Check your Understanding (Probability)

Section 1: Venn Diagram

1. Kesamet Junior College celebrated Staff Fruits Day, where 150 teachers received a free fruit each. The gender of the teachers and types of fruits each teacher received were recorded in the table shown below.

	Male	Female
Apple	12	28
Banana	55	32
Orange	8	15

One of the teachers is selected at random.

M is the event that the teacher selected is a male.
A is the event that the teacher selected received an apple.
B is the event that the teacher selected received a banana.
C is the event that the teacher selected received an orange.

(i) Find the following probabilities.

(a) $\mathrm{P}(M \cup A)$, [2]

(b) $\mathrm{P}(M' \cap B')$. [2]

(c) $\mathrm{P}(M \mid C)$, [2]

(ii) Determine whether the events M and C are independent, justifying your answer. [2]

(ia)	$\mathrm{P}(M \cup A) = \mathrm{P}(M) + \mathrm{P}(A) - \mathrm{P}(M \cap A) = \frac{1}{2} + \frac{4}{15} - \frac{2}{25} = \frac{103}{150}$ Or $\mathrm{P}(M \cup A) = \frac{12+55+8+28}{150} = \frac{103}{150}$

(ib)	$\mathrm{P}(M' \cap B') = \dfrac{28+15}{150} = \dfrac{43}{150}$
(ic)	$\mathrm{P}(M \mid C) = \dfrac{\mathrm{P}(M \cap C)}{\mathrm{P}(C)} = \dfrac{8/150}{23/150} = \dfrac{8}{23}$ or $\mathrm{P}(M \mid C) = \dfrac{\mathrm{n}(M \cap C)}{\mathrm{n}(C)} = \dfrac{8}{23}$
(ii)	$\mathrm{P}(M \mid C) = \dfrac{8}{23}$ but $\mathrm{P}(M) = \dfrac{1}{2}$ $\Rightarrow$ i.e. $\mathrm{P}(M \mid C) \neq \mathrm{P}(M)$ $\therefore$ M and C not independent

2. In the year 2016, there were 620 male students and 520 female students in the JC1 cohort of Hwa Chong Institution (HCI). The number of these students studying H2 Mathematics, H2 Chemistry and H2 Physics are listed in the table below.

	H2 Mathematics	**H2 Chemistry**	**H2 Physics**
Male	610	480	306
Female	512	440	200
Total	1122	920	506

One of the students is chosen at random. Events M, C and F are defined as follows:

M : The student chosen is studying H2 Mathematics.

C : The student chosen is studying H2 Chemistry.

F : The student chosen is a female.

Find

(i) $\mathrm{P}\left(M \cap F'\right)$, [1]

(ii) $\mathrm{P}\left(M \mid F'\right)$, [2]

(iii) $\mathrm{P}\left(C \cup F\right)$ [2]

(iv) Give a reason in each case, state whether the following statements is **True** or **False**.

(a) The events M and F are independent. [2]

(b) The events M and C are not mutually exclusive. [1]

400 students were enrolled into HCI via the Joint Admission Exercise (JAE) in 2016.
It was given that 25% of female students and 40% of male students who study H2 Mathematics were students who enrolled via the JAE.

Three students who were enrolled via the JAE were chosen at random.

(v) Find the probability all three of them are female students who study H2 Mathematics. [2]

(vi) Find the probability that none of them study H2 Mathematics. [2]

2 (i)	Total number of students $=620+520=1140$ $\mathrm{P}\left(M\cap F^{'}\right)=\frac{610}{1140}=\frac{61}{114}$ or 0.535
(ii)	$\mathrm{P}\left(M \mid F^{'}\right)=\frac{\mathrm{P}\left(M\cap F^{'}\right)}{\mathrm{P}\left(F^{'}\right)}$ $=\frac{\frac{610}{1140}}{\frac{620}{1140}}=\frac{610}{620}=\frac{61}{62}$ or 0.984
(iii)	$\mathrm{P}(C\cup F)=\mathrm{P}(C)+\mathrm{P}(F)-\mathrm{P}(C\cap F)$ $=\frac{920}{1140}+\frac{520}{1140}-\frac{440}{1140}$ $=\frac{1000}{1140}$ $=\frac{50}{57}$ or 0.877
(iv) (a)	False. $\mathrm{P}\left(M \mid F^{'}\right)=\frac{61}{62}\neq\frac{1122}{1140}=\mathrm{P}(M)$ M and $F^{'}$ are not independent and hence M and F are not independent OR $\mathrm{P}(M)\times\mathrm{P}\left(F^{'}\right)=\frac{1122}{1140}\times\frac{620}{1140}=\frac{5797}{10830}$ $\mathrm{P}\left(M\cap F^{'}\right)=\frac{61}{114}$ Since $\mathrm{P}(M)\times\mathrm{P}\left(F^{'}\right)\neq\mathrm{P}\left(M\cap F^{'}\right)$ M and F are not independent.

(iv)(b)	True. $\mathrm{P}(M \cap C) \neq 0$ as $1122 + 920 > 1140$ Therefore, events *M* and *C* are not mutually exclusive.
(v)	Number of female JAE students who study Mathematics. $= 0.25(512) = 128$ Required probability = $= \frac{128}{400} \times \frac{127}{399} \times \frac{126}{398} = 0.0322$ OR Required probability $= \frac{\binom{128}{3}}{\binom{400}{3}} = 0.0322$
(vi)	Number of JAE male students who study Mathematics $= (0.40)(610) = 244$ JAE students who do not study Mathematics $= 400 - 128 - 244 = 28$ Required probability = $= \frac{28}{400} \times \frac{27}{399} \times \frac{26}{398} = 0.000309$ OR Required probability $= \frac{\binom{28}{3}}{\binom{400}{3}} = 0.000309$

3. In a certain junior college, each student own exactly one mobile phone: either a Xiphone or a Samsong phone. The number of male and female students who own either a Xiphone or a Samsong phone is shown in the table below.

	Xiphone	Samsong
Male	227	318
Female	263	212

A student from this junior college is chosen at random. Let *F* be the event that the student is a female and *X* be the event that the student owns a Xiphone.

(i) Find

(a) P(F), [2]

(b) $\mathrm{P}(F \cap X)$, [1]

(c) $\mathrm{P}(F \cup X')$. [1]

(ii) Determine whether the events F and X independent, justifying your answer. [2]

(iii) Given that the student selected owns a Samsong, find the probability that the student is a male. [2]

A telco company selects two students from the junior college for a survey. If each student has the same chance of being selected, find the probability that one of the students selected has a Xiphone and the other has a Samsong. [3]

3(ia)	No of females = 263 + 212 Total no. of people = 227 + 263 + 318 + 212 = 1020 $P(F)=\dfrac{475}{1020}\approx 0.46568=0.466$
(ib)	$P(F\cap X)=\dfrac{263}{1020}\approx 0.25784=0.258$
(ic)	$P(F\cup X')=\dfrac{475+318}{1020}=\dfrac{739}{1020}$
	$P(F)\times P(X)=\dfrac{475}{1020}\times\dfrac{490}{1020}\approx 0.223712=0.224$ From (ii) $P(F\cap X)=0.258\neq 0.224$ Hence $P(F\cap X)\neq P(F)\times P(X)$ Hence F and X are **not** independent
	$P(F' \mid X')=\dfrac{P(F'\cap X')}{P(X')}=\dfrac{318}{530}=0.6$
	Prob (one Xiphone and one Samsong) $=2\times\dfrac{490}{1020}\times\dfrac{530}{1019}$ =0.4997=0.500 Alternative: $\dfrac{{}^{490}C_1\,{}^{530}C_1}{{}^{1020}C_1}$

4. For events A and B, it is given that $\mathrm{P}(A'\cap B)=0.2$ and $\mathrm{P}(A\mid B)=0.4$.

(a) Find $\mathrm{P}(A\cap B)$. [3]

It is further given that $\mathrm{P}(A'\cap B')=0.3$.

(b) Determine whether A and B are independent. [4]

4(a)	Let $P(A\cap B)=x$ $\dfrac{x}{x+0.2}=0.4$ $x=0.4x+0.08$ $x=\dfrac{2}{15}$
(b)	$P(A)=1-0.2-0.3=0.5$ $P(B)=0.2+\dfrac{2}{15}=\dfrac{1}{3}$ $P(A)P(B)=(0.5)\left(\dfrac{1}{3}\right)=\dfrac{1}{6}$ $P(A\cap B)=\dfrac{2}{15}$ Since $P(A)P(B)\neq P(A\cap B)$, A and B are not independent.

5. **(i)** A and B are two events such that $P(A'\cap B)=0.33$ and $P(A'\cap B')=0.15$. Show that $P(A)=0.52$. [1]

(ii) Given further that $P(B)=0.45$, find $P(A\cap B)$ and $P(A\cap B')$. [3]

(iii) There is another event C such that $P(C)=0.25$ and $P(A\cup C)=0.7$. Determine whether events A and C are mutually exclusive. [2]

5(i)	$P(A)=1-P(A'\cap B)-P(A'\cap B')$ $=1-0.33-0.15$ $=0.52$
(ii)	$P(A\cap B)=P(B)-P(A'\cap B)$ $=0.45-0.33$ $=0.12$ $P(A\cap B')=P(A)-P(A\cap B)$ $=0.52-0.12$ $=0.4$

	<u>Alternative</u> $\mathrm{P}\left(A\cap B'\right)=1-\mathrm{P}\left(B\right)-\mathrm{P}\left(A'\cap B'\right)$ $=1-0.45-0.15$ $=0.4$ $\mathrm{P}\left(A\cap B\right)=\mathrm{P}\left(A\right)-\mathrm{P}\left(A\cap B'\right)$ $=0.52-0.4$ $=0.12$
(iii)	$\mathrm{P}\left(A\cap C\right)=\mathrm{P}\left(A\right)+\mathrm{P}\left(C\right)-\mathrm{P}\left(A\cup C\right)$ $=0.52+0.25-0.7$ $=0.07$ Since $\mathrm{P}\left(A\cap C\right)=0.07\neq 0$, A and C are not mutually exclusive events.

6. For events A and B, it is given that $\mathrm{P}(A)=0.7$, $\mathrm{P}(B)=0.5$ and $\mathrm{P}(A|B)=0.8$.

(i) State, with a reason, whether A and B are mutually exclusive. [2]

For a third event C, it is given that $\mathrm{P}(A'\cap B'\cap C)=q$.

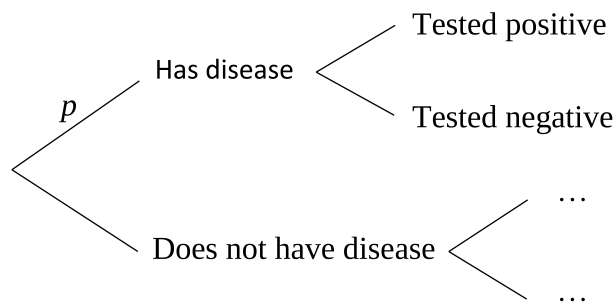
(ii) When $q=0.1$, find $\mathrm{P}(A'\cap B'\cap C')$. [2]

(iii) State the maximum value of q. [1]

6(i)	$\mathrm{P}(A\cap B)=\mathrm{P}(A\|B)\times\mathrm{P}(B)=0.4\neq 0$ Hence A and B are not mutually exclusive.
6(ii)	$\mathrm{P}(A\cup B)$ $=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=0.7+0.5-0.4$ $=0.8$ $\mathrm{P}(A'\cap B'\cap C')$ $=1-\mathrm{P}(A\cup B\cup C)$ $=1-[\mathrm{P}(A\cup B)+\mathrm{P}(A'\cap B'\cap C)]$ $=1-[0.8+0.1]$ $=0.1$
6(iii)	Max q = 0.2

Section 2: Tree Diagram

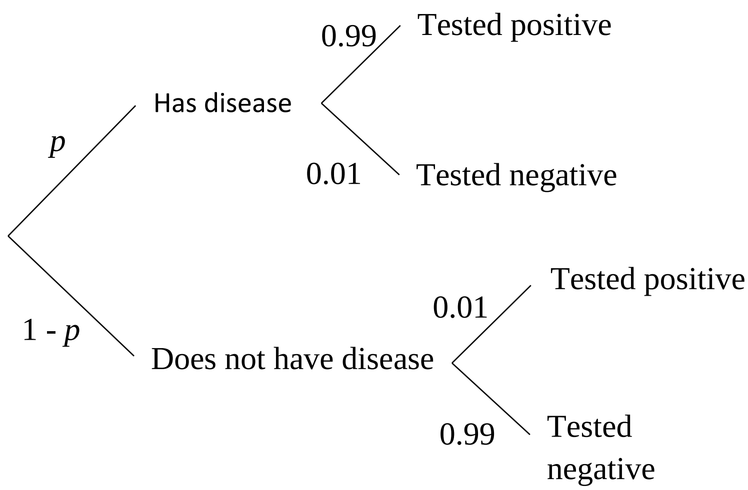
1. The probability that a laboratory mouse has a particular disease is *p*. A test for the disease has a probability of 0.99 of giving a positive result when the mouse has the disease, and a probability of 0.99 of giving a negative result when the mouse does not have the disease.



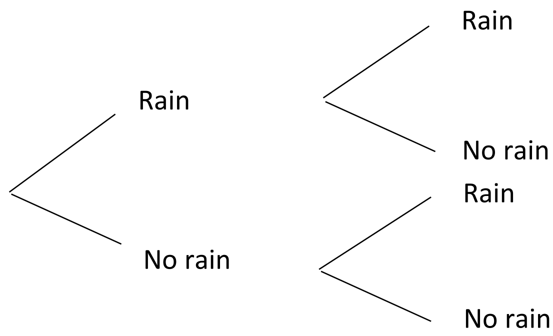
(i) Copy the above tree diagram and complete it using the information provided. [2]

(ii) Show that the probability of a mouse being tested positive is $0.98p + 0.01$. Given that any randomly chosen mouse has no more than 0.75 probability of being tested positive, find the range of values *p* can take. [2]

(iii) There is a 9.75% chance of a mouse not having the disease given that it is tested positive. Find the value of *p*. [3]

1(i)	
(ii)	P(mouse being tested positive) $= p(0.99) + (1-p)(0.01)$ $= 0.98p + 0.01$ (shown) Given: $0.98p + 0.01 \leq 0.75$ $p \leq \frac{0.75 - 0.01}{0.98}$ $\therefore 0 < p \leq 0.755$ (or $p \leq 0.755$)
(iii)	P(does not have disease | tested positive) $= \frac{\text{P(does not have disease and tested positive)}}{\text{P(tested positive)}}$ $= \frac{(1-p)(0.01)}{0.98p + 0.01}$ $\frac{(1-p)(0.01)}{0.98p + 0.01} = 0.0975$ $0.01 - 0.01p = 0.0975(0.98p + 0.01)$ $0.10555p = 0.009025$ $p = 0.0855$ (3 s.f.)

2. The probability of rain on a particular Monday is 0.5. For any subsequent day, the probability of having the same weather as the previous day is 0.2. The information can be represented on the tree diagram:



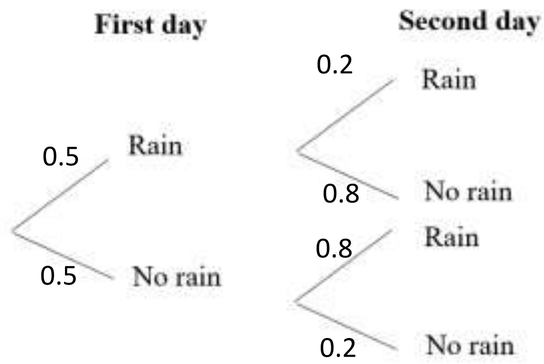
(i) Copy and complete the tree diagram above. [2]

The weather is observed each day from Monday to Wednesday. Find the probability that

(ii) all three days are of the same weather, [2]

(iii) no two consecutive days are of the same weather, [2]

(iv) two consecutive days are of the same weather given that exactly two days are of the same weather. [3]

2(i)	
(ii)	Required probability $= \mathrm{P}(RRR) + \mathrm{P}(NNN)$ $= \left(\frac{1}{2}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$ $= \frac{1}{25}$
(iii)	Required probability $= \mathrm{P}(RNR) + \mathrm{P}(NRN)$ $= \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)$ $= \frac{16}{25}$

(iv)	Required probability $=\dfrac{\mathrm{P}(RRN)+\mathrm{P}(NNR)+\mathrm{P}(NRR)+\mathrm{P}(RNN)}{\mathrm{P}(\text{two of the same weather})}$ $=\dfrac{4\left(\frac{1}{2}\right)\ \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)}{1-\frac{1}{25}}$ $=\dfrac{1}{3}$ **<u>Alternative</u>** Required probability $=\dfrac{\mathrm{P}(RRN)+\mathrm{P}(NNR)+\mathrm{P}(NRR)+\mathrm{P}(RNN)}{\mathrm{P}(\text{two of the same weather})}$ $=\dfrac{4\left(\frac{1}{2}\right)\ \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)}{4\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)+2\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)}$ $=\dfrac{1}{3}$ **<u>Alternative</u>** Required probability $=\dfrac{\mathrm{P}(RRN)+\mathrm{P}(NNR)+\mathrm{P}(NRR)+\mathrm{P}(RNN)}{\mathrm{P}(\text{two of the same weather})}$ $=\dfrac{1-\frac{1}{25}-\frac{16}{25}}{1-\frac{1}{25}}$ $=\dfrac{1}{3}$

3. The school management committee is looking into the records of students who were late for school during the first week of the term. If a student is late on a particular day, then the probability that he is late the next day is $\frac{1}{3}$. If a student is not late on a particular day, then the probability that he is late the next day is $\frac{1}{4}$. It is known that $\frac{3}{10}$ of all students are late on the first day of the term.

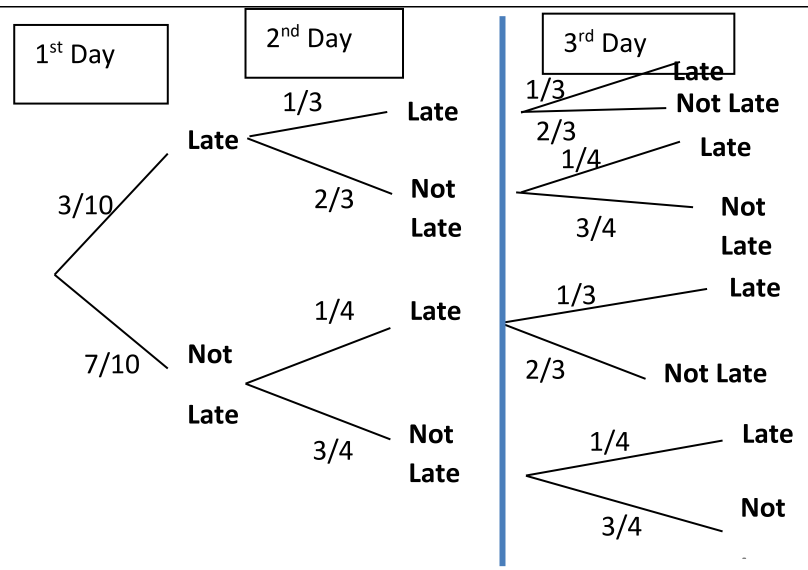
Draw a tree diagram to illustrate this situation on the first and second day of the term. [2]

A student is randomly chosen.

(i) Find the probability that he is late on the second day of the term. [2]

(ii) Find the probability that he is late on the first day given that he is late on the second day. [2]

A student is sent to the Year Head if he is late on at least two of the first three days of the term. Find the probability that he is sent to the Year Head. [2]

<table>
<tr><td>3</td><td>


</td></tr>
<tr><td>(i)</td><td>P(late on 2nd day)
$= \frac{3}{10}\times\frac{1}{3}+\frac{7}{10}\times\frac{1}{4}=\frac{11}{40}$ or 0.275</td></tr>
<tr><td>(ii)</td><td>Let B be event late on 2nd day. Let A be event late on 1st day
$\mathrm{P}(A|B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\frac{\text{P(late on both 1st and 2nd day)}}{\text{P(late on 2nd day)}}$</td></tr>
</table>

	$=\dfrac{\frac{3}{10}\times\frac{1}{3}}{\frac{11}{40}}=\dfrac{4}{11}$ or 0.364
	P(a student is sent to the Year Head) = P(the student is late on at least two of the first three days of the term) = P(LL) + P($L'LL$) + P($LL'L$) or [P(LLL) + P(LLL')] + P($L'LL$) + P($LL'L$) $=\frac{3}{10}\times\frac{1}{3}+\frac{7}{10}\times\frac{1}{4}\times\frac{1}{3}+\frac{3}{10}\times\frac{2}{3}\times\frac{1}{4}=\frac{1}{10}+\frac{7}{120}+\frac{1}{20}$ $=\frac{5}{24}$ or 0.208

4. A family of eight sits in the row in front of the four couples during the game wearing basketball jerseys with a single digit from 1 to 8 printed on the back.

Find the probability that the 8-digit code formed by the digits on the jerseys is

(i) an odd number greater than 40000000, [3]

(ii) greater than 40000000, given that it is an odd number. [2]

4(i)	P(odd number greater than 40000000) $=\dfrac{n(\text{number ends with "1" or "3"})+n(\text{number ends with "5" or "7"})}{n(S)}$ $=\dfrac{{}^5_1C\times 6!\ \times{}^2_1C+{}^4_1C\times 6!\ \times{}^2_1C}{8!}$ $=\dfrac{9}{28}$ (or 0.321 (3sf))
(ii)	$P(\text{number greater than 40000000} \mid \text{odd number})$ $=\dfrac{P(\text{number greater than 40000000} \cap \text{odd number})}{P(\text{odd number})}$ $=\dfrac{P(\text{odd number greater than 40000000})}{P(\text{odd number})}$ $=\dfrac{\frac{9}{28}}{\frac{7!\times{}^4_1C}{8!}}$ $=\dfrac{\frac{9}{28}}{\frac{1}{2}}$ $=\dfrac{9}{14}$ (or 0.643 (3sf))

Section 3: Table of Outcomes

1. The four faces of an unbiased green tetrahedral die are marked 1, 2, 4, 4 while the faces of an unbiased orange tetrahedral die are marked 1, 2, 2, 5. When such a die is thrown, the score is the number on the face on which it lands. The two dice are thrown together and their scores are added. The possibility diagram in the answer space shows some of the totals.

   (i) Copy and complete the possibility diagram. [1]

	Green Die				
Orange Die	+	1	2	4	4
	1				
	2	3			
	2		4		
	5				9

   (ii) Find the probability that the total score is 6. [1]

   (iii) The faces of an unbiased blue tetrahedral die are marked 20, 30, 40, 50. All 3 dice (green, orange and blue) are thrown together and the scores are added. Find the probability that the total is more than 40 but less than 45. [3]

1. (i)

	Green Die				
Orange Die	+	1	2	4	4
	1	2	3	5	5
	2	3	4	6	6
	2	3	4	6	6
	5	6	7	9	9

(ii) P(total score is 6) = 5/16

(iii) P(40 < total of 3 dice < 45)

= P(blue die = 40) . P(1 sum of green and orange die scores 4)

= (1/4)(6/16)

= 3/32