Anglo - Chinese School
(Independent)

FINAL EXAMINATION 2018
YEAR THREE EXPRESS
ADDITIONAL MATHEMATICS
PAPER 1

Friday **5 October 2018** **1 hour 30 minutes**

1 SOLUTION

Let P be (a, b).

$$\frac{a-2}{2}=0 \quad \text{and} \quad \frac{b-1}{2}=2$$

$P = (2, 5)$

Let R be (c, d)

$$\frac{c-2}{2}=6 \quad \text{and} \quad \frac{d-1}{2}=0$$

$R = (14, 1)$

$$C = \left(\frac{2+14}{2}, \frac{5+1}{2}\right) = (h, k)$$

$$= \underline{(8, 3)}$$

2 SOLUTION

(a) $2\sqrt{180}+\sqrt{245}-3\sqrt{125}$ $= 12\sqrt{5}+7\sqrt{5}-15\sqrt{5}$

$$= \underline{4\sqrt{5}}$$

(b) $\dfrac{x}{\sqrt{1-3x}}=\dfrac{1}{2}$

$$2x=\sqrt{1-3x}$$

$$4x^2=1-3x$$

$$4x^2+3x-1=0$$

$$(4x-1)(x+1)=0$$

$$x=\frac{1}{4} \quad or \quad -1 \text{ (NA)}$$

$$\therefore x=\frac{1}{4}$$

3 SOLUTION

(i) **gradient =** $\dfrac{-2-4}{5-1}$

$$= -\frac{3}{2}$$

$$(xy+x)-4=-\frac{3}{2}(y-1)$$

$$xy+\frac{3}{2}y=4-x+\frac{3}{2}$$

$$y(2x+3)=8-2x+3$$

$$y=\frac{11-2x}{2x+3}$$

(ii) $y=\dfrac{11-2x}{2x+3}$

$2(2x+3)=11-2x$

$6x=5$

$\underline{x=\dfrac{5}{6}}$

4 SOLUTION

(i) $\alpha^2+\beta^2=53$ $\alpha^2\beta^2=196$

$(\alpha+\beta)^2-2\alpha\beta=53$ $\alpha\beta=14$ (α and β are lengths and are positive)

$(\alpha+\beta)^2=2(14)+53$

$\alpha+\beta=9$

Equation whose roots are α and β is $\underline{x^2-9x+14=0}$

(ii) Since α and β are roots of $x^2-9x+14=0$ and $(x-7)(x-2)=0$, $\alpha=2$ or 7

Dimensions of the rectangle = 4 cm by 14 cm.

5 SOLUTION

(i) Centre of C_1 = $(1,-4)$

Gradient of PQ = $-\dfrac{1-(-5)}{-4-(-2)}$ = 3 (negative reciprocal of radius at P)

Equation of PQ is: $y+2=3(x+5)$

$\underline{y=3x+13}$

(ii) Radius of C_1 = $\sqrt{1^2+(-4)^2+23}$ = $\sqrt{40}$

Distance between centre of C_1 and R = $\sqrt{(1+3)^2+(-4-1)^2}$ = $\sqrt{41}$ > radius

Hence, R lies outside of C_1.

(iii) Centre of C_2 = $(-7,-4)$

Equation of C_2: $(x+7)^2+(y+4)^2=\left(\sqrt{40}\right)^2$

$x^2+y^2+14x+8y+49+16=40$

$\underline{x^2+y^2+14x+8y+25=0}$ [Note, not necessary to have this form]

6 <u>SOLUTION</u> ***See graph paper***

(ii)(a) $\lg P = b \lg T + \lg a$

Gradient = <u>b = 1.52</u>.

Intercept = lg a = 2.2

<u>a = 158.5</u>

(b) Abnormal Reading: P = 10000

Correct Reading: lg P = 3.75

<u>P = 5623</u> (nearest whole number)

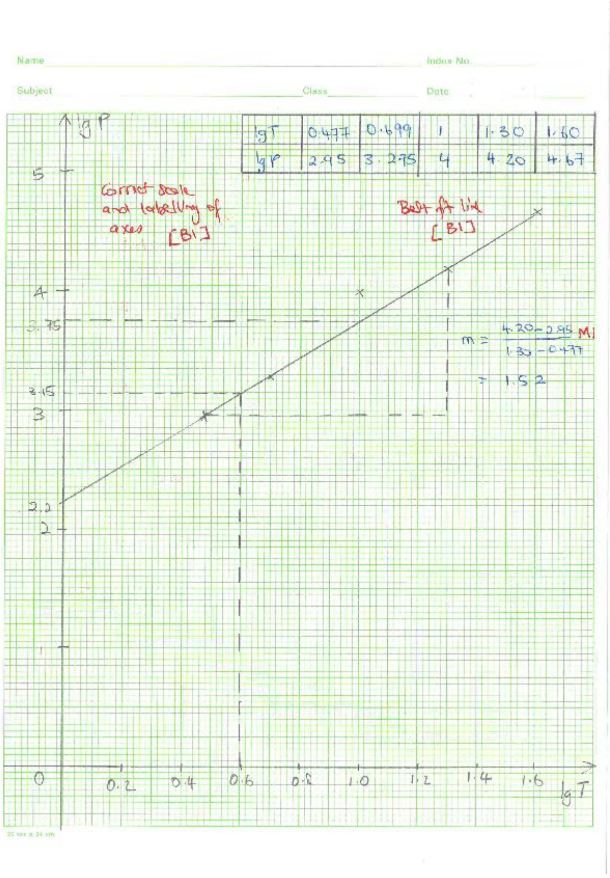
(c) After 4 days, T = 4

lg T = 0.60

lg P = 3.15

P = 1413 (nearest whole number)

The population after 4 days is <u>1413</u>.



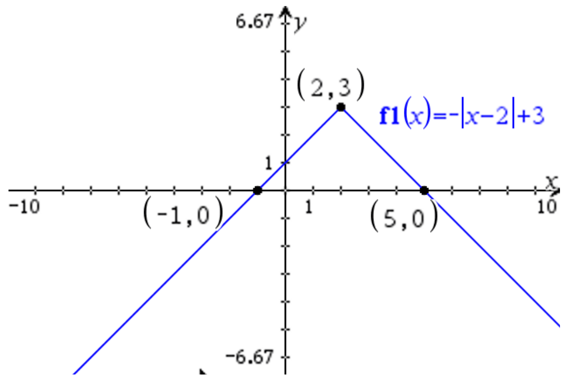
7 SOLUTION

(a)(i) $|x-2|=3$

$x-2=3$ or $x-2=-3$

$\underline{x=5}$ or $\underline{x=-1}$

(ii)



7(a)(iii) $5+4x=|x-2|$

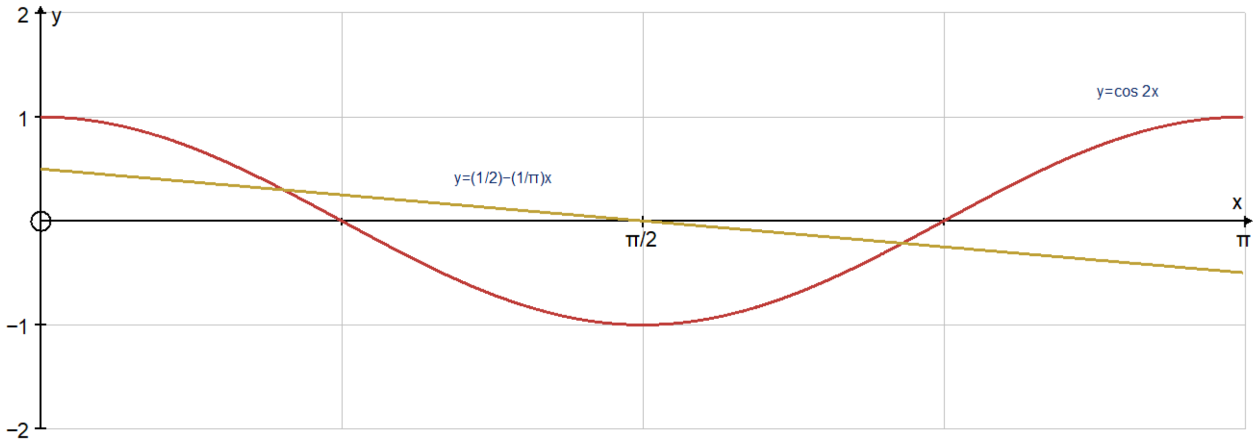
$2+3+4x-|x-2|=0$

$\underline{y=-2-4x}$ is the equation of the line to be inserted.

7(b) $\pi-2x=2\pi\cos 2x$

$y=\frac{1}{2}-\left(\frac{1}{\pi}\right)x$ is to be inserted into the sketch.

There are <u>2 solutions</u>.



8 SOLUTION

(a) $\dfrac{\log_n m \times \log_m n^2}{\log_{m^2} m} = \dfrac{\lg m}{\lg n} \times \dfrac{2\lg n}{\lg m} \times \dfrac{2\lg m}{\lg m}$

$= \underline{4}$

(b) $x^2 y = 2^6$ $\quad$ $xy^2 = 2^7$

$\left(2^r\right)^2 4^s = 2^6$ $\quad$ $2^r \times \left(4^s\right)^2 = 2^7$

$2r + 2s = 6$ - - - (1) $\quad$ $r + 4s = 7$ - - - (2)

2(2) - (1): $6s = 8$

$\underline{s = \dfrac{4}{3}}$ $\quad$ (1): $\underline{r = \dfrac{5}{3}}$

(b) $\dfrac{1}{\log_{(5x-e)} e} = 1 + \ln x$

$\ln(5x - e) = \ln e + \ln x$

$(5x - e) = ex$

$x(5 - e) = e$

$\underline{x = \dfrac{e}{5 - e}}$

END OF PAPER ONE