Name: _____ Tutorial Class: _____

ANGLO-CHINESE JUNIOR COLLEGE

MATHEMATICS DEPARTMENT

MATHEMATICS **8865**

Higher 1

2 May 2023

JC1 CA1(25 Marks) Time allowed**: 45 mins**

1 Find the set of values of k for which $x^2 + x - 1 + k$ is more than 2 for all real values of x. [3]

2 Given that $\alpha > 1$, $\log_\alpha \frac{x}{y} = \frac{9}{2}$ and $\log_\alpha x^5 y^2 = 5$, find the value of $\log_\alpha x$. [4]

3 A piece of metal is heated and then allowed to cool in a room with constant temperature. Its temperature $T\ ^oC$, when it has cooled for time t minutes, is modelled by the formula

$$T = 24 + 72(0.9)^t.$$

(i) Find the initial temperature at which the cooling starts. [1]

(ii) Explain what happens to the temperature for large values of t. [2]

(iii) Sketch the graph of T against t, stating the coordinates of any points of intersections with the axes and the equations of any asymptotes. [2]

(iv) Find the time taken for the temperature of the metal to reach $28\ ^oC$. [1]

4 **(i)** Sketch the graph of $y = 1 + \ln x$, stating the coordinates of any points of intersections with the axes and the equations of any asymptotes. [2]

(ii) By adding a suitable curve to your sketch in part **(i)**, solve

$$\ln x = \frac{11 - x}{x - 10}$$ [4]

5 A company produces three types of sports shoes: Runners, Joggers and Walkers. It is given that the manufacturing cost of a pair of Runners, Joggers and Walkers is \$$R$, \$$J$ and \$$W$ respectively.

The total manufacturing cost for 13 pairs of Runners, 18 pairs of Joggers and 40 pairs of walkers is given by $13R + 18J + 40W = \$2830$.

Given that the manufacturing cost of a pair of Runners, Joggers and Walkers is reduced by 20%, 25% and 10% respectively. With this reduction, the total manufacturing cost for 18 pairs of Runners, 30 pairs of Joggers and 40 pairs of Walkers is \$2934.

It is also given that $10W = 4R + 165$.

(i) Find the manufacturing cost of a pair of Walkers before the manufacturing cost was reduced. [2]

Explain in the context of the question what does the equation $10W = 4R + 165$ represents. [1]

The company wants to increase their profit by investing in a new type of sports shoes, Sprints. An economist predicts that the profit \$$P$ when x pairs of Sprint shoes are produced and sold can be modelled by

$$P = -\frac{x^2}{10} + 70x - 2 .$$

(ii) Find the maximum profit the company can earn from this investment and the corresponding number of pairs of shoes the company needs to manufacture. [2]

(iii) Find the maximum number of pair of shoes the company can manufacture so that the investment is still profitable [1]