$ | 3 |
| mega- | M | 1,000,000 | $10^{6}$ | 6 |
| giga- | G | 1,000,000,000 | $10^{9}$ | 9 |
| tera- | T | 1,000,000,000,000 | $10^{12}$ | 12 |
| peta- | P | 1,000,000,000,000,000 | $10^{15}$ | 15 |
| exa- | E | 1,000,000,000,000,000,000 | $10^{18}$ | 18 |
| zetta- | Z | 1,000,000,000,000,000,000,000 | $10^{21}$ | 21 |
| yotta- | Y | 1,000,000,000,000,000,000,000,000 | $10^{24}$ | 24 |
| ronna- | R | 1,000,000,000,000,000,000,000,000,000 | $10^{27}$ | 27 |
| quetta- | Q | 1,000,000,000,000,000,000,000,000,000,000 | $10^{30}$ | 30 |

For use in Information Technology, prefixes for binary multiples have been adopted by the International Electrotechnical Commission (IEC).

Please refer to website for more information: http://physics.nist.gov/cuu/Units/binary.html

It appears that everyone has been wrong about the kilobyte so you can read it for yourself.

# Tutorial 1 Measurement

## Self-Review Questions

*Use these questions to test your familiarity with the concepts for the topic. These questions should be sufficiently easy such that you can solve them on your own, with a little bit of thinking, without help from the tutors. The solutions to self-review questions are made available on Google Classroom for self-check. Thus your tutor may not go through these questions in class.*

S1. The density of water is 1.00 g $cm^{-3}$. Express this value in kilograms per cubic metres (kg $m^{-3}$).

S2. [N17/I/1] Which list of SI units contains only base units?

**A** kelvin, metre, mole, ampere, kilogram
**B** kilogram, metre, second, ohm, mole
**C** kilogram, newton, metre, ampere, ohm
**D** newton, kelvin, second, volt, mole

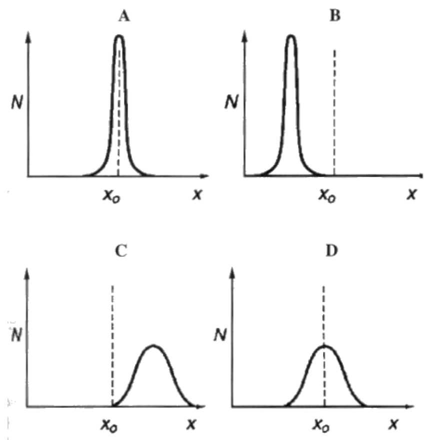
S3. The speed of a car cruising along PIE was 90.0 km $h^{-1}$. Express this value in metres per second.

S4. A light-year is a measure of length which is equal to the distance that light travels in 1 year. The distance from Earth to the star Proxima Centauri is $4.0 \times 10^{16}$ m. Express this distance in light-years. (speed of light, $c = 3.0 \times 10^{8}$ m $s^{-1}$)

S5. [modified N03/I/2] Errors in measurement may be either systematic or random.
Which one of the following involves random error?

**A** not allowing for zero error on a moving-coil voltmeter
**B** not subtracting background count rate when determining the count rate from a radioactive source
**C** stopping a stopwatch at the end of a sprint by a timekeeper
**D** using the value of $g$ as 10 N $kg^{-1}$ when calculating weight from mass

S6. [Adapted from N97/I/2] A quantity is measured many times and the number $N$ of measurements giving a value $x$ is plotted against $x$. The true value of the quantity is $x_o$. Fill in the table with ticks to describe the precision and accuracy of each graph.




| Graph | A | B | C | D |
|---|---|---|---|---|
| Precise | | | | |
| Not precise | | | | |
| Accurate | | | | |
| Not accurate | | | | |

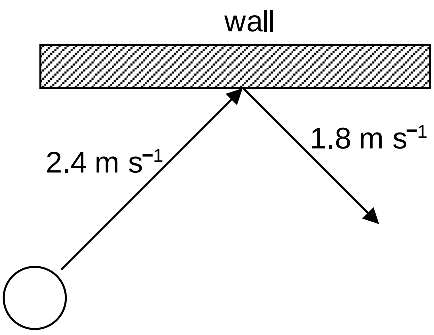
S7. [N20/II/1(b)] The drag force $F$ acting on a cyclist travelling at speed $v$ is given by $F = \frac{1}{2} C_D \rho A v^2$ where $C_D$ is the drag coefficient, $\rho$ is the density of air and $A$ is the frontal area of the cyclist.

Data for the moving cyclist in shown in the table.

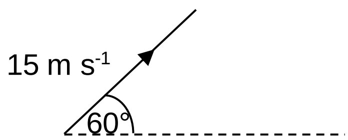
| **Quantity** | **Magnitude** | **Uncertainty** |
|---|---|---|
| $F$ / N | 22 | ± 2 |
| $C_D$ | 0.88 | ± 0.01 |
| $\rho$ / kg m$^{-3}$ | 1.2 | ± 0.1 |
| $A$ / m$^2$ | 0.32 | ± 0.02 |

Determine the speed of the cyclist, with its actual uncertainty. Give your answer to an appropriate number of significant figures.

S8. [2010 C1 LT1] Complete the vector diagram to show the vector representing the change in velocity of a billiard ball after rebounding from the edge of the table. Label the vector Δ$v$.




S9. A stone is thrown with a velocity of 15 m s$^{-1}$ at an angle of 60° to the horizontal as shown.




(a) (i) Explain why the diagram represents the velocity of the stone and not just its speed.

(ii) State whether the magnitude of the initial horizontal component of the velocity of the stone be greater, the same, or less than 15 m s$^{-1}$?

(b) Copy the diagram, and sketch the horizontal and vertical components of the velocity to correct proportion in magnitude.

(c) Calculate the magnitudes of

(i) the initial horizontal component of the velocity.

(ii) the initial vertical component of the velocity.

## Discussion Questions

*These questions are usually more challenging than the self-review questions and require more thinking and they are worth further discussions during tutorial sessions. You should attempt these questions with proper working on foolscap, to your best ability, and prepare to share your work as well as to learn from others their different approaches. Very often, in learning physics, the process of getting the answer is more important than the answer itself.*

Physical Quantities and Units

D1. [SAJC 2007 Prelim] Which of the following could be the correct expression for the velocity *v* of ocean waves in terms of ρ the density of seawater, *g* the acceleration of free fall, *h* the depth of the ocean and $\lambda$ the wavelength?

**A** $\sqrt{g\lambda}$ **B** $\sqrt{g/h}$ **C** $\sqrt{\rho g h}$ **D** $\sqrt{g/\rho}$

D2. [J97/II/29] The experimental measurement of the heat capacity $C$ of a solid as a function of temperature $T$ is to be fitted to the expression $C = \alpha T + \beta T^3$. (Recall heat capacity C is energy that is required to raise the temperature of the object by one unit of temperature, $C = \frac{Q}{\Delta T}$)
What are the possible units of $\alpha$ and $\beta$?

| | $\alpha$ | $\beta$ |
|---|---|---|
| **A** | J | J $K^{-2}$ |
| **B** | J $K^{2}$ | J |
| **C** | J K | J $K^{3}$ |
| **D** | J $K^{-2}$ | J $K^{-4}$ |
| **E** | J | J |

D3. [N99/I/1] Four physical quantities $P$, $Q$, $R$ and $S$ are related by the equation $P = Q - RS$.
Which statement must be correct for the equation to be homogeneous?

**A** $P$, $Q$, $R$ and $S$ all have the same units.
**B** $P$, $Q$, $R$ and $S$ are all scalar quantities.
**C** The product $RS$ has the same units as $P$ and $Q$.
**D** The product $RS$ is numerically equal to $(Q\text{-}P)$.

D4. [09 C1 BT1 Q2]
The power *P* generated by an ideal wind turbine is given by $P = \frac{1}{2} k\, d\, (v - b)^3$

where *k* is a characteristic of the turbine,
*d* is density of the fluid,
*v* is the velocity of the fluid and
*b is* a characteristic of the fluid

The possible units of *k* and *b* are:

| | **Units of *k*** | **Units of *b*** |
|---|---|---|
| **A** | no unit | $m^3$ |
| **B** | $m^2$ | $m^3$ |
| **C** | no unit | m $s^{-1}$ |
| **D** | $m^2$ | m $s^{-1}$ |

D5. [N00/II/1]
(a) The kilogram, the metre and the second are base units. Name two other base units.
(b) Explain why the unit of energy is said to be a *derived* unit.

(c) The density $\rho$ and the pressure P of a gas are related by the expression

$$c = \sqrt{\frac{\gamma P}{\rho}}$$ , where $c$ and $\gamma$ are constants.

(i) 1. Determine the base units of density $\rho$.
2. Show that the base units of pressure P are kg $m^{-1}$ $s^{-2}$.

(ii) Given that the constant $\gamma$ has no unit, determine the unit of $c$.

(iii) Using your answer to (ii), suggest what quantity may be represented by the symbol $c$.

D6. Which one of the following physical quantities, when given in SI unit, is likely to be of the same order of magnitude as the mass of a typical watermelon in SI unit?

**A** Power output of a domestic electric kettle

**B** Human reaction time

**C** Weight of a typical one year old baby

**D** Height of the overhead bridge outside HCI (College) from the road surface

D7. [N21/I/1] What is the best estimate of the weight of a smartphone?

**A** 1.5 cN **B** 1.5 dN **C** 150 cN **D** 150 dN

Errors and Uncertainties

D8. [N12/I/1] A student uses an analogue voltmeter to measure the potential difference across a lamp. The voltmeter is marked every 0.02 V and has a zero error of 0.08 V. The student is not aware of this zero error and writes down a reading of 2.16 V.

Is the reading accurate and is it precise?

| | Accurate | Precise |
|---|---|---|
| **A** | no | no |
| **B** | no | yes |
| **C** | yes | no |
| **D** | yes | yes |

D9. [N02/I/2] An object of mass 1.000 kg is placed on four different balances. For each balance, the reading is taken five times. The table shows the values obtained together with the means. Which balance has the smallest systematic error but is not very precise?

| | balance reading / kg | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | mean / kg |
| **A** | 1.000 | 1.000 | 1.002 | 1.001 | 1.002 | 1.001 |
| **B** | 1.011 | 0.999 | 1.001 | 0.989 | 0.995 | 0.999 |
| **C** | 1.012 | 1.013 | 1.012 | 1.014 | 1.014 | 1.013 |
| **D** | 0.993 | 0.987 | 1.002 | 1.000 | 0.983 | 0.993 |

D10. [N19/I/2] The manufacturers of a digital voltmeter give, as its specification,

'accuracy ± 1% with an additional uncertainty of ± 10 mV'

The meter reads 4.072 V.

How should this reading be recorded, together with its uncertainty?

**A** (4.07 ± 0.01) V **B** (4.07 ± 0.04) V

**C** (4.072 ± 0.052) V **D** (4.07 ± 0.05) V

D11. [HC Prelim03/P1/2] Given that the quantities $L$, $x$ and $y$ are related by the equation $Lx = y^2$, what is the percentage uncertainty in $L$ if the percentage uncertainties in $x$ and $y$ are 1% and 3% respectively?

**A** 2 % **B** 4 % **C** 5 % **D** 7 %

D12. [N22/II/1(a) modified] A mass $m$ is suspended from a vertical spring attached to a fixed support. The mass is pulled down and then released. Ten oscillations are timed using a stop-watch.

The data for the mass and the time, together with the uncertainties, are shown in the table below.

| | |
|---|---|
| **time for 10 oscillations / s** | 6.2 ± 0.2 |
| ***m* / g** | 150 ± 1% |

The period $T$ of the oscillations of the mass is given by: $T = 2\pi\sqrt{\dfrac{m}{k}}$ , where $T$ is in s, $m$ in kg and $k$ is the spring constant in N m$^{-1}$.

Determine the value of $k$ together with its actual uncertainty. Give your answer to an appropriate number of significant figures.

D13. [N10/I/3 modified] A wire of uniform circular cross-section has diameter $d$ and length $L$. A potential difference $V$ between the ends of the wire gives rise to a current $I$ in the wire.

The resistivity $\rho$ of the material of the wire is given by the expression

$$\rho = \frac{\pi d^2 V}{4LI}$$

In one particular experiment, the following measurements are made.

$d$ = (1.20 ± 0.01) cm
$I$ = (1.50 ± 0.05) A
$L$ = (100 ± 1) cm
$V$ = (5.0 ± 0.1) V

(a) Explain which of the four measurements gives rise to the least uncertainty in the value for the resistivity.

(b) Determine the value of the resistivity together with its actual uncertainty. Give your answer to an appropriate number of significant figures.

D14. [N12/I/2] The equation connecting object distance *u*, image distance *v* and focal length *f* for a lens is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

A student measures values of *u* and *v*, with their associated uncertainties. These are

$u$ = 50 mm ± 3 mm
$v$ = 200 mm ± 5 mm

He calculates the value of *f* as 40 mm. What is the uncertainty in this value?

**A** ± 2.1 mm **B** ± 3.4 mm **C** ± 4.5 mm **D** ± 6.8 mm

Scalars and Vectors

D15. [N10/I/2] A boat changes its velocity from 8 m $s^{-1}$ due north to 6 m $s^{-1}$ due east. What is its change in velocity?

**A** 2 m $s^{-1}$ at a direction of 37° east of north
**B** 2 m $s^{-1}$ at a direction of 53° east of north
**C** 10 m $s^{-1}$ at a direction of 37° east of south
**D** 10 m $s^{-1}$ at a direction of 53° west of south

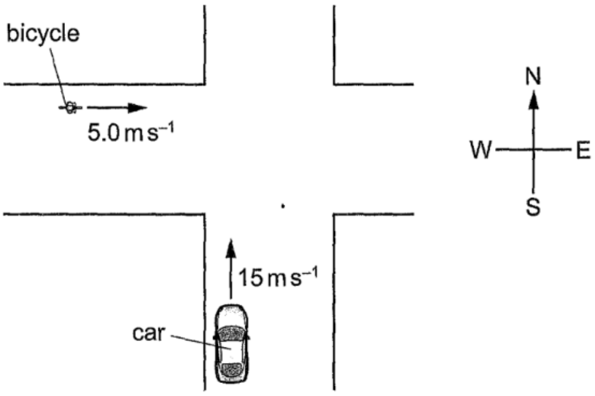
D16. [J89/II/8c] A car changes its velocity from 30 m $s^{-1}$ due East to 25 m $s^{-1}$ due South.
(i) Draw a vector diagram to show the initial and final velocities and the change in velocity.
(ii) Calculate the change in speed.
(iii) Calculate the change in velocity.

D17. [N16/I/3] An aircraft flies with an airspeed of 700 km/h through a 250 km/h jet-stream wind from the west. The pilot wishes to fly directly north from Australia towards Changi airport in Singapore. To achieve this, the pilot points the aircraft away from the north direction.

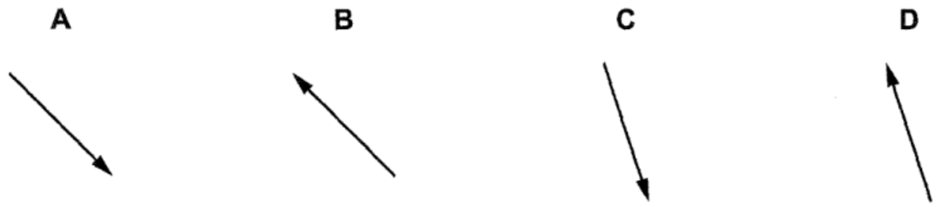
What is the speed of the aircraft in the direction of north relative to the ground?

**A** 450 km/h **B** 650 km/h **C** 740 km/h **D** 950 km/h

D18. [N20/I/5] A car and a bicycle are equal distances from a crossroads. The car is travelling north with a speed of 15 m $s^{-1}$. The bicycle is travelling east with a speed of 5.0 m $s^{-1}$.




At this instant, which arrow represents the velocity of the bicycle relative to the car?




**Numerical answers**

S1. $1.00 \times 10^3$ kg $m^{-3}$
S3. 25.0 m $s^{-1}$
S4. 4.2 light years
S7. 11 ± 1 m $s^{-1}$
S9. (c)(i) 7.5 m $s^{-1}$; (c)(ii) 13 m $s^{-1}$
D12. 15 ± 1 N $m^{-1}$
D13. (b) $(38 \pm 3) \times 10^{-3}$ Ω cm
D16. (ii). – 5 m $s^{-1}$; (iii) 39 m $s^{-1}$ 140° clockwise with respect to the initial velocity