Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2015
YEAR THREE EXPRESS
ADDITIONAL MATHEMATICS
PAPER 2

5 October 2015 **1 hour 30 minutes**

PAPER REVIEW

1

Solution:

$$2^{x+2}3^{x-4} = 5^{1-2x}$$

$$2^x 3^x \left(4 \times \frac{1}{81}\right) = \left(\frac{1}{25}\right)^x (5)$$

$$2^x 3^x 25^x = \frac{5 \times 81}{4}$$

$$150^x = \frac{5 \times 81}{4}$$

$$x = \frac{\lg\left(\frac{5 \times 81}{4}\right)}{\lg 150}$$

$$\underline{x = 0.922} \text{ (to 3 sf)}$$

Most students can get up to this line, but many could get no further. The mistake comes from $2^x 3^x = 5^x$
Some students started by logging both sides of the equation, but brought down the powers without bracketing them:
$x + 2\lg 2 + x - 4\lg 3 = 1 - 2x\lg 5$

And getting
$2x + 2x\lg 5 = 1 - 2\lg 2 + 4\lg 3$

2 **Solution:**

Students have either made
1) e^o =0 thus getting k to be 20, and then cannot solve (iii)
or
2) $4e^o$= 4e, thus getting k to be 237.46….

$$D = \frac{k}{1+4e^{-0.14t}}$$

(i) When $t = 0$, $D = 20$: $20 = \frac{k}{1+4e^{0}}$

$$\underline{k = 100}$$

(ii) When $t = 10$, $D = \frac{100}{1+4e^{-1.4}}$

= 50 (nearest integer)

$\therefore$ There will be 50 deer after 10 years.

(iii) When D = 70, $70 = \frac{100}{1+4e^{-0.14t}}$

$$4e^{-0.14t} = \frac{100}{70} - 1$$

$$t = \frac{\ln\frac{3}{28}}{-0.14}$$

$$t = 16.0 \text{ (to 3 sf)}$$

The population will take 16 years to reach a population of 70.

3

(i) Solution:

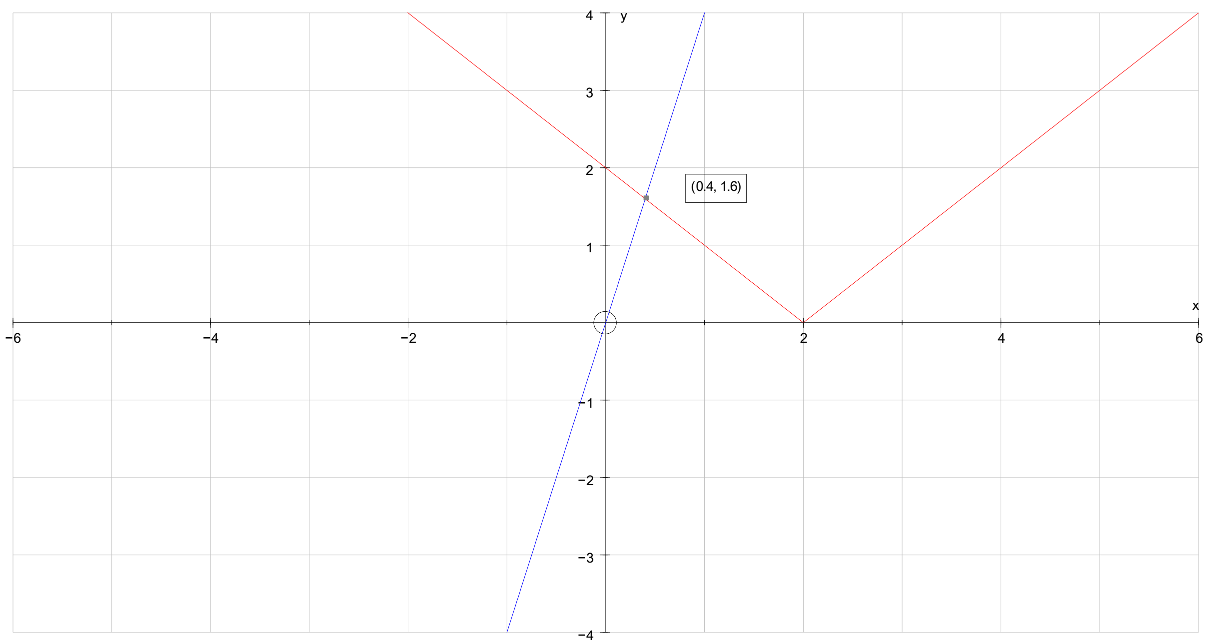
Students forget to check that x >0

$2 - x = 4x$ **M1** or $2 - x = -4x$

$\underline{x = \frac{2}{5}}$ $x = -\frac{2}{3}$ (NA)

(ii) Solution:

$y = |2 - x|$ $\qquad\qquad$ $y = 4x$



Several students didn't label the point of intersection and did not realize that the range of values for x is obtained by referring to the sketch.

Accurate sketch of the graphs of $y = |2 - x|$ and $y = 4x$

Label of points of intersection $\left(\frac{2}{5},\ 1\frac{3}{5}\right)$

(iii) Solution: $\underline{x < \frac{2}{5}}$

4

Solution:

(i) When $y = 0$, $x = \mathrm{m} - 5$ $\therefore P \ is \ (m-5,\ 0)$.

(ii) When $x = 0$, $y = n + 3$ $\therefore Q \ is \ (0,\ n+3)$.

Midpoint of $PQ = \left(\dfrac{m-5}{2}, \dfrac{n+3}{2}\right)$

(iii) Gradient of perpendicular bisector of $PQ = \dfrac{1}{2}$

Gradient of $PQ = -2$

$$\frac{n+3-0}{0-(m-5)} = -2$$
$$n+3 = 2m-10$$
$$n = 2m-13 \quad \text{------(1)}$$

Since the midpoint of PQ lies on $y = \dfrac{1}{2}x + 3$,

$$\frac{n+3}{2} = \frac{1}{2} \times \frac{m-5}{2} + 3$$
$$2n+6 = m-5+12$$

(1): $2(2m-13) = m+7-6$
$$3m = 27$$
$$\underline{m = 9}$$

(1): $\underline{n = 5}$

Many students multiply by the LCM of the denominators and got lost in the expansion, before they let x=0 and y=0.

Students proved (i) with hardly any problems, but (ii) became complicated when students changed $\tan^2 x$ *to* $\dfrac{\sin^2 x}{\cos^2 x}$

Several students assumed that the RHS of (iii) is the same as the LHS of (ii) and thus equated 2secx to -1, instead of -4.

5 **(i)** **Solution:**

$$\text{LHS} = \frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x}$$
$$= \frac{(1-\sin x)^2 + \cos^2 x}{\cos x(1-\sin x)}$$
$$= \frac{1-2\sin x + \sin^2 x + \cos^2 x}{\cos x(1-\sin x)}$$
$$= \frac{2-2\sin x}{\cos x(1-\sin x)}$$
$$= \frac{2(1-\sin x)}{\cos x(1-\sin x)}$$

= **2sec*x*** = RHS **(proven)**

Students attempted to find the equation of PQ to intersect with the equation of its perpendicular bisector to equate the point of intersection with the midpoint of PQ – and got lost because they expanded (m-5)(n+3) and didn't know how to factorize this back to simplify.

(ii) Solution:

LHS $= (\sin x+1)(\sin x-1)(\sin^2 x+\tan^2 x+\cos^2 x)$

$= (\sin^2 x-1)(1+\tan^2 x)$

$= (-\cos^2 x)(\sec^2 x)$

$= -1 =$ **RHS** (proven)

(iii) Solution:

$$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=4(\sin x+1)(\sin x-1)(\sin^2 x+\tan^2 x+\cos^2 x)$$

$$2\sec x=4(-1)$$

$$\cos x=-\frac{1}{2}$$

$$\alpha=60^\circ$$

$$\underline{x=120^\circ \ \text{or} \ 240^\circ}$$

6

Solution:

(a) (i) $x^4+x^2+x+1\equiv(x^2+A)(x^2-1)+Bx+C$

When $x = 0$, $1 = -A + C$ ------(1)

When $x = 1$, $4=B+C$ -------(2)

When $x = -1$, $2=-B+C$ -------(3)

(method mark given for method of substitution or comparison of coefficients)

Most students did long division to find the remainder instead of understanding that f(x)=(x-a)g(x) +R(x).

(2) + (3): $C = 3$

(2): $B = 1$

(1): $A = 2$

(answer mark given for answers for A and B)

(ii) $x^4+x^2+x+1\equiv(x^2+2)(x^2-1)+x+3$

Remainder = $x+3$

Solution:

(b) $$\frac{x^3+7x+2}{x^2-1}=x+\frac{8x+2}{(x-1)(x+1)}$$

(by long division)

$$\frac{8x+2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$8x+2 = A(x+1) + B(x-1)$$

$x=1: \quad 10 = 2A \qquad A = 5$

$x=-1: \quad -6 = -2B \qquad B = 3$

(method of substitution or comparison of coefficients)

$$\frac{x^3+7x+2}{x^2-1} = \underline{x + \frac{5}{x-1} + \frac{3}{x+1}}$$

Several students changed $\sin^2 x$ to $\frac{\tan^2 x}{\sec^2 x}$ - the long way

Several students split into partial fractions immediately without realizing that the degree of the polynomial numerator is higher than that of the denominator.

7

Solution:

(a) $\dfrac{3+2\sin^2 x}{\cos^2 x} = 3\sec^2 x + 2\tan^2 x$

$= 3(1+\tan^2 x) + 2\tan^2 x$

$= \underline{3+5a^2}$

Solution:

(b) $2\cos x = \cot x$

$2\cos x = \dfrac{\cos x}{\sin x}$

$2\sin x \cos x - \cos x = 0$

$(2\sin x - 1)\cos x = 0$

$\sin x = \dfrac{1}{2} \qquad$ or $\qquad \cos x = 0$

$\alpha = \dfrac{\pi}{6}, \dfrac{\pi}{2}$

$\underline{x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}}$

Several students divided by cos *x* instead of factorizing it out and letting $\cos x = 0$

Many students are not aware the trig ratio of special angles as in this question and gave answers in radians but not in pi. Some even gave answers in degrees.

8 **<u>Scale:</u> Many students chose a scale that is time-consuming to plot the points, and encounter the possibility of points being plotted wrongly.**

$2y = 35\sqrt{x}$ **A few students still didn't know to change this to $\frac{y}{\sqrt{x}} = 17.5$ and draw the horizontal line to meet the line they drew. However, several forgot that the value on the horizontal axis does not give the *x* value but the $x\sqrt{x}$ value.**

The abnormal as well as the correct readings should be given in *x* and *y*, not $\frac{y}{\sqrt{x}}$ and $x\sqrt{x}$ coordinates.

For the graph of $\frac{y}{x^2}$ is plotted against $\frac{\sqrt{x}}{x^2}$, the student needs only re-arrange the given equation and see that q is the gradient of the new line and give its value as obtained in b(i).

End of Paper