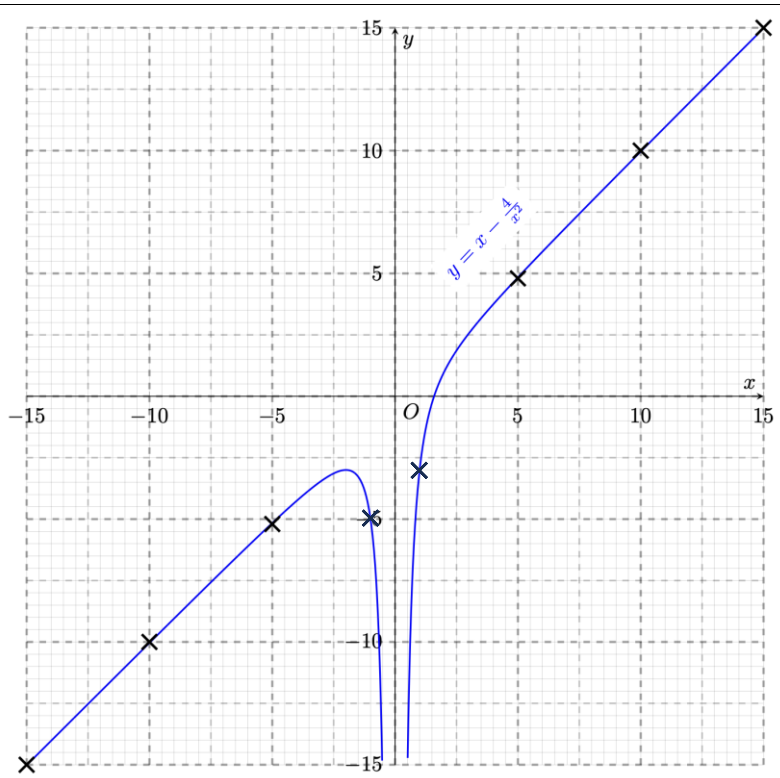
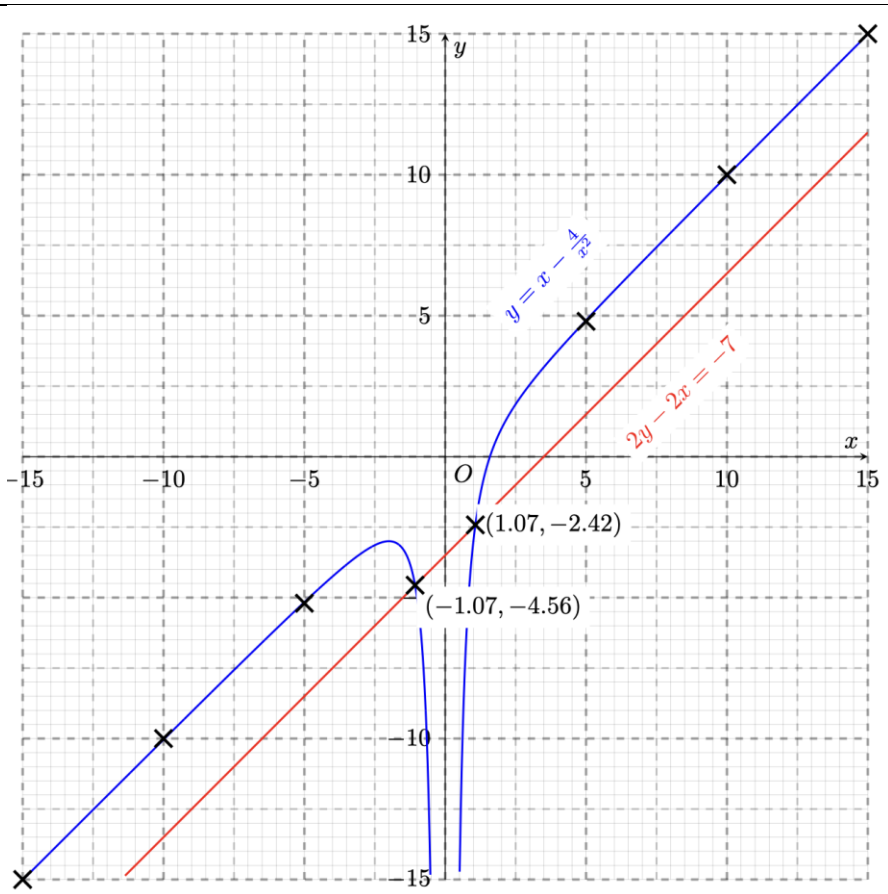
Paper 2

Qn No.	Solutions
1ai	$2x^2 - 6x - 12$ $= 2(x^2 - 3x - 6)$ $= 2[(x - 1.5)^2 - 2.25 - 6]$ $= 2[(x - 1.5)^2 - 8.25]$ $=2(x - 1.5)^2 - 16.5$
1aii	$2x^2 - 6x - 12 = 0$ $2(x - 1.5)^2 - 16.5 = 0$ $2[(x - 1.5)^2 = 16.5$ $(x - 1.5)^2 = 8.25$ $x - 1.5 = 2.87 \quad or \quad x - 1.5 = -2.87$ $x = 4.37 \quad or \quad x = -1.37$
1b	$\frac{3x+1}{2x^2+11x+12} - \frac{1}{x+4}$ $= \frac{3x+1}{(2x+3)(x+4)} - \frac{1}{x+4}$ $= \frac{3x+1}{(2x+3)(x+4)} - \frac{(2x+3)}{(2x+3)(x+4)}$ $=\frac{3x+1-2x-3}{(2x+3)(x+4)}$ $=\frac{x-2}{(2x+3)(x+4)}$
2	$2x + y = 3x + 5y + 3$ $x = -4y - 3 \quad -(1)$ $4x + 5y - 7 = x + y$ $3x + 4y = 7 \quad -(2)$ Sub (1) into (2) $3(-4y - 3) + 4y = 7$ $-12y - 9 + 4y = 7$ $-8y = 16$ $y = -2$ Sub $y = -2$ into (1) $x = -4(-2) - 3$

	$x = 5$
3a	$(x-3)(2x+8) = -12$ $2x^2 + 8x - 6x - 24 + 12 = 0$ $2x^2 + 2x - 12 = 0$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3 \ \ or \ x = 2$
3b	$\frac{4}{2x-3} - \frac{3}{x+2} = 1$ $\frac{4(x+2)-3(2x-3)}{(2x-3)(x+2)} = 1$ $4(x+2) - 3(2x-3) = (2x-3)(x+2)$ $4x + 8 - 6x + 9 = 2x^2 + x - 6$ $2x^2 + 3x - 23 = 0$ $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-23)}}{2(2)}$ $x = \frac{-3 \pm \sqrt{193}}{4}$ $x = \ 2.72 \ or \ -4.22$
4a	TSA of cone $= \pi r^2 + \pi r l$ $= \pi(2y)^2 + \pi(2y)(l)$ $= 2\pi y(2y + l)$
4b	TSA of hemisphere $= \pi r^2 + \frac{1}{2}(4\pi r^2)$ $= \pi(3y)^2 + 2\pi(3y)^2$ $= 9\pi y^2 + 18\pi y^2$ $= 27\pi y^2$ $2\pi y(2y + l) = 27\pi y^2$ $2y + l = \frac{27y}{2}$ $l = \frac{27y}{2} - 2y$ $l = \frac{23y}{2} \ or \ 11.5y$
4c	Vol of hemisphere $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = 729$ $\frac{2}{3}\pi(3y)^3 = 729$ $18\pi y^3 = 729$ $y \ = \ 2.3448$

	height of cone $= \sqrt{(l^2 - r^2)}$ $= \sqrt{\left(\left(\frac{23}{2}(2.3448)^2\right)^2 - (2 \times 2.3448)^2\right)}$ = 26.554 cm Vol of cone = $\frac{1}{3}\pi r^2 h$ $= \frac{1}{3}(\pi)(2 \times 2.3448)^2(26.554)$ $= 611.547 \approx 612cm^3\ (to\ 3s.f.)$
5a	$2:\ 100000$ $1:\ 50000$
5b	$1\ cm:\ 50000\ cm$ $1\ cm:\ 0.5\ km$ Area 1 cm^2 : 0.25 km^2 456 cm^2 : 114 km^2
6ai	sin *ABE* = sin *EBC* $=\frac{2}{\sqrt{8}}$
6aii	cos *ABE* $= -cos\ EBC$ $=-\frac{3}{\sqrt{8}}$
6b	Area $= \left(\frac{1}{2}\right)(AB)(BE)sinABE$ $= \left(\frac{1}{2}\right)(4)\left(\sqrt{8}\right)\left(\frac{2}{\sqrt{8}}\right)$ = 4 cm^2
6c	$\frac{AB}{AC} = \frac{4}{7}$ $\frac{Area\ ABE}{Area\ ACD} = \frac{16}{49}$ Area *ABE* = $4 \div 16 \times 49 = 12.25$ cm^2
7a	When $x = -10$, $y = -10 - \frac{4}{(-10)^2} = -10.0$

7b	
7ci	
7cii	-1.07 or 1.07 ($\pm$0.1)
7ciii	$2y - 2x = -7$: (equation 1) $y = x - \frac{4}{x^2}$: (equation 2)

	Sub (2) into (1): $2\left(x-\frac{4}{x^2}\right)-2x=-7$ $2x-\frac{8}{x^2}-2x=-7$ $-\frac{8}{x^2}=-7$ $8=7x^2$ $7x^2-8=0$ $A\ =\ 7, B=-8$
8a	$\angle POQ=180°-60°=120°$ $\angle OQP=\frac{180°-120°}{2}=30°$ (base ∠ of iso △) $\angle TQP=90°-30°=60°$
8b	$\angle PRQ=\frac{120°}{2}=60°$ (∠ at centre = 2 ∠ at circumference)
8c	$OPR=180°-60°-20°-(2\times 30°)=40°$ (sum of ∠ of △)
8d	Obtuse $\angle QOP=360°-120°=240°$ Area of major sector $=\frac{240°}{360°}\times\pi(5)^2\quad=\left(\frac{50}{3}\pi\right)\ cm^2$ Area of $\triangle POQ=\frac{1}{2}(5)(5)\sin 1\,20°$=10.825 Total Area = 63.2 cm^2
9a	$Length=\sqrt{[-6-(-4)]^2+[-2-(-7)]^2}$ $=\sqrt{(-2)^2+5^2}$ $=5.39$
9b	$m_{AB}=m_{CD}=\frac{-2-(-7)}{-6-(-4)}=\frac{5}{-2}$ $\overrightarrow{OC}=\overrightarrow{BC}+\overrightarrow{OB}$ $=\begin{pmatrix}8\\-2\end{pmatrix}+\begin{pmatrix}-4\\-7\end{pmatrix}$ $=\begin{pmatrix}4\\-9\end{pmatrix}$ $C(4,-9)$ $y=-2.5x+c$ $-9=-2.5(4)+c$ $c=1$ $y=-\frac{5}{2}x+1\ or\ 2y=-5x+2$
9ci	$\overrightarrow{XC}=\frac{1}{2}\overrightarrow{AC}$ $=\frac{1}{2}\left[\overrightarrow{OC}-\overrightarrow{OA}\right]$

	$= \frac{1}{2}\left[\begin{pmatrix} 4 \\ -9 \end{pmatrix} - \begin{pmatrix} -6 \\ -2 \end{pmatrix}\right]$ $= \begin{pmatrix} 5 \\ -7/2 \end{pmatrix}$
9cii	$\overrightarrow{OX} = \overrightarrow{OC} - \overrightarrow{XC}$ $= \begin{pmatrix} 4 \\ -9 \end{pmatrix} - \begin{pmatrix} 5 \\ -7/2 \end{pmatrix}$ $= \begin{pmatrix} -1 \\ -11/2 \end{pmatrix}$
9d	$\frac{2}{1}$ *or* $2:1$
10a	$(35 \times 20) + (45 \times 39) + (55 \times 16) + (65 \times 20) + (75x)$ $= 50.1(20 + 39 + 16 + 20 + x)$ $4635 + 75x = 50.1(95 + x)$ $4635 + 75x = 4759.5 + 50.1x$ $24.9x = 124.5$ $x = 5$
10b	Std Deviation $= 11.6$ min
10c	The **male participants** ran faster than the females participants as their **mean time** was shorter. The **female participants** were **more consistent** in their running speed as their **standard deviation** was lesser than that of the males.
11a	$T_5 = 7^2 + 17 = 66$
11b	The sum of 2 odd numbers or the sum of 2 even numbers will always be an even number.
11c	$T_n = (n+2)^2 + 5 + 3(n-1)$ $= n^2 + 4n + 4 + 5 + 3n - 3$ $= n^2 + 7n + 6$
11d	$T_{p+1} - T_p$ $= (p+1)^2 + 7(p+1) + 6 - [p^2 + 7p + 6]$ $= p^2 + 2p + 1 + 7p + 7 + 6 - p^2 - 7p - 6$ $= 2p + 8$
11e	$2p + 8 = 4$ $2\text{p} = -4 \Rightarrow \text{ p} = -2$ Since p cannot be negative, consecutive terms of the sequence cannot have a difference of 4 .
12ai	Let *X* be North of *H* Angle *PHX* = 360° – 306° = 54° Bearing of H from P = 180° – 54° = 126° (int angles)
12aii	Bearing of L from P = 126° + 124° = 250°
12aiii	tan θ = $\frac{500}{2500}$ angle of elevation = 11.3°

<table>
<tr><td>12b</td><td>$HL^2 = 2.5^2 + 3^2 - 2(2.5)(3)\cos 124^o$
HL = 4.86 km</td></tr>
<tr><td>12c</td><td>Let X be the point where LX is the shortest distance to HPQ
$\cos 56^o = \frac{XP}{2.5}$
XP = 1.39798

HX = 1.39798 + 3 = 4.39798
Time = $\frac{4397.98\ m}{4.5\ m/s}$ = 977.33 s = 16.289 min

It left harbour at 0753 hours</td></tr>
<tr><td></td><td></td></tr>
<tr><td>13a</td><td>$\frac{3}{4} \times 0.88 = 0.66$ (shown)
$\frac{1}{2} \times 0.88 = 0.44$ (shown)</td></tr>
<tr><td>13b</td><td>$0.5 \times 0.5 \times 0.22 \times 0.9$
$= 0.0495$</td></tr>
<tr><td>13c</td><td>
<table>
<tr><th>Time</th><th>Color</th><th>Target Size</th><th>Probability (Hit + Capture)</th></tr>
<tr><td>0s</td><td>Red</td><td>2</td><td>Target 2
$= 0.66 \times 0.8 \times 0.4 \times 0.7 = 0.14784$</td></tr>
<tr><td>1s</td><td>Green</td><td>3</td><td>Target 3
$= 0.44 \times 0.95 \times 0.9 \times 0.7 = 0.26334$</td></tr>
<tr><td>2s</td><td>Yellow</td><td>4</td><td>Target 4
$= 0.22 \times 0.5 \times 0.7 \times 0.7 = 0.0539$</td></tr>
<tr><td>3s</td><td>Orange</td><td>1</td><td>Target 1
$= 0.88 \times 0.7 \times 0.5 \times 0.7 = 0.2156$</td></tr>
</table>
∴ Maximum probability happens at green Target 2

Sam should wait for 1 second for the target to change from red Target 2 to green Target 3, with maximum probability of 0.26334

∴
</td></tr>
<tr><td></td><td></td></tr>
</table>