

南华中学

NAN HUA HIGH SCHOOL

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PRELIMINARY EXAMINATION 2022

Subject	:	Mathematics
Paper	:	4048/02
Level	:	Secondary Four Express
Date	:	18 August 2022
Duration	:	2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in. Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue, or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question. The total of the marks for this paper is 100.

For Examiner's Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOTAL

Compound interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone =
$$\pi rl$$

Surface area of a sphere = $4\pi r^2$
Volume of a cone = $\frac{1}{3}\pi r^2 h$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Area of triangle $ABC = \frac{1}{2}absinC$
Arc length = $r\theta$, where θ is in radians
Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all questions.

1 (a) Simplify
$$\frac{3a^2}{5b^3} \div \left(\frac{12c}{25ab}\right)^2$$
.
 $\frac{3a^2}{5b^3} \div \left(\frac{12c}{25ab}\right)^2$
 $= \frac{3a^2}{5b^3} \div \frac{144c^2}{625a^2b^2}$
 $= \frac{3a^2}{5b^3} \times \frac{625a^2b^2}{144c^2}$
 $= \frac{125a^4}{48bc^2}$

(b) n is a positive integer.

Show that, for all n, $(3n+2)^2 - (3n-2)^2$ is a multiple of 3. Answer

$$(3n+2)^{2} - (3n-2)^{2}$$

= $(3n+2+3n-2)[3n+2-(3n-2)]$
= $(6n)(4)$
= $24n$
= $3(8n)$
Since *n* is a positive integer, $3(8n)$ is a multiple of 3.

[2]

(c) Solve the equation
$$\frac{2}{x-4} + \frac{7x}{3x-2} = 1$$
.
 $\frac{2}{x-4} + \frac{7x}{3x-2} = 1$
 $\frac{2(3x-2) + 7x(x-4)}{(x-4)(3x-2)} = 1$
 $2(3x-2) + 7x(x-4) = (x-4)(3x-2)$
 $4x^2 - 8x - 12 = 0$
 $x^2 - 2x - 3 = 0$
 $(x-3)(x+1) = 0$
 $x = 3 \text{ or } x = -1$

Answer $x = \dots$ [4]

(d) Use the quadratic formula to solve the equation.

$$3x^{2} - 7x - 4 = 0$$

$$3x^{2} - 7x - 4 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^{2} - 4(3)(-4)}}{2(3)}$$

$$= 2.81 \text{ or } -0.475$$

Answer $x = \dots$ [2]

- 2 A box of chocolate contains 7 dark chocolates and 5 milk chocolates. Sufyan takes a chocolate, selected at random, from the box and eats it. LeLe then takes a chocolate, selected at random, from the box.
 - (i) Draw a tree diagram to show the probabilities of the possible outcomes.

Answer



(ii) Find, as a fraction in its simplest form, the probability that

(a) Sufyan and LeLe both picked dark chocolates,

P(both dark chocolate) = $\frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

(b) LeLe picked a milk chocolate,

P(LeLe picked milk chocolate) =
$$\frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{4}{11}$$

= $\frac{5}{12}$

(c) at least one milk chocolate was chosen.

P(at least one milk chocolate was chosen) = $1 - \frac{7}{12} \times \frac{6}{11}$ = $\frac{15}{22}$

3



In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

 $2\overrightarrow{OB} = 3\overrightarrow{BC}, \ \overrightarrow{OD} = 3\overrightarrow{OA}, \ \text{and} \ 2\overrightarrow{AX} = \overrightarrow{XB}.$

(i) Express, as simply as possible, in terms of **a** and **b**,

(a) \overrightarrow{BA} ,

Answer
$$\overrightarrow{BA} = \dots \quad \mathbf{a} - \mathbf{b} \quad \dots \quad [1]$$

(b)
$$OX$$
,
 $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$
 $= \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$
 $= \mathbf{a} + \left(-\frac{1}{3}\right)(\mathbf{a} - \mathbf{b})$
 $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

$$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$$
$$= 3\mathbf{a} - \frac{5}{3}\mathbf{b}$$

Answer $\overrightarrow{CD} = \dots$ [1]

(ii) It is given that CM : MD is 10 : 9.

(a) Express \overrightarrow{OM} , as simply as possible, in terms of **a** and **b**.

$$\frac{CM}{MD} = \frac{10}{9}$$
$$\overline{OM} = \frac{10}{9}\mathbf{b} + \frac{10}{19}\left(3\mathbf{a} - \frac{5}{3}\mathbf{b}\right)$$
$$= \frac{30}{19}\mathbf{a} + \frac{15}{19}\mathbf{b}$$

(b) Show that *O*, *X* and *M* are collinear.

Answer

$$\overrightarrow{OM} = \frac{15}{19}\mathbf{b} + \frac{30}{19}\mathbf{a}$$
$$= \frac{45}{19} \left(\frac{1}{3}\mathbf{b} + \frac{2}{3}\mathbf{a}\right)$$
$$= \frac{45}{19} \overrightarrow{OX}$$

Hence, OM // OX and O is a common point. Thus O, X and M are collinear.

[2]

(c) Given that MY is parallel to OD, express \overrightarrow{MY} in terms of **a** and **b**.

$$\frac{OM}{OX} = \frac{45}{19}$$
$$\Rightarrow \frac{XM}{OX} = \frac{YM}{OA} = \frac{26}{19}$$
$$\overrightarrow{YM} = \frac{26}{19}\overrightarrow{OA}$$
$$\overrightarrow{MY} = -\frac{26}{19}\mathbf{a}$$

(iii) Find the ratio of

Answer $\overrightarrow{MY} = \dots$ [1]

Answer [1]





In the diagram, OAPB is a sector of circle with centre O and radius 9 cm. Angle AOB = 1.4 radians.

C is the centre of the circle enclosed inside the sector.

OCP is a straight line and the circle touches the sector at points P, Q and R.

(i) Show that the radius of the enclosed circle is 3.526 cm, correct to 3 decimal places.

Answer

Let the radius of the enclosed circle be r cm $\angle CRO = 90^{\circ} \text{ (tangent } \bot \text{ radius)}$ $\angle ROC = \frac{1.4}{2} \text{ (tangents from external point)}$ = 0.7 rad $\sin \angle ROC = \frac{RC}{CO}$ $\sin 0.7 = \frac{r}{9-r}$ $9 \sin 0.7 - r \sin 0.7 = r$ $r = \frac{9 \sin 0.7}{1 + \sin 0.7}$ = 3.5262 (4 dp)= 3.526 (3 dp) (ii) Calculate the area of the shaded region.

$$\angle RCO = \pi - \frac{\pi}{2} - 0.7 \ (\angle \text{ sum of } \Delta)$$

$$= \frac{\pi}{2} - 0.7$$

$$\angle RCP = \pi - \left(\frac{\pi}{2} - 0.7\right) \text{ (adjacent } \angle \text{s on a straight line)}$$

$$= \frac{\pi}{2} + 0.7$$

area of sector $RCP = \frac{1}{2} \left(\frac{\pi}{2} + 0.7\right) (3.526)^2$

$$= 14.116 \text{ cm}^2 \text{ (5sf)}$$

area of $\Delta RCO = \frac{1}{2} (3.526) (9 - 3.526) \sin\left(\frac{\pi}{2} - 0.7\right)$

$$= 7.3812 \text{ cm}^2 \text{ (5sf)}$$

area of sector $OAP = \frac{1}{2} (0.7) (9)^2$

$$= 28.35 \text{ cm}^2$$

area of shaded region = 28.35 - 7.3812 - 14.116

$$= 6.85 \text{ cm}^2 \text{ (3sf)}$$

5



The diagram shows a right circular cone cut from a solid steel sphere. Point O is the centre of the sphere with radius 12 cm.

(a) Given the circumference of the base of the cone is 50 cm, show that the height of the cone is 21.0 cm, corrected to 3 significant figures.

Answer

radius of cone =
$$\frac{50}{2\pi}$$

= 7.9577 cm (5sf)
length from *O* to base of cone = $\sqrt{12^2 - 7.9577^2}$
= 8.9819 cm (5sf)
height of cone = 12 + 8.9819
= 20.9819
= 21.0 cm (3sf)

(b) Find the curved surface area of the cone.

slant height of cone =
$$\sqrt{20.9819^2 + 7.9577^2}$$
 or $\sqrt{21.0^2 + 7.9577^2}$
= 22.440 cm = 22.457 cm
curved surface area = $\pi (7.9577)(22.440)$
= 561 cm² (3sf)

[3]

(c) After the cone is cut from the steel sphere, the remaining steel is melted down to form part of a solid right circular cone as shown in the following diagram.



The cone comprises 3 layers of equal heights, h cm. The top and bottom layers are cast from the remaining steel. The centre section is made from acrylic.

Find the volume of the acrylic used to make the centre layer of the solid cone.

volume of cone in sphere $=\frac{1}{3}\pi (7.9577)^2 (20.9819)$	or $\frac{1}{3}\pi (7.9577)^2 (21.0)$
=1391.4 cm ³	$=1392.6 \text{ cm}^{3}$
volume of remaining steel = $\frac{4}{3}\pi (12)^3 - 1391.4$	or $\frac{4}{3}\pi(12)^3 - 1392.6$
$= 5846.8 \text{ cm}^3$	=5845.6 cm ³
ratio of volumes A : $A+B$: $A+B+C$: $A+$	- <i>C</i>
h^3 : $\left(2h ight)^3$: $\left(3h ight)^3$	
1 : 8 : 27 : 20	
volume of acrylic = $\frac{7}{20} \times 5846.8$	
$= 2050 \text{ cm}^3$	

- 6 Plane A travels at an average speed of x km/h for 3 hours 20 minutes and then at an average speed of y km/h for 1 hour 10 minutes.
 The plane travels a total distance of 3700 km.
 - (a) Write down an equation in x and y to represent this information and show that it simplifies to 20x + 7y = 22200.

Answer

$$x \times 3\frac{1}{3} + y \times 1\frac{1}{6} = 3700$$
$$\frac{10x}{3} + \frac{7y}{6} = 3700$$
$$20x + 7y = 22200 \text{ (shown)}$$

[1]

Plane *B* travels at an average speed of x km/h for 2 hours 30 minutes and then at an average speed of y km/h for 1 hour 50 minutes. It travels 350 km lesser than Plane *A*.

(b) Write down an equation in x and y to represent this information.

$$x \times 2\frac{1}{2} + y \times 1\frac{5}{6} = 3700 - 350$$
$$\frac{5x}{2} + \frac{11y}{6} = 3350$$
$$15x + 11y = 20100$$

(c) Solve these two equations to find the value of x and the value of y.

Answer

$$20x + 7y = 22200$$

$$x = \frac{22200 - 7y}{20} - (1)$$

$$15x + 11y = 20100 - (2)$$
sub (1) into (2):
$$15\left(\frac{22200 - 7y}{20}\right) + 11y = 20100$$

$$x = \frac{22200 - 7600}{20} = 900$$

Answer x = [3]

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7 In the diagram, P, Q, R, S and T are points on the circle with centre O.
VX is a tangent to the circle at P.
Line SR is parallel to the diameter of the circle TQ.
W lies on TQ and PR.



It is given that $\angle OPR = 45^{\circ}$ and $\angle PRQ = 30^{\circ}$.

(a) (i) Find angle *TOP*.Give a reason for each step of your working.

$$\angle POQ = 60^{\circ} (\angle \text{ at centre} = 2\angle \text{s at circumference})$$

 $\angle TOP = 180^{\circ} - 60^{\circ} (\text{adjacent } \angle \text{s on a straight line})$
 $= 120^{\circ}$

(ii) Find angle TPV.

Give a reason for each step of your working.

$$\angle OPT = \frac{180^{\circ} - 120^{\circ}}{2} \text{ (base } \angle \text{s of isosceles } \Delta\text{)}$$
$$= 30^{\circ}$$
$$\angle OPV = 90^{\circ} \text{ (tangent } \perp \text{ radius)}$$
$$\angle TPV = 90^{\circ} - 30^{\circ}$$
$$= 60^{\circ}$$

Answer Angle $TPV = \dots$ [2]

(iii) Find angle *TSR*. Give a reason for each step of your working.

$$\angle TSR = 180^\circ - 45^\circ - 30^\circ (\angle s \text{ in opposite segments})$$

= 105°

Answer Angle $TSR = \dots$ [1]

(b) Show that *SRWT* is a parallelogram.

Answer

$$\angle TWR = 60^{\circ} + 45^{\circ} (\text{exterior } \angle \text{ of } \Delta)$$

= 105°
= $\angle TSR$
$$\angle STW = 180^{\circ} - 105^{\circ} (\text{interior } \angle s, SR / /TW)$$

= 75°
$$\angle SRW = 360^{\circ} - 75^{\circ} - 105^{\circ} - 105^{\circ} (\angle \text{ sum of quadrilateral})$$

= 75°

Since opposite angles of a quadrilateral are equal, SRWT is a parallelogram.

[3]

8 The variables x and y are connected by the equation $y = \frac{1}{3}x(11-x^2)$.

Some corresponding values of *x* and *y*, correct to 1 decimal places, are given in the following table.

x	-4	-3	-2	-1	0	1	2	3
У	6.7	-2	-4.7	-3.3	0	3.3	р	2

(a) Calculate the value of p. p = 4.7

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Answer p = \dots [1]
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(b) On the grid opposite, draw the graph of
$$y = \frac{1}{3}x(11-x^2)$$
 for $-4 \le x \le 3$.
[3]

(c) Use your graph to find the solution of $\frac{1}{3}x(11-x^2) = 3$ in the range $-4 \le x \le 3$. x = -3.67, 0.88, 2.79

(d) By drawing a tangent, find the gradient of the curve at the point (-1, -3.3).

Gradient = 2.33



(e) On the same axes, draw the graph of 5y + 2x = 7 for $-4 \le x \le 3$.

[1]

(f) (i) Show that the points of intersection of the line and the curve give the solutions of the equation $5x^3 - 61x + 21 = 0$. Answer 5y + 2x = 7 $y = \frac{-2x + 7}{5}$ -(1) $y = \frac{1}{3}x(11 - x^2)$ -(2) sub (1) into (2): $\frac{-2x + 7}{5} = \frac{1}{3}x(11 - x^2)$ $-6x + 21 = 55x - 5x^3$

 $5x^3 - 61x + 21 = 0$ (shown)

[2]

(ii) Use your graphs to solve the equation $5x^3 - 61x + 21 = 0$. x = -3.6537 or 0.34771

Answer x = [1]

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9 60 potato plants produce 5 to 10 potatoes each.
 The mass of potatoes produced by each plant were measured.
 The sumulative frequency surve below shows the distribution of the mass

The cumulative frequency curve below shows the distribution of the masses of the potatoes produced by each plant.



- (i) Use the curve to estimate
 - (a) the median mass, median mass = 3.1 kg

Answer kg [1]

(b) the interquartile range. interquartile range = 3.6 - 2.4 = 1.2 kg

Answer kg [1]

(ii) It was stated that 20% of the potato plants were considered premium plants as they produced greater mass of potatoes.

Find the least mass of potatoes produced for the plant to be 'premium'.

 $\frac{100-20}{100} \times 60 = 48$ least mass = 3.8 kg

Answer kg [2]

(iii) The potatoes produced by another group of 60 plants have the same median but smaller interquartile range.

Describe how the cumulative frequency curve will differ from the given curve.

Answer		The curve will be steeper.	
]
	••••		
			[1]

10

(a)



ABL is a park on horizontal ground. A is 1390 m from L on a bearing of 076° . B is 960 m from L on a bearing of 138° .

(i) Find *AB*.

$$\angle ALB = 138^{\circ} - 76^{\circ}$$

 $= 62^{\circ}$
 $AB = \sqrt{960^{2} + 1390^{2} - 2(960)(1390)\cos 62^{\circ}}$
 $= 1265.2 \text{ m (5sf)}$
 $= 1270 \text{ m (3sf)}$

Answer m [2]

(ii) Find angle *LAB*.

 $\sin \angle LAB = \frac{\sin 62^{\circ}}{1265.2} \times 960$ $\angle LAB = 42.064^{\circ} \text{ (3dp)}$ $= 42.1^{\circ} \text{ (1dp)}$

Alternatively,

$$\angle LAB = \cos^{-1} \left[\frac{1390^2 + 1265.2^2 - 960^2}{2(1390)(1265.2)} \right]$$

(iii) Find the area of triangle *LAB*.

area =
$$\frac{1}{2}(1390)(960) \sin 62^{\circ}$$

= 589100 m² (5sf)
= 589000 m² (3sf)

Alternatively, Area of $\Delta LAB = \frac{1}{2}(1265.2)(1360) \sin 42.064^{\circ}$

Answer $\dots m^2$ [2]

(iv) Find the bearing of *B* from *A*.

 $\angle LAN_{1} = 180^{\circ} - 76^{\circ} \text{ (interior } \angle s, NL / / N_{2}A)$ = 104° bearing of *B* from $A = 360^{\circ} - 104^{\circ} - 42.064^{\circ} \text{ (} \angle s \text{ at a point)}$ = 213.936° (3dp) = 213.9° (1dp)

 $\angle LAC = 76^{\circ} \text{ (alternate } \angle \text{s, } NL / / N_2 A \text{)}$ Bearing of *B* from $A = 180^{\circ} + (76^{\circ} - 42.064^{\circ})$ $= 213.939^{\circ} \text{ (3 dp)}$ = 213.9 (1 dp)

Alternatively,

Answer° [2]

(b) T is the top of a tower at L. The greatest angle of depression from T to the path AB is 5.06°.

Calculate the height of the tower.

shortest distance from *L* to $AB = 589100 \div \frac{1}{2} \div 1265.2$ = 931.24 m (5sf) $\tan 5.06^\circ = \frac{LT}{931.24}$ $LT = 931.24 \times \tan 5.06^\circ$ LT = 82.5 m (3sf)

Alternatively, Shortest distance from *L* to $AB = 1390 \times \sin 42.063^{\circ}$ Shortest distance from *L* to $AB = 960 \times \sin 75.936^{\circ}$

Answer m [3]

(c) Jonah goes on a jog along the edge of the park at a speed of 8.5 km/h. He starts from *L* towards *A* then to *B* before going back to *L*.

Calculate the time he takes to jog. Give your answer in minutes and seconds, corrected to the nearest 10 seconds.

total distance = 960 + 1390 + 1265.2= 361.2 mtime taken = $3.615 \div 8.5$ = 0.42532 h (5sf)= 25.519 min= 25 min 30 s (nearest 10 seconds)

Answer seconds [3]

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11 Mr Ho would like to replace all 60 units of air conditioners in his office building. He is considering between two brands of air conditioners.

Brand	Samsing	Potasonic	
Price of each air conditioner unit (after GST)	S\$1388	S\$740	
Power consumption per air conditioner unit	3 kW	3.5 kW	
Servicing Frequency	Twice a year	Four times a year	
Total cost for servicing 60 units	\$2100 per	servicing	
Warranty*	Two years	Three years	
*Warranties cover servicing and maintenance of air conditioner units with free replacement of parts for the stated duration.			

Information about the two brands of air conditioners is shown in the table below.

Electricity Tariff from 1 July 2022 is shown in the table below.

SPgroup Empowering the Future of Energy	Tariff (without GST)	Tariff with GST				
kWh* charge (¢ per kWh)	30.17	32.28				
* kWh (kilowatt-hour) is a unit of energy equal to one kilowatt (kW) of power sustained for one hour						

The usage of air conditioner units in Mr Ho's office building is shown in the table below.

Days	Usage Time/unit	Number of units used	
Monday	8 hours	50	
Tuesday to Friday	6 hours each day	60	
Saturday and Sunday	No usage		

(a) In view of public holidays, he estimates that the company operates for 51 weeks per year.

Find the usage of all air conditioner units, in hours, for a year.

 $51 \times (50 \times 8 + 60 \times 6 \times 4) = 93840$ h

Answer h [2]

(b) Based on the usage of his office building, which air conditioner model will have a lower cost after 4 years of use?

Justify your decision with calculations.

Answer

	Samsing	Potasonic
Total cost of air	1388×60	740×60
conditioners	=\$83280	=\$44400
Total cost of	$3000 \times 93840 \div 1000$	3500×93840÷1000
electricity over 4	×4×0.3228	×4×0.3228
years	= \$363498.624	= \$424081.728
Cost of servicing	2×2×2100	4×2100
over 4 years	=\$8400	=\$8400
Total cost over 4	\$455178.624	\$476881.728
years		

Samsing air conditioner model will have a lower total cost after 4 years.

(c) State one assumption you have made in the above calculations.

Answer

 Price of electricity does not change over 4 years No spare parts needed after warranty ends 	
 Aircon usage remains the same for 4 years GST remains the same for 4 years 	• • • • • •
	J
	[1]

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