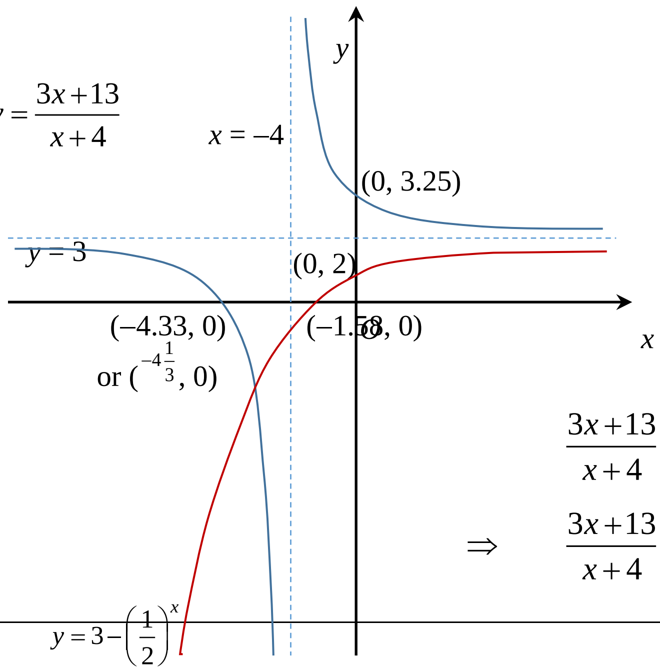
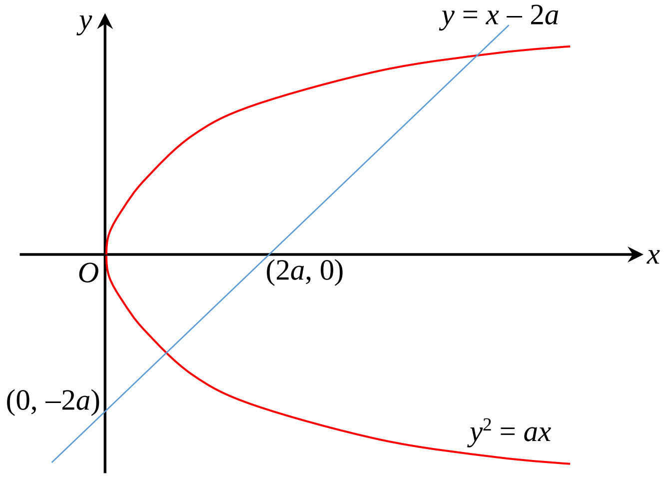
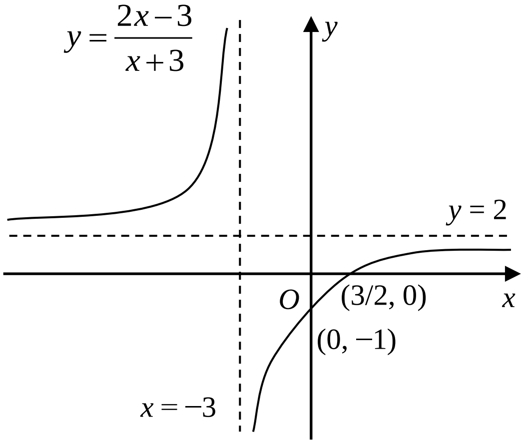
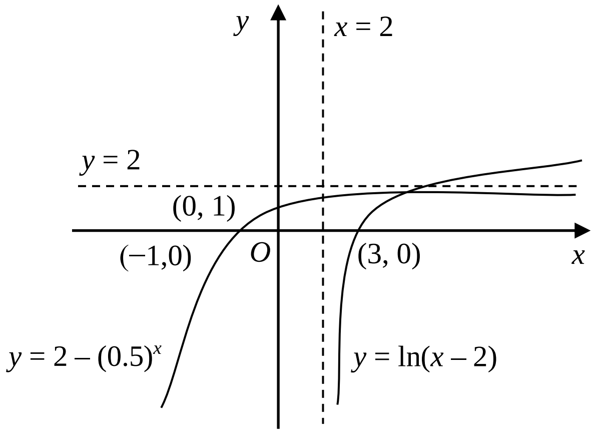
2023 JC1 H1 REVISION SET A
COMPLETE SOLUTIONS for ACJC CA1

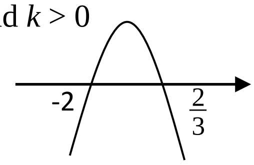
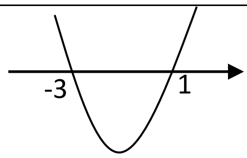
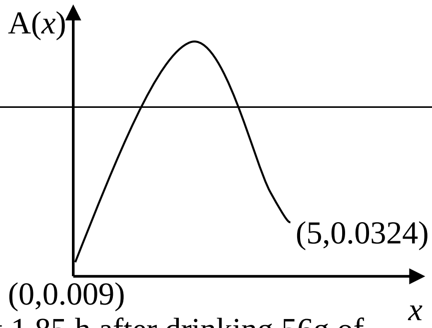
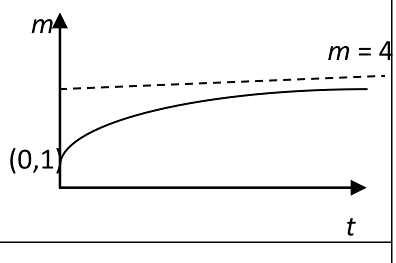
	ACJC 2015 CA1 [24 April 2015]
1	$4x^2 + 3 = 2kx$ has two real distinct roots $\Rightarrow 4x^2 - 2kx + 3 = 0$ has real, distinct roots Discriminant = $(-2k)^2 - 4(4)(3) > 0$ $\Rightarrow\ 4k^2 - 48 > 0 \qquad \Rightarrow\ k^2 - 12 > 0$ $\Rightarrow\ (k - \sqrt{12})(k + \sqrt{12}) > 0$ $\Rightarrow\ k < -\sqrt{12}$ or $k > \sqrt{12}$ (or $k < -2\sqrt{3}$ or $k > 2\sqrt{3}$)
2 (a)	Given $M = \lg\left(\frac{I}{S}\right)$, let intensity in Alaska be I_A. Then intensity in Iceland is $4I_A$. Magnitude in Iceland $= \lg\left(\frac{4I_A}{S}\right) = \lg 4 + \lg\left(\frac{I_A}{S}\right) = \lg 4 + 8.3$ $= 8.9$ (1 decimal place)
2 (b)	In Italy, $7.1 = \lg\left(\frac{I}{S}\right)$ In Alaska, $8.3 = \lg\left(\frac{I_A}{S}\right)$ $\lg\left(\frac{I}{S}\right) - \lg\left(\frac{I_A}{S}\right) = \lg\left(\frac{I}{S} \times \frac{S}{I_A}\right)$ $7.1 - 8.3 = \lg\left(\frac{I}{I_A}\right)$ $-1.2 = \lg\left(\frac{I}{I_A}\right) \Rightarrow 10^{-1.2} = \frac{I}{I_A}$ Ratio is $I : I_A = 10^{-1.2} : 1$ or $1 : 10^{1.2}$ <u>Method 2:</u> In Italy, $7.1 = \lg\left(\frac{I}{S}\right)$ In Alaska, $8.3 = \lg\left(\frac{I_A}{S}\right)$ $I = \left(10^{7.1}\right)S$ $I_A = \left(10^{8.3}\right)S$ $\frac{I}{I_A} = \frac{1}{10^{1.2}}$ $I : I_A = 1 : 10^{1.2}$
3	$y = 3 - \left(\frac{1}{2}\right)^x$ and $y = \frac{3x+13}{x+4} = 3 + \frac{1}{x+4}$  $\frac{3x+13}{x+4} - 3 + \left(\frac{1}{2}\right)^x \geq 0$ $\Rightarrow \frac{3x+13}{x+4} \geq 3 - \left(\frac{1}{2}\right)^x$

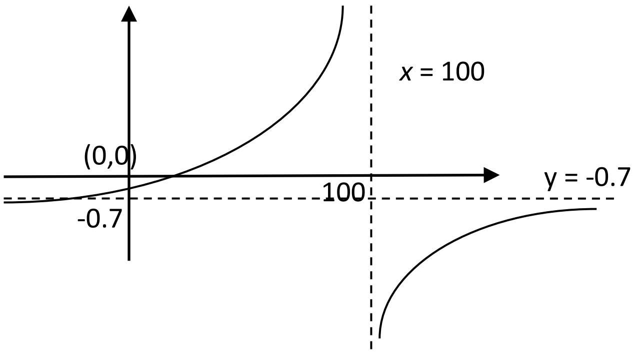
	Intersection at (–4.05 9956, –13.67894). Range of values of x is $x \le -4.06$ or $x > -4$.
4	$y = x - 2a$ and $y^2 = ax$ 
4 (i)	At intersection, $(x-2a)^2 = ax$ $x^2 - 4ax + 4a^2 = ax$ $x^2 - 5ax + 4a^2 = 0$ $(x-a)(x-4a) = 0$ $x = a$ or $x = 4a$ Points of intersection are $(a, -a)$ and $(4a, 2a)$. Length of $AB = \sqrt{(4a-a)^2+(2a+a)^2}$ $= \sqrt{9a^2+9a^2} = \sqrt{18a^2}$ $= a\sqrt{9\times 2} = 3a\sqrt{2}$ $\therefore AB = 3a\sqrt{2}$ units
4 (ii)	Coordinates of C are $(2.5a, 0.5a)$ or $\left(\frac{5}{2}a, \frac{1}{2}a\right)$ Radius of circle is $\frac{1}{2}AB = \frac{1}{2}\left(3a\sqrt{2}\right)$ Equation of circle is $\left(x-\frac{5}{2}a\right)^2 + \left(y-\frac{1}{2}a\right)^2 = \frac{9(2)}{4}a^2$ $\Rightarrow \left(x-\frac{5}{2}a\right)^2 + \left(y-\frac{1}{2}a\right)^2 = \frac{9}{2}a^2$

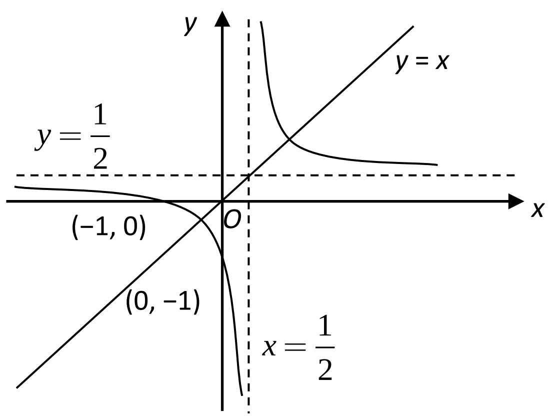
	ACJC 2015 CA1 [24 April 2015]
	or any equivalent form, e.g. $(2x-5a)^2+(2y-a)^2=18a^2$

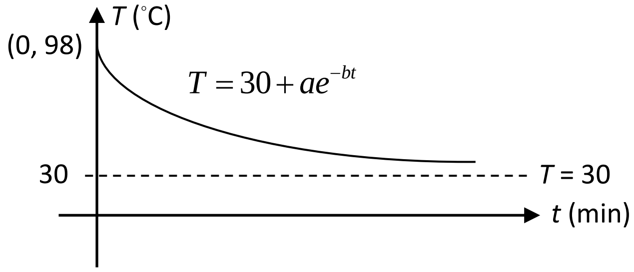
	2016 JC1 H1 Mathematics CA1 [21 April 2016]
1	
2	$e^{2x}+2e^2=3e^{x+1} \quad \Rightarrow \quad u^2+2e^2=3eu$ $\Rightarrow u^2-3eu+2e^2=0 \Rightarrow (u-e)(u-2e)=0$ $\therefore \quad e^x=e \quad \text{or} \quad e^x=2e$ $x=1 \quad \text{or} \quad e^{x-1}=2$ $x=1 \quad \text{or} \quad x-1=\ln 2$ i.e. $x=1+\ln 2$
3	$kx^2+k+3>4x \quad \Rightarrow \quad kx^2-4x+k+3>0$ No intersections with the x-axis so NO real roots and curve has a minimum point $\Rightarrow$ discriminant < 0 and coefficient of $x^2>0$ $D<0 \quad \& \quad k>0$ $(-4)^2-4k(k+3)<0 \Rightarrow 4-k^2-3k<0$ $\Rightarrow k^2+3k-4>0 \Rightarrow (k+4)(k-1)>0$ $\Rightarrow k<-4$ or $k>1 \quad \& \quad k>0$ $\therefore \quad k>1$
4	Let the cost of a ticket in each category be x, y, z. $10x + 4y + 5z = 320$ $9x + 6y + 4z = 352.5$ $7x + 5y + 3z = 282.5$ By GC, $x = 12.50$, $y = 30$, $z = 15$ Total cost for Lim family = 5(12.5) + 10(30) + 5(15) = \$437.50
5(i)	$n=Ae^{1.5t}$ When $t=0$, $n=A$.

5(ii)	When $n = 50A$, $50A = Ae^{1.5t}$ $e^{1.5t} = 50$ $1.5t = \ln 50$ $\therefore \quad t = \frac{2}{3}\ln 50$ or 2.61 (3 s.f.)
6 (i)	
6 (ii)	By GC, $x = 9.38$ (3 s.f.)
6 (iii)	For $2-(0.5)^x < \ln(x-2)$, $x > 9.38$ (3 sf)

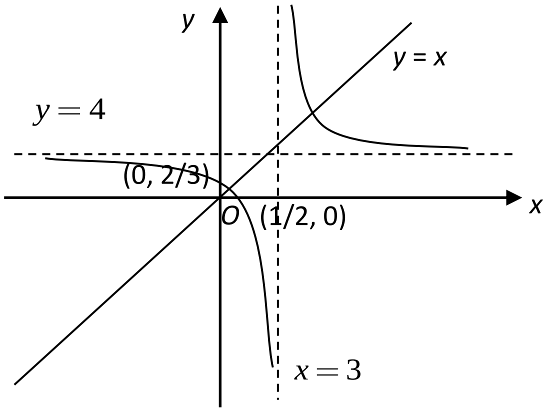
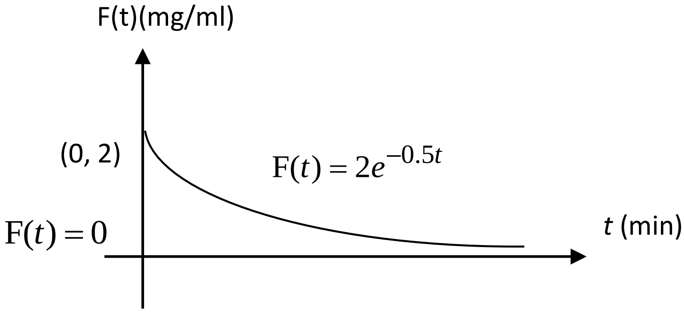
	2017 JC1 H1 Mathematics CA1 [21 April 2017]
1	$kx^2 + (k-2)x + k > 0$ D = discriminant = $(k-2)^2 - 4k^2 < 0$ and $k > 0$ $-3k^2 - 4k + 4 < 0$ and $k > 0$ $(-3k+2)(k+2) < 0$ and $k > 0$ $k < -2$ or $k > \frac{2}{3}$ and $k > 0$ $\Rightarrow k > \frac{2}{3}$ 
2	$(\ln x)^2 + \ln x^2 - 3 \geq 0$ Let $u = \ln x$ $u^2 + 2u - 3 \geq 0$ $(u+3)(u-1) \geq 0$ $u \leq -3$ or $u \geq 1$ $\Rightarrow \ln x \leq -3$ or $\ln x \geq 1$ $\Rightarrow x \leq e^{-3}$ or $x \geq e$ but $x > 0$ $\Rightarrow 0 < x \leq e^{-3}$ or $x \geq e$ 
3	$-2x^2 + 400x = 120x + q$ $-2x^2 + 280x - q = 0$ If company cannot break even then equation has no real roots. D = discriminant = $(280)^2 - 4(-2)(-q) < 0$ $(280)^2 < 8q$ $q > 9800$
4	 (ii) Using GC alcohol content is a maximum at <u>1.85 h</u> after drinking 56g of alcohol (iii) The period in which the 77kg man is legally drunk is $1.11 < x < 2.73$
5	(i) $t \to \infty$, $m \to 4$ Mass of chemical in the long term is 4 g. (ii) m = 2.56 = $(2 - e^{-0.1t})^2$ $2 - e^{-0.1t} = \pm 1.6$ $e^{-0.1t} = 0.4$ or $e^{-0.1t} = 3.6$ -0.1 t = ln 0.4 or $-$ 0.1 t = ln 3.6 t = $-$ 10 ln 0.4 or t = $-$10 ln 3.6 (NA because $t \geq 0$) t = 10 ln (2.5) (iii) When $t = 0$, $m = 1$ Asymptotes $m = 4$. Intersects with the y – axis at (0,1) 
6	(a) $y = \frac{0.7x}{100-x} = -0.7 + \frac{170}{100-x}$ Horizontal asymptote $y = -0.7$ and $x = 100$ When $x = 0$, $y = 0$. Curve passes through (0,0)

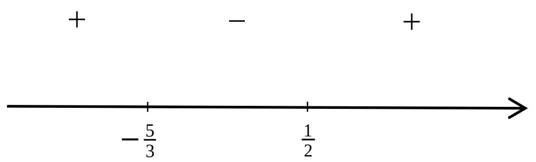
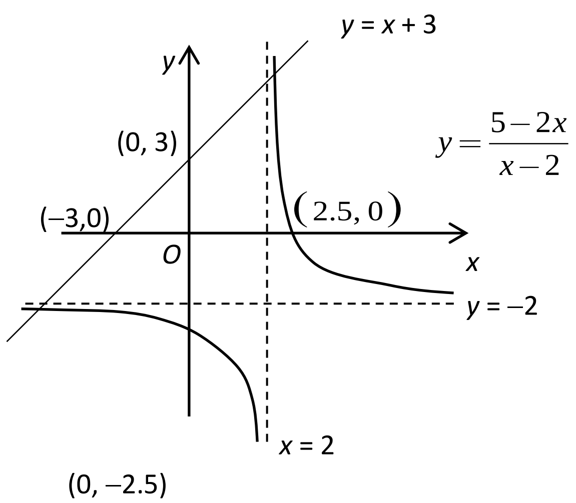
	 (b) No. Because as *x* tends to 100 the cost *y* approaches infinity **OR** No. Because when *x* is 100, *y* is undefined **OR** NO. The cost to remove the pollutant is relatively low at first but skyrocketed as we get closer and closer to removing 100 percent of all the pollutants.

	2018 JC1 H1 Mathematics CA1 [26 April 2018]
1	$3^x - 6\left(3^{-x}\right) = 5$ Let $y = 3^x$, $y - \dfrac{6}{y} = 5$ $y^2 - 5y - 6 = 0$ $(y-6)(y+1) = 0$ $y = 3^x = 6$ or $y = 3^x = -1$ (rejected since $3^x > 0$) $x\ln 3 = \ln 6$ $x = \dfrac{\ln 6}{\ln 3}$
2	$(k+2)x^2 + kx + 5 = x + 4 \Rightarrow (k+2)x^2 + (k-1)x + 1 = 0$ $b^2 - 4ac > 0 \Rightarrow (k-1)^2 - 4(k+2)(1) > 0$ $k^2 - 2k + 1 - 4k - 8 > 0$ $k^2 - 6k - 7 > 0$ $(k-7)(k+1) > 0$ $k < -1$ or $k > 7$ The curve has a minimum point $\Rightarrow k + 2 > 0 \Rightarrow k > -2$ $k > -2$ and $k < -1$ or $k > 7$ Hence $-2 < k < -1$ or $k > 7$
3	Let the number of Chocolate, Strawberry and Vanilla ice cream tubs be *c*, *s*, *v* respectively. $c + s + v = 60$ ----- (1) $16c + 14s + 12v = 860$ ----- (2) $v - s = \dfrac{1}{3}c \Rightarrow c + 3s - 3v = 0$ ----- (3) By GC, $c = 30,\ s = 10,\ v = 20$
	New amount with membership $= 30(0.8\times\$16) + 10(0.9\times\$14) + 20(0.95\times\$12) + \$10 = \$748$ Amount saved $= \$860 - \$748 = \$112$
4 (i)	$y = \dfrac{x+1}{2x-1}$ 
4 (ii)	Suitable graph added is $y = x$.

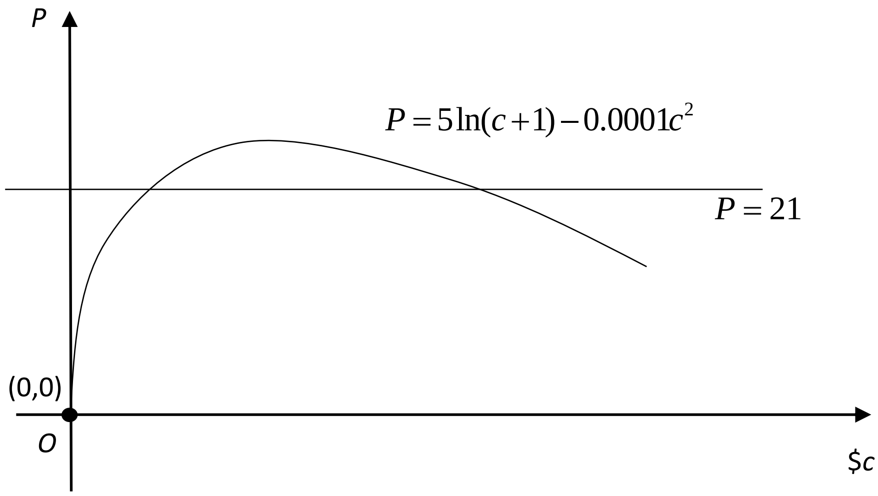
	At points of intersection, $\frac{x+1}{2x-1}=x$ $x+1=x(2x-1)$ $2x^2-2x-1=0$ $x=\frac{2\pm\sqrt{(-2)^2-4(2)(-1)}}{2(2)}=\frac{2\pm\sqrt{12}}{4}=\frac{1\pm\sqrt{3}}{2}$ For $\frac{x+1}{2x-1}\le x$, $\frac{1-\sqrt{3}}{2}\le x<\frac{1}{2}$ or $x\ge\frac{1+\sqrt{3}}{2}$
5 (i)	When t = 0, $98=30+a\Rightarrow a=68$
5 (ii)	$T=63,\ t=12\Rightarrow 63=30+68\mathrm{e}^{-12b}$ $\Rightarrow \mathrm{e}^{-12b}=\frac{33}{68}$ $\Rightarrow b=-\frac{1}{12}\ln\frac{33}{68}$ or 0.0603 (3sf)
5 (iii)	
5 (iv)	The temperature of the pot of soup in the long run is 30°C.

	2019 JC1 H1 Mathematics CA1 [9 May 2019]
1	$2\log_2(x+3)-\log_2(1+x)=3$ $\log_2(x+3)^2-\log_2(1+x)=3$ $\log_2\frac{(x+3)^2}{(1+x)}=3$ $x^2+6x+9=2^3(1+x)$ $x^2+6x+9-8-8x=0$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$
2	$-3x^2+kx-4<0$ $D<0$ $k^2-4(-3)(-4)<0$ $k^2-48<0$ $(k+\sqrt{48})(k-\sqrt{48})<0$ $-\sqrt{48}<k<\sqrt{48}$ Greatest integer value of k = 6
3	Let x, y and z be the unit cost of craft paper, marker and glue stick. $3.21x+4.28y+5.35z=26.75$ --------(1) $5.136x+4.28y+1.712z=26.12$ --------(2) $2.889x+1.926y+0.963z=12.91$ --------(3) OR $3x+4y+5z=\frac{26.75}{1.07}$ --------(1) $6x+5y+2z=\frac{26.12}{1.07\times0.8}$ --------(2) $3x+2y+z=\frac{12.91}{1.07\times0.9}$ --------(3) By GC, $x=\$1.65,\ y=\$3.70,\ z=\$1.05$

4	$y=\frac{a(1-2x)}{x+b}=-2a-\frac{5a}{x+b}$ Asymptotes: $x=-b=3$ $y=-2a=4$ $b=-3$ $a=-2$  $0.298<x<3$ or $x>6.70$
5	$y=mx-3$ --------(1) $y=-x^2+3x-28$ --------(2) Solving (1) & (2) $mx-3=-x^2+3x-28$ $x^2+(m-3)x+25=0$ L is tangential to C, D $=0$ $(m-3)^2-4(1)(25)=0$ $(m-3)^2-100=0$ $[(m-3)-10][(m-3)+10]=0$ $(m+7)(m-13)=0$ $m=-7$ or 13
6	$\text{F}(t)=2e^{-0.5t}$ When t = 0, $\text{F}(0)=2$ mg/ml Initial concentration of drug is 2mg/ml
	$\text{F}(t)=2e^{-0.5t}$ When F(t) = 1, $1=2e^{-0.5t}$ $e^{-0.5t}=0.5$ $-0.5t=\ln 0.5$ $t=1.3863\,hours$ $=83.178$minutes $t=83$minutes 

	2020 JC1 H1 Mathematics Quiz 1 [9 June 2020]
1	$x^2+2kx+k^2>x \Rightarrow x^2+(2k-1)x+k^2>0$ Since $x^2+(2k-1)x+k^2$ is always positive, there are no real roots $\therefore b^2-4ac<0$ $(2k-1)^2-4(1)(k^2)<0$ $4k^2-4k+1-4k^2<0$ $k>\frac{1}{4}$
2	$5-6x^2<7x$ $\Rightarrow \quad 6x^2+7x-5>0$ $\Rightarrow \quad (3x+5)(2x-1)>0$  $\therefore \quad x<-\frac{5}{3}$ or $x>\frac{1}{2}$. Replace x by e^x, Since $e^x>0$ for all x, $e^x<-\frac{5}{3}$ has no solution. $e^x>\frac{1}{2} \quad \Rightarrow \quad x>-\ln 2$.
3	

	$\frac{1}{x-2} > x+5 \Rightarrow \frac{1}{x-2} - 2 > x+3$ $\frac{1-2(x-2)}{x-2} > x+3$ $\frac{5-2x}{x-2} > x+3$ Insert a line $y = x + 3$. The two graphs intersect at $(-5.14, -2.14)$ and $(2.14, 5.14)$ $\therefore x < -5.14$ or $2 < x < 2.14$
4(a)	Let x, y and z be the number of original flavor, chocolate and salted yolk cakes sold per day. $x + y + z = 150$ $150x + 80y + 50z = 15000$ Using GC, $x = \frac{300}{7} + \frac{3}{7}z$ ………………..(1) $y = \frac{750}{7} - \frac{10}{7}z$ ………………(2) Since $z \le 20$, $x = \frac{300}{7} + \frac{3}{7}z$ $\le \frac{300}{7} + \frac{3}{7}(20) = 51.4$ Hence $x = 50$ or 51. But if $x = 50$, from (1), $z = \frac{50}{3}$ (NA) since x, y, z are integers. Therefore $x = 51$. When $x = 51$, from (1), $z = 19$. Sub $z = 19$ into (2): get $y = 80$. ACafe bakes 80 chocolate cakes every day. Total sales from cakes $= (2.8 \times 51) + (3.5 \times 80) + (5 \times 19)$ $= 517.80$

<table>
<tr><td>4(b)</td><td>


(ii) Cost of overheads increase with an increase in the number of bakes, hence daily profit also increases.

(iii) From the graph, maximum daily profit is \$22 80 (nearest 100) when *c* = \$157.6.

(iii) Adding *P* = 21 on the same graph, intersection is at *c* = 73.238 and 261.89.

For profit to be more than \$2100, 73.238 < *c* < 261.89.
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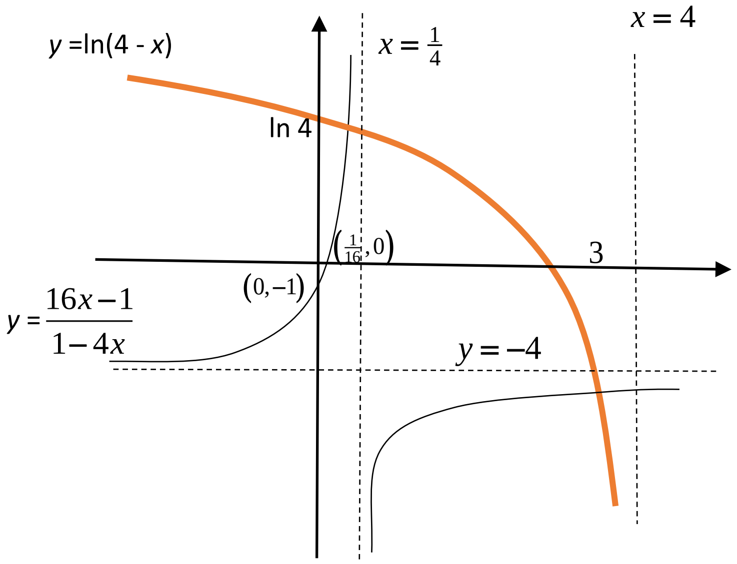
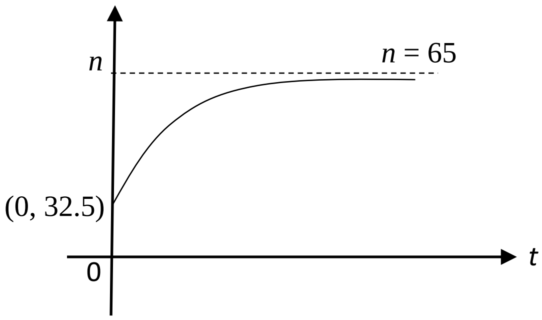
	2021 JC1 H1 Mathematics CA1 [11 May 2021]
1	$y=(k-6)x^2-8x+1$ Since the curve has a min point, $(k-6)>0 \quad \therefore\ k>6$ and Since the curve cuts the *x*-axis at two points, $b^2-4ac>0$ $(-8)^2-4(k-6)(1)>0$ $64-4k+24>0$ $88>4k$ $\therefore 22>k$ Range of values of *k* is $6<k<22$
2	$\ln\left(\frac{e^{2x}-e^x}{6}\right)=0$ $\frac{e^{2x}-e^x}{6}=1$ $e^{2x}-e^x-6=0$ Let $u=e^x$. $u^2-u-6=0$ $(u-3)(u+2)=0$ $u=3$ or $u=-2$ (NA) $e^x=3$ $x=\ln 3$

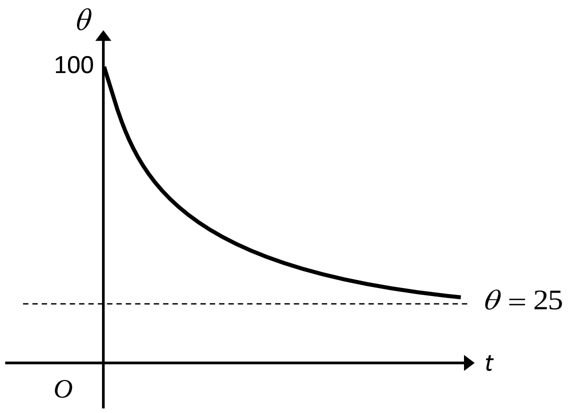
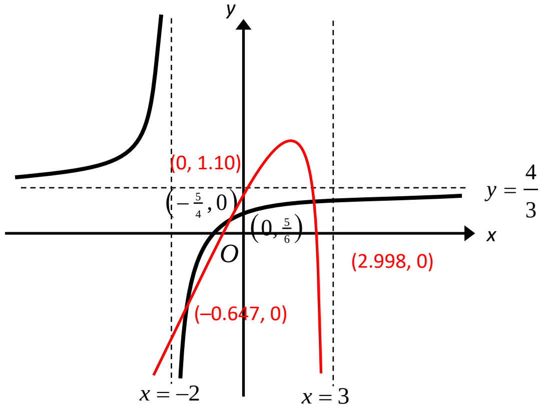
3	Let the price (in \$) for each cup of Peach Tea, Milk Tea and Green Tea be P, M and G respectively. $45P+36M+55G=244.50$ $55P+46M+50G=276.50$ $0.75(85P)+0.8(66M)+0.9(60G)=313.65$ By GC, $P=\$2.20,\quad M=\$1.75,\quad G=\$1.50$
4	(i) $\frac{105}{x}$ (ii) $\frac{105}{x-25}-\frac{105}{x}=2$ multiply throughout by $x(x-25)$: $105x-105(x-25)=2x(x-25)$ $105x-105x+2625=2x^2-50x$ $2x^2-50x-2625=0$ (iii) $2x^2-50x-2625=0$ $x=-25.8$ (rejected since $x>0$), $x=50.824$ Required answer $=\frac{105}{x-25}=\frac{105}{50.824-25}=4.066=4$ (to the nearest minute)
5(i)	$y=\frac{16x-1}{1-4x}=-4+\frac{3}{1-4x}$ Asymptotes: $y=-4$ and $x=\frac{1}{4}$ When $x=0$, $y=-1$ When $y=0$, $x=\frac{1}{16}$ Axial intercepts : $(0,-1)$ and  Equation of graph C_2 is $y=\ln(4-x)$

	For $y=\ln(4-x)$, asymptote $x=4$ When $x=0,\ y=\ln 4$ When $y=0,\ 4-x=1$ i.e $x=3$ Axial intercepts: (0,ln4) and (3,0) From GC the solutions of the equation $\ln(4-x)=\dfrac{16x-1}{1-4x}$ are 0.110 and 3.99
6(i) **(ii)**	When $t=0$, $n=\dfrac{65}{(e^0+1)}=\dfrac{65}{2}=32.5$ thousands = 32500 y- intercept (0, 32.5) 
(iii) **(iv)**	$t\to\infty$, $e^{-0.5t}\to 0$ and $n\to 65$ thousands For large values of t, approximate size of population is 65000 $n=\dfrac{65}{e^{-0.5t}+1}>64$ **<u>Method 1:</u>** Using GC When $n=64,\ t=8.31777$ Least number of days for the size of the population to first exceed 64000 is 9 days **<u>Method 2:</u>** $(e^{-0.5t}+1)<\dfrac{65}{64}$ $e^{-0.5t}<\dfrac{1}{64}$ $-0.5t<\ln\dfrac{1}{64}$ $t>8.31777$ Least number of days for the size of the population to first exceed 64000 is 9 days.

2022 H1 CA1 solutions for students

Qn	Solutions
1	Let the price of set meals A, B and C be a, b and c respectively. $12a+10b+8c=616$ $3(12a)-2(8c)=248$ $a+b+c=63$ From GC, $a=18,\ b=20,\ c=25$ The price of set meal A is \$18.
2	For point of intersection between the line and the curve,

	$3x-4k=kx^2+kx \Rightarrow kx^2+(k-3)x+4k=0$ Since the line does not intersect the curve, $kx^2+(k-3)x+4k=0$ has no real roots. $(k-3)^2-4k(4k)<0$ $k^2-6k+9-16k^2<0$ $5k^2+2k-3>0$ $(5k-3)(k+1)>0$ $k<-1$ or $k>\frac{3}{5}$
	$kx^2+kx>3x-4k \Rightarrow kx^2+(k-3)x+4k>0$ $D<0$ & $k>0$ Hence $k>\frac{3}{5}$.
3(i)	$\theta=25+Ae^{-kt}$ When $t=0, \theta=100 \Rightarrow 100=25+Ae^0 \Rightarrow A=75$.
(ii)	When $t=10, \theta=50$, $\therefore 50=25+75e^{-k(10)}$ $\Rightarrow \frac{25}{75}=e^{-10k}$ $\Rightarrow \ln\left(\frac{1}{3}\right)=-10k$ $\Rightarrow k=-\frac{1}{10}\ln\left(\frac{1}{3}\right) \quad \left(\text{or } \frac{1}{10}\ln 3\right)$

(iii)	
4(i)	$x=-2$ is vertical asymptote implies that denominator is 0 when $x=-2$ $\therefore a(-2)+b=0 \Rightarrow b=2a$. Alternatively, Asymptote: $ax+b=0 \Rightarrow x=-\dfrac{b}{a}=-2 \Rightarrow b=2a$ $\therefore y=\dfrac{4x+5}{ax+2a}$ Sub in $x=1, y=1$ $1=\dfrac{4+5}{a+2a} \Rightarrow 3a=9 \Rightarrow a=3$.
(ii) (iii)	$y=\dfrac{4x+5}{3x+6}$ 
(iv)	$4x+5=2x(3x+6)+(3x+6)\ln(3-x)$ $\dfrac{4x+5}{3x+6}=2x+\ln(3-x)$ From GC, $x=2.992$