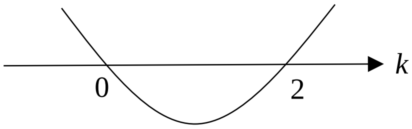
2022 J2 H1 Mathematics Prelim Exam Solutions with Comments

Section A: Pure Mathematics [40 marks]

1 **(a)** Find the set of values of k for which the graphs of $y = 2x^2 - kx + k$ and $y = kx$ intersect at most once. [4]

Qn	Solutions	Comments
1(a)	Equate the equations of the two graphs, $2x^2 - kx + k = kx$ $2x^2 - 2kx + k = 0$ Since the graphs intersect at most once, $(-2k)^2 - 4(2)(k) \le 0$ $4k^2 - 8k \le 0$ $4k(k-2) \le 0$  $0 \le k \le 2$ $\{k \in \mathbb{R} : 0 \le k \le 2\}$	

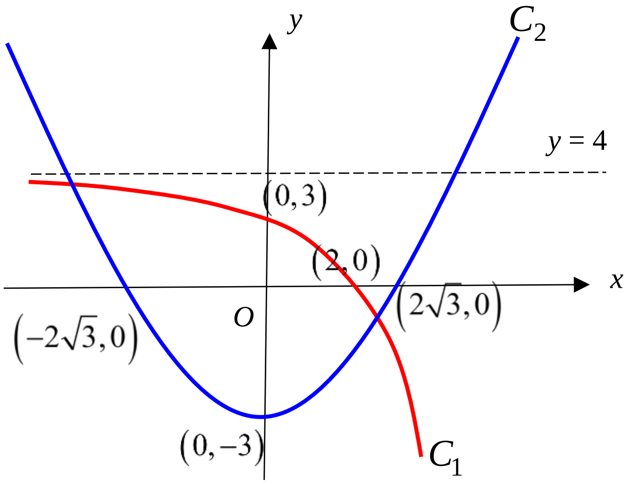
1 **(b)** MARS Café serves three types of coffee: Flat, Cappuccino and Macchiato. On 23 December 2021, a customer paid \$17.30 for two cups of Flat, one cup of Cappuccino and three cups of Macchiato after using a \$10 cash voucher. On the same day, the Café owner recorded a total sale of \$1151.30 from selling 84 cups of Flat, 123 cups of Cappuccino and 65 cups of Macchiato. The total price of three cups of Macchiato is equivalent to the total price of five cups of Flat.

Express this information as 3 linear equations and hence find the price of one cup of Macchiato. [4]

Qn	**Solutions**	**Comments**
1(b)	Let \$*F*, \$*C* and \$*M* represent the cost of one cup of Flat, Cappuccino and Macchiato, respectively. $2F+C+3M=27.30$ (1) $84F+123C+65M=1151.30$ (2) $-5F+3M=0$ (3) Using the GC, $F=3.30$ $C=4.20$ $M=5.50$ The cost of one cup of Macchiato is \$5.50.	

2 The curve C_1 has equation $y = 4 - 2^x$. The curve C_2 has equation $y = \frac{x^2}{4} - 3$.

(i) Sketch C_1 and C_2 on the same diagram, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]

Qn	Solutions	Comments
2(i)		

(ii) Find the x-coordinates of the points of intersection of C_1 and C_2, giving your answers correct to 4 decimal places. [1]

Qn	Solutions	Comments
2 (ii)	Using GC, $x = -5.2818$ or 2.4570 (4 decimal places)	

(iii) Write down as an integral an expression for the area of the region bounded by the curves C_1 and C_2, the line $x = -5$ and the y-axis. Evaluate this integral, giving your answer correct to 3 decimal places. [2]

Qn	Solutions	Comments
2 (iii)	$\int_{-5}^{0} (4-2^x) - \left(\frac{x^2}{4} - 3\right) \mathrm{d}x = 23.186$ (3 decimal places)	

3 **(i)** Differentiate $\ln\left(5x\mathrm{e}^{x^2-3}\right)$. [3]

Qn	Solutions	Comments
3(i)	$\ln\left(5x\mathrm{e}^{x^2-3}\right)=\ln 5+\ln x+\ln \mathrm{e}^{x^2-3}$ $=\ln 5+\ln x+x^2-3$ $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(5x\mathrm{e}^{x^2-3}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln 5+\ln x+x^2-3\right)$ $=\frac{1}{x}+2x$	

(ii) Integrate $\left(\frac{1}{2\sqrt{x}}+x\right)^2$. [3]

Qn	Solutions	Comments
3 (ii)	$\int\left(\frac{1}{2\sqrt{x}}+x\right)^2\,\mathrm{d}x=\int\frac{1}{4x}+2\left(\frac{1}{2\sqrt{x}}\right)x+x^2\ \mathrm{d}x$ $=\int\frac{1}{4x}+\sqrt{x}+x^2\ \mathrm{d}x$ $=\frac{1}{4}\ln x+\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+\frac{x^3}{3}+C$ $=\frac{1}{4}\ln x+\frac{2}{3}x^{\frac{3}{2}}+\frac{1}{3}x^3+C$	

4 A curve has equation $y = 5 + x + 3x^2 - x^3$.

(i) The point *P* is on the curve such that the *x*-coordinate of *P* is positive. The tangent to the curve at *P* is parallel to the line $y = -8x + 3$. Find the equation of the tangent at *P*, giving your answer in the form $ax + by + c = 0$ where *a*, *b* and *c* are integers. [4]

Qn	Solutions	Comments
4(i)	$\frac{dy}{dx} = 1 + 6x - 3x^2$ $\frac{dy}{dx} = -8$ $1 + 6x - 3x^2 = -8$ $3x^2 - 6x - 9 = 0$ $x^2 - 2x - 3 = 0$ $(x+1)(x-3) = 0$ $x = -1$ or $x = 3$ (N.A. $x > 0$) When $x = 3$, $y = 5 + 3 + 3(3^2) - 3^3$ $= 8$ Equation of tangent at the point *P* is $y - 8 = -8(x - 3)$ $y = -8x + 32$ $8x + y - 32 = 0$	

(ii) The tangent at *P* meets the *x*-axis at point *T*. Find the area of triangle *PTO*, where *O* is the origin. [2]

Qn	Solutions	Comments
4 (ii)	When $y = 0$, $0 = -8x + 32$ $8x = 32$ $x = 4$ Area of triangle *POT* = $\frac{1}{2} \times 4 \times 8 = 16$	

5 A fish farm which supplies sea bass to restaurants in Singapore started operations in January 2001. Based on an expert model, the population of sea bass in the fish farm, P thousands, at time t years is given by

$$P = N - 15e^{-0.2t}, \text{ where } N \text{ is a positive real number.}$$

The fish farm had a population of 15000 sea bass when it started operations.

(i) Show that $N = 30$. [1]

Qn	**Solutions**	**Comments**
5(i)	When $t = 0$, $P = 15$, $N - 15e^{-0.2(0)} = 15$ $N - 15(1) = 15$ $N = 30$	

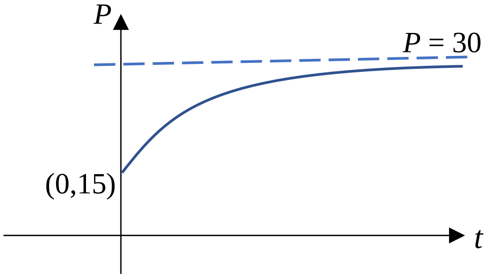
(ii) Find $\dfrac{dP}{dt}$. [2]

Qn	**Solutions**	**Comments**
5 (ii)	$\dfrac{dP}{dt} = -15(-0.2)e^{-0.2t} = 3e^{-0.2t}$	

(iii) Using the expression for $\dfrac{dP}{dt}$, explain why the sea bass population in the fish farm will keep increasing. [1]

Qn	**Solutions**	**Comments**
5 (iii)	$\dfrac{dP}{dt} = 3e^{-0.2t}$ Since $e^{-0.2t} > 0$ for all values of t. $\dfrac{dP}{dt}$ is always positive. Therefore, population of sea bass in the fish farm is always increasing.	

(iv) Sketch the graph of P against t. [2]

Qn	Solutions	Comments
5 (iv)		

(v) The fish farm is designed to have a maximum capacity of 25000 fishes. Determine the year in which the fish farm is estimated to reach its maximum capacity. [1]

Qn	Solutions	Comments
5(v)	Using G.C to solve $30-15\mathrm{e}^{-0.2t}=25,\ t=5.4930$ The fish farm will reach maximum capacity in the year **2006** **Alternative,** $30-15e^{-0.2t}\leq 25$ $t\leq 5\ln 3$ $t\leq 5.49306$ The fish farm will reach maximum capacity in the year **2006.**	

The owner of the fish farm is also interested in modelling his revenue, R thousand dollars per year, at any point in time. The model he uses is

$$R=\frac{t^3}{3}-\frac{9t^2}{2}+14t+15, \quad \text{for } 0\le t\le 5.$$

(vi) Use differentiation to find the stationary value of R and justify whether this value is a minimum or maximum. [4]

<table>
<tr><th>Qn</th><th>Solutions</th><th>Comments</th></tr>
<tr><td>5
(vi)</td><td>

$R=\frac{t^3}{3}-\frac{9t^2}{2}+14t+15$

$\frac{dR}{dt}=t^2-9t+14$

$=(t-2)(t-7)$

For stationary points, $\frac{dR}{dt}=0$,

$(t-2)(t-7)=0$

$t=2$ or 7 (rejected since $0\le t\le 5$)

By first derivative test,

<table>
<tr><td>x</td><td>1.99</td><td>2</td><td>2.01</td></tr>
<tr><td>$\frac{dR}{dt}$</td><td>0.0501</td><td>0</td><td>–0.0499</td></tr>
</table>

The stationary value of R is maximum at t = 2.

Alternative

By second derivative test,

$\frac{d^2R}{dt^2}=2t-9$

When t = 2, $\frac{d^2R}{dt^2}=-5<0$

The stationary value of R is maximum at t = 2.

When t = 2,

$R=\frac{2^3}{3}-\frac{9(2^2)}{2}+14(2)+15=\frac{83}{3}=27.6667\approx 27.7$ (3 s.f.)

Therefore, the stationary value of R = 27.7 is maximum when t = 2.

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</table>

(vii) Use your calculator to find the value of $\int_0^5 \left(\frac{t^3}{3} - \frac{9t^2}{2} + 14t + 15 \right) \mathrm{d}t$. In the context of the question, what does this value represent? [2]

Qn	**Solutions**	**Comments**
5 **(iv)**	$\int_0^5 \left(\frac{t^3}{3} - \frac{9t^2}{2} + 14t + 15 \right) \mathrm{d}t = 114.58 \approx 115$ The total revenue of the fish farm owner at the end of 5 years after the fish farm started operations in January 2001 is 115 thousand dollars.	

Section B: Probability and Statistics [60 marks]

6 A and B are events such that $\mathrm{P}(A \mid B) = 0.3$, $\mathrm{P}(B \mid A) = 0.6$ and $\mathrm{P}(A \cup B) = 0.72$.

(i) Find $\mathrm{P}(A \cap B)$. [3]

Qn	Solutions	Comments
6(i)	$\mathrm{P}(A\mid B)\mathrm{P}(B) = \mathrm{P}(A\cap B)$ $0.3\mathrm{P}(B) = \mathrm{P}(A\cap B)$ $\mathrm{P}(B) = \dfrac{\mathrm{P}(A\cap B)}{0.3} \qquad (1)$ $\mathrm{P}(B\mid A)\mathrm{P}(A) = \mathrm{P}(A\cap B)$ $0.6\mathrm{P}(A) = \mathrm{P}(A\cap B)$ $\mathrm{P}(A) = \dfrac{\mathrm{P}(A\cap B)}{0.6} \qquad (2)$ $\mathrm{P}(A\cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A\cap B)$ $0.72 = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A\cap B) \qquad (3)$ Sub (1) and (2) into (3), $0.72 = \dfrac{\mathrm{P}(A\cap B)}{0.6} + \dfrac{\mathrm{P}(A\cap B)}{0.3} - \mathrm{P}(A\cap B)$ $0.432 = \mathrm{P}(A\cap B) + 2\mathrm{P}(A\cap B) - 0.6\mathrm{P}(A\cap B)$ $0.432 = 2.4\mathrm{P}(A\cap B)$ $\mathrm{P}(A\cap B) = 0.18$	

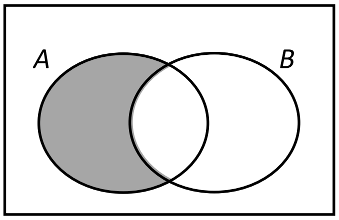
(ii) Determine whether the events A and B are independent. [2]

Qn	Solutions	Comments
6 (ii)	$\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)} = 0.6$ $\mathrm{P}(A) = \dfrac{0.18}{0.6} = \dfrac{3}{10}$ Since P(A) = P(A|B), events A and B are independent. **Alternative**	

	$\frac{P(A \cap B)}{P(A)} = 0.6$ $P(A) = \frac{0.18}{0.6} = \frac{3}{10}$ $\frac{P(A \cap B)}{P(B)} = 0.3$ $P(B) = \frac{0.18}{0.3} = \frac{6}{10} = \frac{3}{5}$ $P(A) \cdot P(B) = \frac{3}{10} \times \frac{3}{5}$ $P(A \cap B) = 0.18$ $P(A) \cdot P(B) = P(A \cap B)$ Therefore, events *A* and *B* are independent events.	

(iii) Describe in words what is meant by the event represented by the shaded region in the Venn diagram and find its probability.

[2]

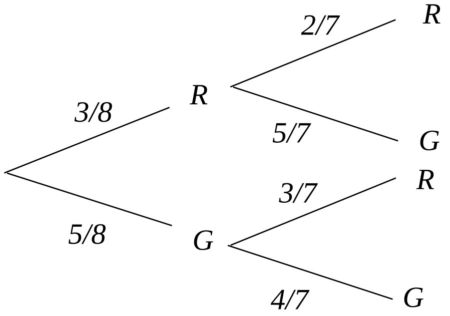


Qn	Solutions	Comments
6 (iii)	Event *A* occurs but not event *B*. $\text{Required probability} = P(A \cap B')$ $= P(A) - P(A \cap B)$ $= 0.3 - 0.18$ $= 0.12$	

7 Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar at random, without replacement.

Find the probability that

(i) none of the marbles are green, [1]

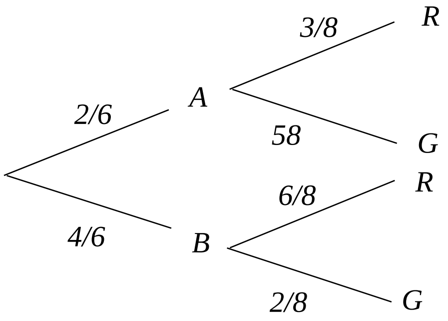
Qn	Solutions	Comments
7 **(i)**	 P(none of the marbles are green) = $\frac{3}{8}\times\frac{2}{7}=\frac{3}{28}=0.10714\approx 0.107(3\text{ sf})$ **Alternative** P(none of the marbles are green) = $\frac{{}^3C_2}{{}^8C_2}=\frac{3}{28}=0.10714\approx 0.107(3\text{ s.f.})$	

(ii) at least one of the marbles is green. [2]

Qn	Solutions	Comments
7 **(ii)**	P(at least one of the marbles is green) = $=1-\mathrm{P}(RR)$ $=1-\frac{3}{8}\cdot\frac{2}{7}=\frac{25}{28}$ $=0.89286\approx 0.893(3\text{ sf})$ **Alternative,** P(at least one of the marbles is green) = $=\mathrm{P}(GG)+\mathrm{P}(GR)+\mathrm{P}(RG)$ $=\frac{5}{8}\cdot\frac{4}{7}+\frac{5}{8}\cdot\frac{3}{7}+\frac{3}{8}\cdot\frac{5}{7}=\frac{25}{28}=0.89286\approx 0.893(3\text{ sf})$	

All the marbles are put back into jar A. Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

(iii) Find the probability that a green marble is drawn. [2]

Qn	Solutions	Comments
7 (iii)	 P(a green marble is drawn) = $\frac{2}{6}\cdot\frac{5}{8}+\frac{4}{6}\cdot\frac{2}{8}=\frac{3}{8}=0.375$	

(iv) Given that a red marble is drawn, find the probability that it was drawn from jar A. [2]

Qn	Solutions	Comments
7(iv)	$=\frac{\mathrm{P}\left(\text{a red marble is drawn from jar A}\right)}{\mathrm{P}\left(\text{A red marble is drawn}\right)}$ $=\frac{\mathrm{P}\left(\text{a red marble is drawn from jar A}\right)}{1-\mathrm{P}\left(\text{no red marbles drawn}\right)}$ $=\frac{\frac{1}{3}\cdot\frac{3}{8}}{1-\frac{3}{8}}$ $=\frac{1}{5}$ **Alternative,**	

	$\mathrm{P}\left(\text{drawn from Jar A} \mid \text{red marble is drawn}\right)$ $= \dfrac{\mathrm{P}\left(\text{a red marble is drawn from jar A}\right)}{\mathrm{P}\left(\text{a red marble is drawn}\right)}$ $== \dfrac{\mathrm{P}\left(\text{a red marble is drawn from jar A}\right)}{\mathrm{P}\left(\text{B, red}\right)+\mathrm{P}\left(\text{A , red}\right)}$ $= \dfrac{\frac{1}{3}\cdot\frac{3}{8}}{\frac{1}{3}\cdot\frac{3}{8}+\frac{2}{3}\cdot\frac{6}{8}}$ $= \dfrac{1}{5}$	

8 **(a)** Different arrangements of all the letters of the word COPYRIGHTABLE are formed.

(i) Find the number of different arrangements that can be formed consisting of the words COPY and TABLE. [1]

Qn	Solutions	Comments
8(a) (i)	Number of different arrangements $= 6! = 720$	

(ii) Find the number of different arrangements that can be formed such that no vowel is next to another vowel. [2]

Qn	Solutions	Comments
8(a) (ii)	Number of different arrangements $= 9! \times {}^{10}C_4 \times 4! = 1\,828\,915\,200.$ **Alternative** Number of different arrangements $= 9! \times {}^{10}P_4 = 1\,828\,915\,200.$	

(b) Passwords are formed using 5 letters from the word COPYRIGHTABLE. Find the number of different passwords that can be formed with at least 3 vowels. [3]

Qn	Solutions	Comments
8 (b)	Number of different passwords $= {}^4C_3 \times {}^9C_2 \times 5! + {}^4C_4 \times {}^9C_1 \times 5! = 18\,360.$	

9 A technician repairs two types of electronic devices, laptops and tablets. The times taken by the technician to repair laptops are normally distributed with mean 2 hours and standard deviation 0.55 hours. The times taken by the technician to repair tablets are normally distributed with mean 1.5 hours and standard deviation 0.3 hours. The times taken to repair the electronic devices are independent of each other.

(i) Find the probability that the time taken by the technician to repair a laptop is more than 2.5 hours. [1]

Qn	**Solutions**	**Comments**
9 (i)	Let X be the time taken by the technician to repair a laptop. $X \sim \mathrm{N}\left(2, 0.55^2\right)$ $\mathrm{P}\left(X > 2.5\right) = 0.18165 \approx 0.182$	

(ii) Find the probability that the time taken by the technician to repair a laptop exceeds the total time taken to repair two tablets. [3]

Qn	**Solutions**	**Comments**
9 (ii)	Let Y be the time taken by the technician to repair a tablet. $Y \sim \mathrm{N}\left(1.5, 0.3^2\right)$ $\mathrm{E}\left(X - Y_1 - Y_2\right) = 2 - 1.5 - 1.5 = -1$ $\mathrm{Var}\left(X - Y_1 - Y_2\right) = 0.55^2 + 0.3^2 + 0.3^2 = 0.4825$ $X - Y_1 - Y_2 \sim \mathrm{N}\left(-1, 0.4825\right)$ $\mathrm{P}\left(X - Y_1 - Y_2 > 0\right) = 0.0750$ (3 sig. fig.)	

On any workday, the technician will repair two laptops and three tablets. The technician earns \$50 per hour for repairing a laptop and \$35 per hour for repairing a tablet.

(iii) Find the probability that the total earnings of the technician on a workday is between \$300 and \$400. [4]

Qn	**Solutions**	**Comments**
9 (iii)	Let S be the total earnings of the technician for repairing 2 laptops and 3 tablets.	

	$S=50\left(X_1+X_2\right)+35\left(Y_1+Y_2+Y_3\right)$ $\mathrm{E}\left(S\right)=\left(50\right)\left(2\right)\left(2\right)+\left(35\right)\left(3\right)\left(1.5\right)=357.5$ $\mathrm{Var}\left(S\right)=\left(50^2\right)\left(2\right)\left(0.55^2\right)+\left(35^2\right)\left(3\right)\left(0.3^2\right)=1843.25$ $S\sim \mathrm{N}\left(357.5,1843.25\right)$ $\mathrm{P}\left(300<S<400\right)=0.74865\approx 0.749$	

The technician repairs another type of electronic device, mobile phones. The times taken by the technician to repair mobile phones are recorded. The records show that 25% of the mobile phones take more than 1.5 hours to be repaired and 10% of the mobile phones take less than 0.5 hours to be repaired.

(iv) Assuming that the times taken by the technician to repair mobile phones are normally distributed, find the mean and variance of the distribution. [4]

Qn	Solutions	Comments
9 **(iv)**	Let *M* be the time taken by the technician to repair a mobile phone. $M\sim \mathrm{N}\left(\mu,\sigma^2\right)$ $\mathrm{P}\left(M>1.5\right)=0.25$ $\mathrm{P}\left(Z>\frac{1.5-\mu}{\sigma}\right)=0.25$ $\frac{1.5-\mu}{\sigma}=0.67449$ $1.5=0.67449\sigma+\mu \qquad (1)$ $\mathrm{P}\left(M<0.5\right)=0.1$ $\mathrm{P}\left(Z<\frac{0.5-\mu}{\sigma}\right)=0.1$ $\frac{0.5-\mu}{\sigma}=-1.2816$ $0.5=-1.2816\sigma+\mu \qquad (2)$ Using GC to solve (1) and (2), $\sigma=0.51122\approx 0.511$ $\mu=1.1552\approx 1.16$ The mean of the distribution is 1.16 hours and the variance is 0.261 hours^2.	.

10 A confectionary produces gummy candies in different colours. The probability that any randomly chosen gummy candy is red is 0.6. The gummy candies are sold in packets of 24. The number of red gummy candies in a packet is the random variable *X*.

(i) Find the probability that, in a randomly selected packet of gummy candies, at most half of the gummy candies are red. [1]

Qn	Solutions	Comments
10 **(i)**	Let *X* be the number of red gummy candies, in a packet of 24. $X \sim \mathrm{B}(24, 0.6)$ $\mathrm{P}(X \le 12) = 0.21302 \approx 0.213$ (3 s.f.)	

(ii) Find the probability that the number of red gummy candies in a packet is more than the expected value of *X*. [3]

Qn	Solutions	Comments
10 **(ii)**	Expected value of *X* $= (0.6)(24) = 14.4$ $\mathrm{P}(X > 14.4) = 1 - \mathrm{P}(X \le 14)$ $= 0.48908$ ≈ 0.489	

A packet of gummy candies is accepted if at most half of the gummy candies are red.

Two packets of gummy candies are selected one after another.

(iii) Find the probability that the first packet of gummy candies selected is accepted and the second packet of gummy candies selected is not accepted. [2]

Qn	Solutions	Comments
10 **(iii)**	Required probability $= (0.21302)(1 - 0.21302) = 0.16764 \approx 0.168(3 \text{ s.f.})$	

A customer buys 12 packets of gummy candies.

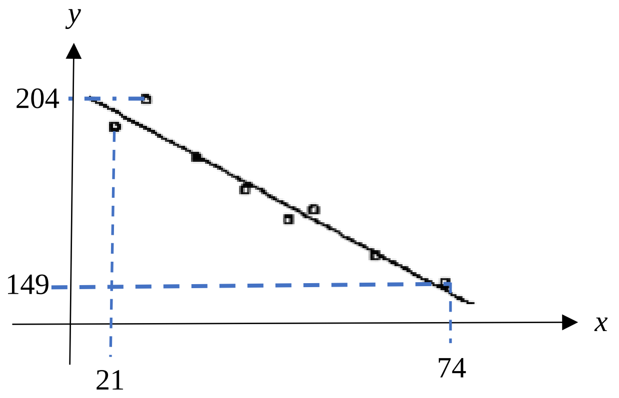
(iv) Find the probability that, for exactly 4 of these packets, at most half of the gummy candies are red. [2]

Qn	**Solutions**	**Comments**
10 (iv)	Let Y be the number of packets of gummy candies, out of 12 packets, where at most half of the gummy candies are red. $Y \sim \mathrm{B}(12, 0.21302)$ $\mathrm{P}(X = 4) = 0.14997 \approx 0.150 (3\text{ s.f.})$	

11 The following table shows the age, x years, and the maximum heart rate, y bpm (beats per minute), for a random sample of 8 adult males.

Adult male	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
x	63	34	21	49	26	53	74	42
y	157	187	196	168	204	171	149	177

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

Qn	Solutions	Comments
11 **(i)**		

(ii) Find the product moment correlation coefficient and comment on its value. Interpret this value in the context of the data. [3]

Qn	Solutions	Comments
11 **(ii)**	$r = -0.97356 = -0.974$ As $r \approx -0.947$ is close to -1 and the points in the scatter diagram seem to lie close to a straight line with negative gradient, thus indicating a **strong negative linear relationship** between the age of adult males (x) and the maximum heart rate (y). This means that as x increases, y tends to decrease at a **constant rate.**	

(iii) Find the equation of the regression line of y on x, in the form $y = mx + c$, giving the values of m and c correct to 3 significant figures. Sketch this line on your scatter diagram. [2]

Qn	Solutions	Comments
11 **(iii)**	$y = -1.0094x + 221.80$ $y = -1.01x + 222$	

(iv) Calculate an estimate of the maximum heart rate of a randomly chosen 61-year-old adult male. [1]

Qn	Solutions	Comments
11 **(iv)**	When x = 61, y = 160.23 ≈ 160 The estimated maximum heart rate of a randomly chosen 61-year-old adult male is 160.	

(v) Explain why you would expect this estimate to be reliable. [1]

Qn	Solutions	Comments
11 **(v)**	The scatter plot diagram shows that the linear model is an appropriate fit, r is close to –1 and $x = 61$ lies in the data range $21 \leq x \leq 74$, therefore the estimate for $x = 61$ is reliable.	

12 A manufacturer states that a jar of jam that he produces has a mean mass of 200 grams. A food standards inspector wishes to investigate whether the mean mass of these jars of jam is actually less than 200 grams. A random sample of 70 jars of jam is taken. The masses of the jars of jam, *x*, in grams, are summarised by

$$\sum x = 13950, \qquad \sum x^2 = 2781000.$$

(i) Determine the conclusion of the investigation if the food standards inspector carries out a test at the 5% significance level. [7]

Qn	**Solutions**	**Comments**
12 (i)	Let μ be the population mean mass of the jar of jam, in grams. $H_0: \mu = 200$ $H_1: \mu < 200$ Level of significance: 5% Test Statistics: Since n = 70 is large, by Central Limit Theorem, $\bar{X}$ is approximately normally distributed. When H_0 is true, $Z = \dfrac{\bar{X} - 200}{\frac{s}{\sqrt{n}}} \sim N(0,1)$ approximately Computations: $n = 70$ $\bar{x} = \dfrac{\sum x}{n} = \dfrac{13950}{70} = 199.29 \approx 199$ $s^2 = \dfrac{1}{n-1}\left(\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}\right)$ $= \dfrac{1}{70-1}\left(2781000 - \dfrac{13950^2}{70}\right) = 13.975 \approx 14.0$ p-value = $0.056027 \approx 0.0560$ Conclusion: Since p-value $\approx 0.0560 > 0.05$, H_0 is not rejected at the 5% level of significance.	

	There is insufficient evidence to conclude that the mean mass of the jars of jam is less than 200 grams. The food standards inspector will conclude that the mass of the jars of jam is not less than 200 grams.	

The manufacturer now produces a new recipe of jam and finds that the population standard deviation is 15.3 grams.

A new random sample of 70 jars of jam is taken and the mean mass of the jars of jam in this sample is *m* grams. A test at the 10% significance level shows that there is sufficient evidence to suggest that the population mean mass of the jars of jam differs from 200 grams.

(ii) Find the range of possible values of *m*. [4]

Qn	Solutions	Comments
12 (ii)	$H_0: \mu = 200$ $H_1: \mu \neq 200$ Level of significance: 10% Test Statistics: Since *n* = 70 is large, by Central Limit Theorem, the same mean mass is approximately Normally Distributed. When H_0 is true, $Z = \dfrac{\bar{X} - 200}{\frac{15.3}{\sqrt{n}}} \sim \mathrm{N}(0,1)$ approximately Since a test at the 10% significance level shows that there is sufficient evidence to suggest that the population mean mass of the jars of jam differ from 200 grams. H_0 is rejected at the 10% level of significance. $z \leq -1.64485$ or $z \geq 1.64485$ $\dfrac{m-200}{\frac{15.3}{\sqrt{70}}} \leq -1.64485$ or $\dfrac{m-200}{\frac{15.3}{\sqrt{70}}} \geq 1.64485$ $m \leq 196.99$ or $m \geq 203.01$ $m \leq 197$ or $m \geq 203$ (3 sig. fig.)	