Name:______________________________ **Class:**__________

JURONG JUNIOR COLLEGE
Preliminary Examination

MATHEMATICS
Higher 2

9740/01
28 August 2012
3 hours

Additional materials: Answer Paper
List of Formulae (MF15)
Cover Page

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together, with the cover page in front.

This document consists of **6** printed pages.

1 Without using a calculator, solve the inequality $\dfrac{4x}{x-3} \geq 1$.

Hence find the range of values of x that satisfies $\dfrac{4|x|}{|x|-3} \geq 1$. [5]

2 A sequence $u_0,\ u_1,\ u_2,\ \ldots$ is such that $u_0 = 0$ and

$$u_n = u_{n-1} - 2^{-n}\left(n^2 - 4n + 2\right), \quad \text{for all } n \in \mathbb{Z}^+.$$

(i) Use the method of mathematical induction to prove that $u_n = n^2 2^{-n}$, for all $n \in \mathbb{Z}^+$. [4]

(ii) Find $\displaystyle\sum_{n=1}^{N}\left[-2^{-n}\left(n^2 - 4n + 2\right)\right]$. [2]

(iii) State the sum to infinity of the series in part **(ii)**. [1]

3 **(a)** **(i)** Differentiate $\sqrt{x^2 - 1}$ with respect to x. [1]

(ii) Hence find $\displaystyle\int x\cos^{-1}\left(\frac{1}{x}\right)\mathrm{d}x$. [3]

(b) Use the substitution $u = \dfrac{1}{x}$ to find the exact value of $\displaystyle\int_3^6 \frac{1}{x\sqrt{x^2 - 9}}\,\mathrm{d}x$. [4]

4 Let R be the region bounded by the curves $y = 4\sqrt{x}$ and $y = x^2 + 1$.

(i) Find the area of region R. [3]

(ii) If the line $x = c$ divides region R into two equal areas, find the value of c. [3]

(iii) Find the volume of the solid generated when region R is rotated through 2π radians about the y-axis. [3]

5 **(i)** Given that $y = \dfrac{1}{2}\ln\left(1 + \tan x\right)$, show that

$$2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x.$$

Hence write down the first two non-zero terms in the Maclaurin series for y. [4]

(ii) Given that the first two non-zero terms in the Maclaurin series for y are equal to the first two non-zero terms in the series expansion of $\dfrac{x}{a+bx}$, where a and b are constants, find a and b. [4]

6 A curve C has parametric equations

$$x = \cos\left(e^t\right), \quad y = \sin\left(e^t\right), \text{ where } t \le \ln\frac{\pi}{2}.$$

(i) Sketch the curve C, showing clearly its axial intercepts. [2]

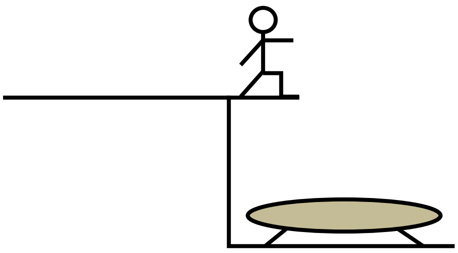
(ii) Show that any normal to the curve C passes through the origin. [4]

(iii) The equation of the normal to the curve C at the point P is $y = x$.

Find the equation of the tangent at P. [4]

7 **(a)** A particular set of Russian dolls consists of seven dolls. The heights of the seven dolls are in arithmetic progression with common difference d. If the height of the tallest doll is four times the height of the shortest one, and the sum of the heights of all the dolls in the set is 70 cm, find d. [4]

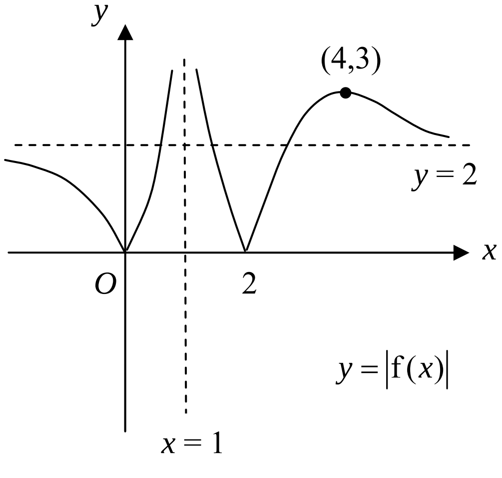
(b)

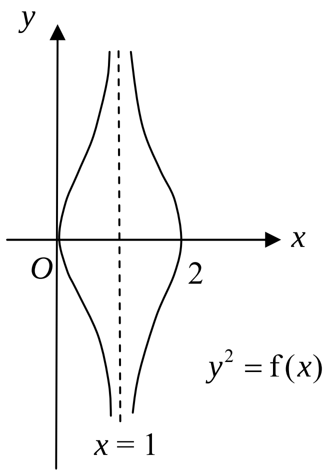


Bob stepped off a platform, fell vertically, and bounced on a trampoline. The time interval between any particular bounce and his next bounce is 90% of the time interval between that particular bounce and the preceding bounce. The interval between his first and second bounces is 4 seconds.

Given that the interval between the k th bounce and the $(k + 1)$ th bounce is the first such interval less than 0.4 seconds, find k. Also, find the total time from the first bounce to the k th bounce, giving your answer to the nearest second. [5]

8 (a) The diagrams below show the graphs of $y = |\mathrm{f}(x)|$ and $y^2 = \mathrm{f}(x)$ for a function f.





Sketch, on separate diagrams, the graphs of

(i) $y = \mathrm{f}(x)$, [2]

(ii) $y = \mathrm{f}(|x|)$, [2]

stating the equations of any asymptotes and the coordinates of any axial intercepts and turning points.

(b) The curve C has equation $y = \dfrac{ax^2 + 3x + b}{x}$, where a and b are constants.

(i) Given that $y = x + 3$ is an oblique asymptote of C, determine the value of a. [1]

(ii) Given also that C does not cut the x-axis, show that $b > \dfrac{9}{4}$. [2]

(iii) Sketch the curve C for b = 4, stating clearly the coordinates of any stationary points and the equations of any asymptotes. [3]

Deduce the range of values of k for which $\dfrac{ax^2 + 3x + 4}{x(kx + 3)} = 1$ has two real roots. [1]

9 The functions f and g are defined by

$$\mathrm{f} : x \mapsto x - \frac{1}{x}, \quad x < 0,$$
$$\mathrm{g} : x \mapsto \sin x, \quad 0 \le x \le 2\pi.$$

(i) Explain why f^{-1} exists. [1]

(ii) Define f^{-1} in a similar form. [3]

(iii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$, giving the coordinates of any points where the curves cross the x- and y- axes. [3]

(iv) Show that the composite function fg does not exist. [1]

(v) The function h is defined by

$$\mathrm{h} : x \mapsto \sin x, \ \pi < x < 2\pi.$$

Define fh and state the range of fh. [3]

10 (a) The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, such that $|\mathbf{a}| = |\mathbf{b}|$. The point P with position vector $\mathbf{p}$ lies on AB such that $\mathbf{b} \bullet \mathbf{p} = \mathbf{a} \bullet \mathbf{p}$.

(i) Show that AB is perpendicular to OP. [2]

(ii) Determine the position vector of the point R in terms of $\mathbf{a}$ and $\mathbf{b}$, where R is the reflection of O about the line AB. [2]

(iii) Give the geometrical meaning of $|\mathbf{a} \times \mathbf{b}|$. [1]

(b) The line l_1 passes through the points C and D with coordinates $(10, 8, 3)$ and $(11, 22, a)$ respectively, where a is a constant. Find a vector equation of the line l_1 in terms of a. [1]

Another line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} b \\ -10 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix}, \ \mu \in \mathbb{R}, \text{ where } b \text{ is a constant.}$$

Given that l_1 and l_2 are perpendicular and intersect at a point, find the values of a and b. [4]

[Turn over

11 **(a)** **(i)** Show that for all complex numbers z with $r > 0$ and $-\pi < \theta \leq \pi$,

$$\left(z - r\mathrm{e}^{\mathrm{i}\theta}\right)\left(z - r\mathrm{e}^{-\mathrm{i}\theta}\right) = z^2 - \left(2r\cos\theta\right)z + r^2.$$ [2]

(ii) Solve the equation $z^4 = -81$, expressing the solutions in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

(iii) Hence express $z^4 + 81$ as the product of two quadratic factors with real coefficients, giving each factor in non-trigonometrical form. [3]

(b) The complex number w has modulus 4 and argument $-\dfrac{\pi}{6}$.

Find the exact value of the modulus and argument of p, where $p = \dfrac{w^*}{(1-\mathrm{i})^2}$. [2]

Given that p^n is real, find the possible values of n where n is a positive integer. [2]