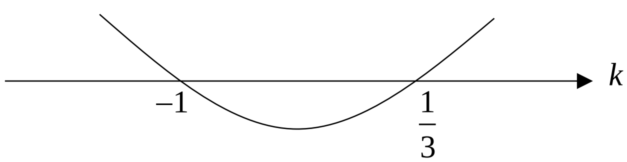
Q1 Solution

$kx^2+(k-1)x+k>0$ for all real values of x So, graph has no real roots and is U-shaped Discriminant < 0 $b^2-4ac<0$ and $k>0$ $(k-1)^2-4(k)(k)<0$
$-3k^2-2k+1<0$ $3k^2+2k-1>0$ $(3k-1)(k+1)>0$ Alternatively, If $3k^2+2k-1=0$, $k=\dfrac{-2\pm\sqrt{2^2-4(3)(-1)}}{2(3)}$ $=-1$ or $\dfrac{1}{3}$
 $k<-1$ (rej $\because$ $k>0$) or $k>\dfrac{1}{3}$ Set of values of k is $\left\{k\in\mathbb{R}:k>\dfrac{1}{3}\right\}$

Q2 Solution

Let the polynomial be $y=ax^3+bx^2+cx+d$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=3ax^2+2bx+c$
At $(1,-13)$, $a+b+c+d=-13$ --- (1) $3a+2b+c=0$ -------- (2)
At $(-1,7)$, $-a+b-c+d=7$ --- (3) $3a-2b+c=-12$ --- (4)
Solving (1), (2), (3) and (4), $a=2$, $b=3$, $c=-12$, $d=-6$

Hence the polynomial is $2x^3+3x^2-12x-6$

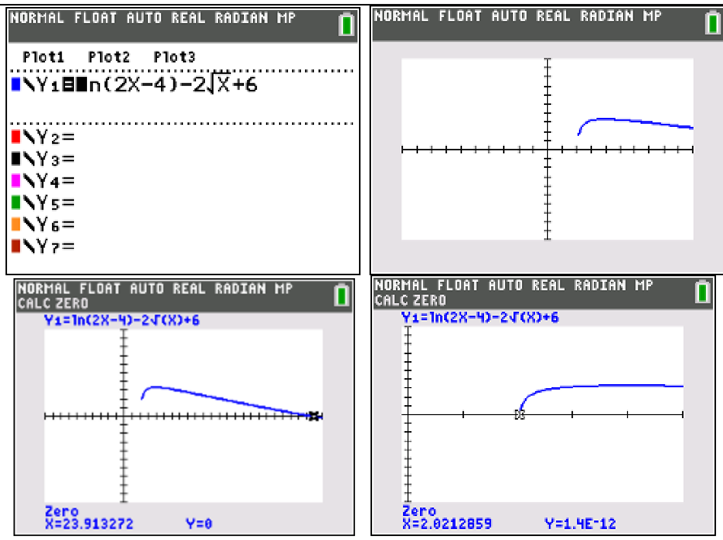
Q3 <u>Solution</u>

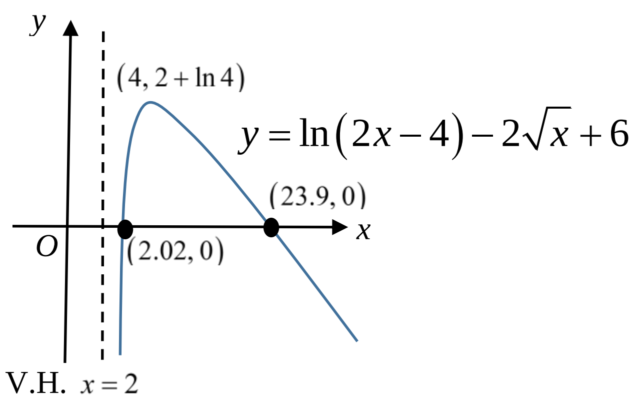
(a) (i)	$\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\frac{e^{2x}}{x+2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(e^{2x}\right)-\ln(x+2)\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(2x-\ln(x+2)\right)=2-\frac{1}{x+2}$
	<u>Alternative Method (Quotient rule)</u> $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\frac{e^{2x}}{x+2}\right)$ $=\frac{x+2}{e^{2x}}\times\left(\frac{(x+2)2e^{2x}-e^{2x}}{(x+2)^2}\right)$ $=2-\frac{1}{x+2}$
(a) (ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(1+\frac{2}{x}\right)^n=n\left(1+\frac{2}{x}\right)^{n-1}\left(-\frac{2}{x^2}\right)=-\frac{2n}{x^2}\left(1+\frac{2}{x}\right)^{n-1}$
(b)	$\frac{1}{\sqrt{x}-\sqrt{x-2}}=\frac{1}{\sqrt{x}-\sqrt{x-2}}\times\frac{\sqrt{x}+\sqrt{x-2}}{\sqrt{x}+\sqrt{x-2}}=\frac{\sqrt{x}+\sqrt{x-2}}{\left(\sqrt{x}\right)^2-\left(\sqrt{x-2}\right)^2}=\frac{\sqrt{x}+\sqrt{x-2}}{2}$
	$\int\frac{1}{\sqrt{x}-\sqrt{x-2}}dx=\frac{1}{2}\int\sqrt{x}+\sqrt{x-2}dx$ $=\frac{1}{2}\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+\frac{(x-2)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]+C$, where C is an arbitrary constant $=\frac{1}{3}\left[x^{\frac{3}{2}}+(x-2)^{\frac{3}{2}}\right]+C$

Q4 **Solution**

(i)	$y=\ln(2x-4)-2\sqrt{x}+6$ $\frac{dy}{dx}=\frac{2}{2x-4}-2\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$ $=\frac{1}{x-2}-\frac{1}{\sqrt{x}}$
	For stationary points, set $\frac{dy}{dx}=0$ So, $\frac{1}{x-2}-\frac{1}{\sqrt{x}}=0$ $\frac{1}{x-2}=\frac{1}{\sqrt{x}}$ $\sqrt{x}=x-2$
	Method 1 Squaring both sides, $\left(\sqrt{x}\right)^2=(x-2)^2$ $x=x^2-4x+4$ $x^2-5x+4=0$ $(x-1)(x-4)=0$ $x=1$ or $x=4$ Since $x>2$, so $x=4$
	Method 2 $\sqrt{x}-x+2=0$ Let $y=\sqrt{x}$, then $y-y^2+2=0$ $y^2-y-2=0$ $(y+1)(y-2)=0$ So, $y=\sqrt{x}=2$ or $y=\sqrt{x}=-1$ (no solution) Hence $x=4$
	$\frac{1}{x-2}-\frac{1}{\sqrt{x}}=0$ $\frac{1}{x-2}=\frac{1}{\sqrt{x}}$
	When $x=4$, $y=\ln(2\times4-4)-2\sqrt{x}+6=\ln4-4+6=2+\ln4$ So turning point is $(4,2+\ln4)$

(ii)





(iii)

At $x=9$,

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x-2}-\frac{1}{\sqrt{x}}=\frac{1}{9-2}-\frac{1}{\sqrt{9}}=\frac{1}{7}-\frac{1}{3}=\frac{3-7}{21}=-\frac{4}{21}$$

and $y=\ln(2\times 9-4)-2\sqrt{9}+6=\ln 14-6+6=\ln 14$

Method 1

Substituting point and gradient into $y=mx+c$

$$\ln 14=\left(-\frac{4}{21}\right)(9)+c$$

$$c=\ln 14+\left(\frac{4}{21}\right)(9)=\ln 14+\frac{12}{7}$$

Hence equation of tangent at $x=9$ is

$$y=-\frac{4}{21}x+\frac{12}{7}+\ln 14$$

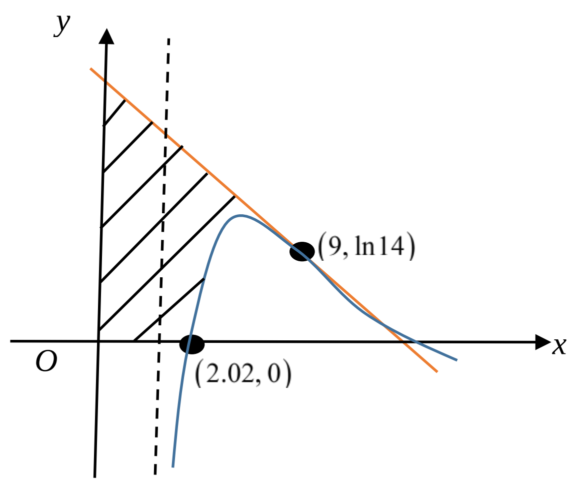
Method 2

Hence equation of tangent at $x=9$ is

$$y-\ln 14=-\frac{4}{21}(x-9)$$

$$y=-\frac{4}{21}x+\frac{4}{21}\times 9+\ln 14$$

$$y=-\frac{4}{21}x+\frac{12}{7}+\ln 14$$

<table>
<tr><td>(iv)</td><td>


V.H. $x=2$
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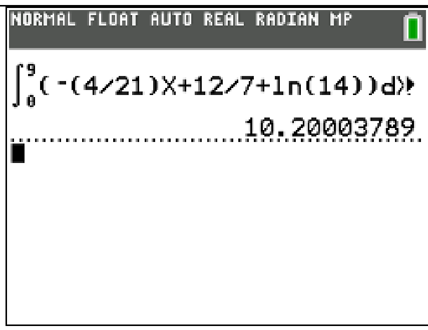
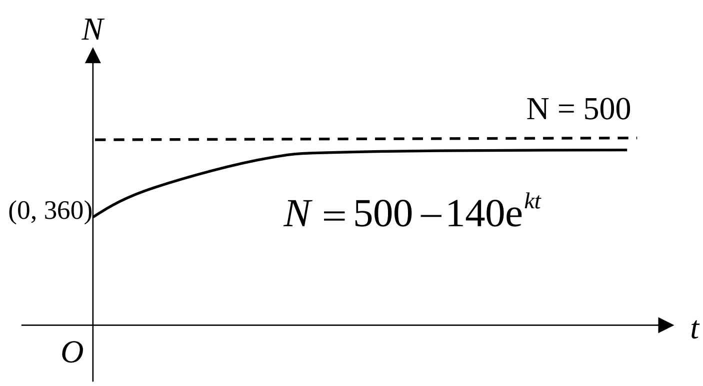
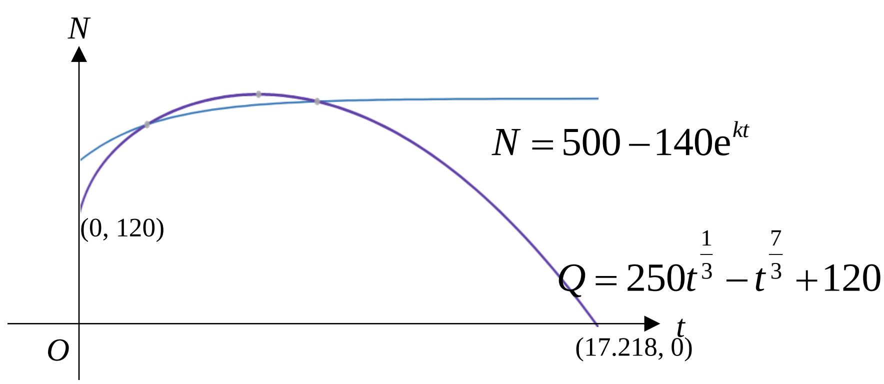
<tr><td></td><td>


</td></tr>
<tr><td></td><td>
Area
= area under tangent – area under curve
$=\int_0^9 -\frac{4}{21}x+\frac{12}{7}+\ln 14 \ \mathrm{d}x-\int_{2.0212859}^9 \ln(2x-4)-2\sqrt{x}+6 \ \mathrm{d}x$
$=10.2$ (to 3 s.f.)
</td></tr>
</table>

Q5 **Solution**

(i)	Since N is in hundreds, So, $N = 481$
	$481 = 500 - 140e^{k(5)}$ $e^{k(5)} = \frac{19}{140}$ $k = \frac{1}{5}\ln\left(\frac{19}{140}\right)$ $k = -0.399 = -0.4$ (1 d.p.)
(ii)	$\frac{dN}{dt} = 56e^{-0.4t}$
	$\frac{dN}{dt}$ represents the rate of change of recorded influenza cases with respect to time. Or $\frac{dN}{dt}$ represents the rate of increase of recorded influenza cases with respect to time. Or $\frac{dN}{dt}$ implies that for every increase of 1 day, the number of recorded influenza cases increase by $56e^{-0.4}$ hundreds.
(iii)	
(iv)	As t becomes large, $e^{-0.4t} \to 0$, $N \to 500$ So, number of recorded influenza cases will tend to 50 000.
	No, it is not realistic as it is not possible to have the number of cases stay at a constant as sick people should get well eventually. (or any other reasonable reason.)

<table>
<tr><td>(v)</td><td>$\frac{\mathrm{d}Q}{\mathrm{d}t}=\frac{250}{3}t^{-\frac{2}{3}}-\frac{7}{3}t^{\frac{4}{3}}$</td></tr>
<tr><td></td><td>For max/min values, $\frac{\mathrm{d}Q}{\mathrm{d}t}=0$
$\frac{250}{3}t^{-\frac{2}{3}}-\frac{7}{3}t^{\frac{4}{3}}=0$</td></tr>
<tr><td></td><td>$\frac{250}{3}t^{-\frac{2}{3}}=\frac{7}{3}t^{\frac{4}{3}}$
$250t^{-\frac{2}{3}}=7t^{\frac{4}{3}}$
$250=7t^2$
$t=\sqrt{\frac{250}{7}}$ (since $t>0$)</td></tr>
<tr><td></td><td>
<table>
<tr><td>t</td><td>5.97</td><td>$\sqrt{\frac{250}{7}}$</td><td>5.98</td></tr>
<tr><td>$\frac{\mathrm{d}Q}{\mathrm{d}t}$</td><td>0.05203</td><td>0</td><td>-0.03266</td></tr>
</table>

Hence, $t=\sqrt{\frac{250}{7}}$ days gives max value of Q.</td></tr>
<tr><td>(vi)</td><td>
</td></tr>
<tr><td>(vii)</td><td>x-coordinates of intersection: 2.3655512, 7.874977
$\therefore 2.37\le t\le 7.87$ days</td></tr>
</table>

Q6 **<u>Solution</u>**

(i)	Consider AB as one group. Number of ways $= 7! \times 2!$ Arrange 7 objects Adam and Bernice can swap places $= 10080$
(ii)	A J S J S J S J or J S J S J S J A Number of ways $= \left(4! \times 3!\right) \times 2$ Arrange 4 Juniors Arrange remaining 3 Seniors Adam can be on the left or right of row. $= 288$
(iii)	Probability $= \dfrac{\text{nbr of ways to select 1 Junior and 4 Seniors}}{\text{nbr of ways to select 5 members}}$ $= \dfrac{{}^4C_1 \times {}^4C_4}{{}^8C_5}$ $= \dfrac{1}{14}$
	<u>Alternative Method</u> Probability $= \dfrac{4}{8} \times \dfrac{4}{7} \times \dfrac{3}{6} \times \dfrac{2}{5} \times \dfrac{1}{4} \times 5 = \dfrac{1}{14}$

Q7 Solution

<table>
<tr><td>(i)</td><td>

(i) Let X be the random variable denoting number of passengers rejected by the facial recognition scanner out of 50 passengers.

Then $X \sim \mathrm{B}(50, p)$

Given $\mathrm{Var}(X) = 3.68$

$\Rightarrow np(1-p) = 3.68$

$\Rightarrow 50p(1-p) = 3.68$

$\Rightarrow p - p^2 = 0.0736$

$\Rightarrow p^2 - p + 0.0736 = 0$

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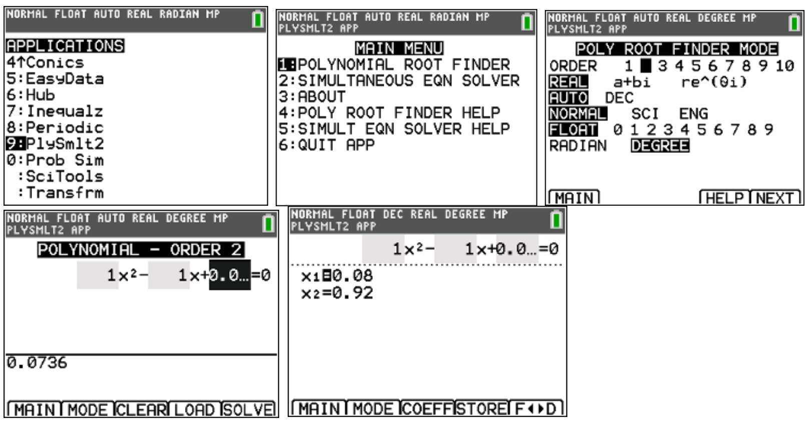
<u>Method 1</u> <u>(Using GC to find the roots)</u>



Since $p < 0.5$, $p = 0.08$

</td></tr>
<tr><td></td><td>

<u>Method 2 (factorize to find the roots)</u>

$p^2 - p + 0.0736 = 0$

$p^2 - p + \dfrac{46}{625} = 0$

$625p^2 - 625p + 46 = 0$

$(25p - 2)(25p - 23) = 0$

So, $p = \dfrac{2}{25}$ or $p = \dfrac{23}{25}$ (reject since $p < 0.5$)

Hence, $p = 0.08$

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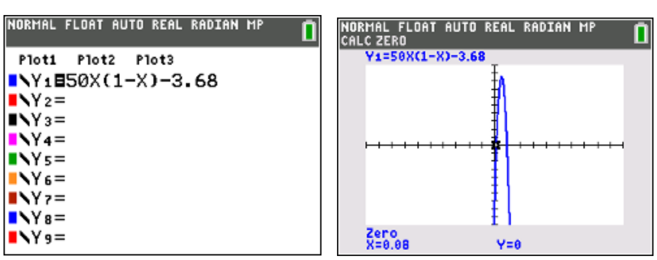
<u>Method 3 (using GC graph to find the roots)</u>



Using GC, since $p < 0.5$, $p = 0.08$

</td></tr>
</table>

(ii) Let Y be the random variable denoting number of passengers rejected by the iris scanner out of 100 passengers.

Then $Y \sim \mathrm{B}(100, 0.07)$

P(plane departs on time)

$= \mathrm{P}(Y < 13)$

$= \mathrm{P}(Y \leq 12) = 0.9775924796 \approx 0.978$ (to 3 s.f.)

(iii) Let W be the random variable denoting number of planes delayed out of 75 planes.

Then $W \sim \mathrm{B}(75, \mathrm{P}(Y \geq 13))$

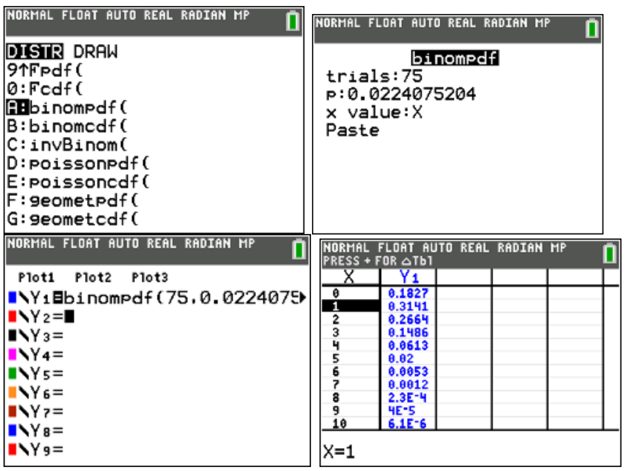
$W \sim \mathrm{B}(75, 1 - 0.9775924796)$

$W \sim \mathrm{B}(75, 0.0224075204)$

P(more than 5 planes delayed) $= \mathrm{P}(W > 5) = 1 - \mathrm{P}(W \leq 5)$

$= 0.0068237351 \approx 0.00682$ (to 3 s.f.)

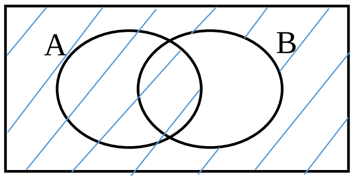
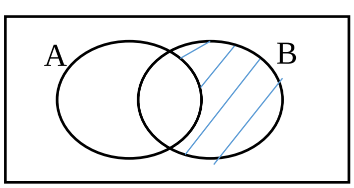
(iv) $W \sim \mathrm{B}(75, 0.0224075204)$



w	$\mathrm{P}(W = w)$
0	0.1827
1	0.3141
2	0.2664

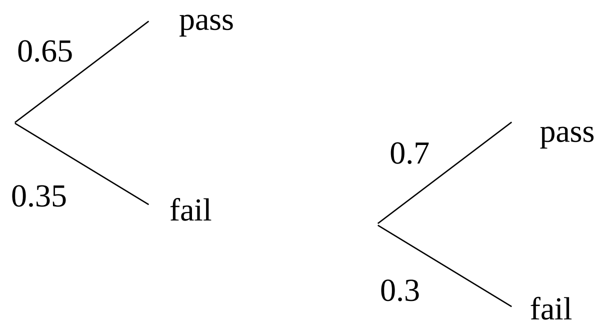
Hence the most likely number of planes delayed is 1.

Q8 **Solution**

(i)	 $(A \cup B')$
	Method 1 $\mathrm{P}(A \cup B') = 1 - \mathrm{P}(B) + \mathrm{P}(A \cap B)$ $1 - \mathrm{P}(B) + \mathrm{P}(A \cap B) = \frac{13}{20}$ $1 - \left(a + \frac{11}{40}\right) + \frac{1}{20} = \frac{13}{20}$ $1 - \frac{13}{20} + \frac{1}{20} = a + \frac{11}{40}$ $a = 1 - \frac{12}{20} - \frac{11}{40} = \frac{1}{8}$
	Method 2 $\mathrm{P}(B) = [1 - \mathrm{P}(A \cup B')] + \mathrm{P}(A \cap B)$ $a + \frac{11}{40} = \left[1 - \frac{13}{20}\right] + \frac{1}{20}$ $a = \frac{1}{8}$
	Method 3 $\mathrm{P}(A \cup B') = \mathrm{P}(A) + \mathrm{P}(B') - \mathrm{P}(A \cap B')$ $= \mathrm{P}(A) + \mathrm{P}(B') - [\mathrm{P}(A) - \mathrm{P}(A \cap B)]$ $\frac{13}{20} = a + \left[1 - \left(a + \frac{11}{40}\right)\right] - \left[a - \frac{1}{20}\right]$ $\frac{13}{20} = \frac{29}{40} + \frac{1}{20} - a$ $a = \frac{31}{40} - \frac{13}{20} = \frac{5}{40} = \frac{1}{8}$
(ii)	 $(A' \cap B)$
	$\mathrm{P}(A' \cap B)$ is the probability of **only** event B occurring. or the probability of only event B occurring and event A not occurring.

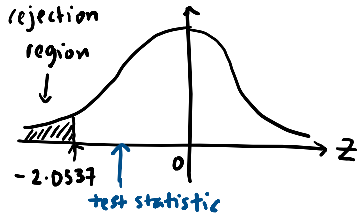
	$\mathrm{P}(A'\cap B)=\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=a+\frac{11}{40}-\frac{1}{20}$ $=\frac{1}{8}+\frac{9}{40}$ $=\frac{14}{40}=\frac{7}{20}$
(iii)	$\mathrm{P}(A\cap B)=\frac{1}{20}$ $\mathrm{P}(A)\times\mathrm{P}(B)=\frac{1}{8}\times\left(\frac{1}{8}+\frac{11}{40}\right)=\frac{1}{8}\times\frac{2}{5}=\frac{1}{20}$ Yes, *A* and *B* are independent.
	Since *A* and *B* are independent, then *A*' and *B* are also independent. So $\mathrm{P}(A'\mid B)=\mathrm{P}(A')=\frac{7}{8}$
	[Check: $\mathrm{P}(A'\mid B)=\frac{\mathrm{P}(A'\cap B)}{\mathrm{P}(B)}=\frac{\frac{7}{20}}{\frac{2}{5}}=\frac{7}{8}$ $=\mathrm{P}(A')$]

Q9 Solution

(i)	
(ii)	P(did not qualify) = P(did not pass first song and did not pass second song) $= 0.35 \times 0.3$ $= \dfrac{21}{200}$ or ≈ 0.105
(iii)	$\mathrm{P}(\text{sing twice} \mid \text{qualify})$ $= \dfrac{\mathrm{P}(\text{sing twice and qualify})}{\mathrm{P}(\text{qualify})}$ $= \dfrac{\mathrm{P}(\text{1st song did not pass followed by 2nd song passed})}{1 - \mathrm{P}(\text{did not qualify})}$ $= \dfrac{0.35 \times 0.7}{1 - 0.105}$ $= \dfrac{49}{179}$ or ≈ 0.274 (3 s.f.)
	Alternative Method $\mathrm{P}(\text{sing twice} \mid \text{qualify})$ $= \dfrac{\mathrm{P}(\text{sing twice and qualify})}{\mathrm{P}(\text{qualify})}$ $= \dfrac{\mathrm{P}(\text{1st song did not pass followed by 2nd song passed})}{\mathrm{P}(\text{1st song passed}) + \mathrm{P}(\text{1st song did not pass followed by 2nd song passed})}$ $= \dfrac{0.35 \times 0.7}{0.65 + 0.35 \times 0.7}$ $= \dfrac{49}{179}$ or ≈ 0.274 (3 s.f.)
(iv)	Probability $= \mathrm{P}(\text{qualify after 2nd song}) \times \left[\mathrm{P}(\text{did not qualify})\right]^2 \times 3$ $= (0.35 \times 0.7) \times (0.105)^2 \times 3$ $= 0.00810$ (3s.f.)

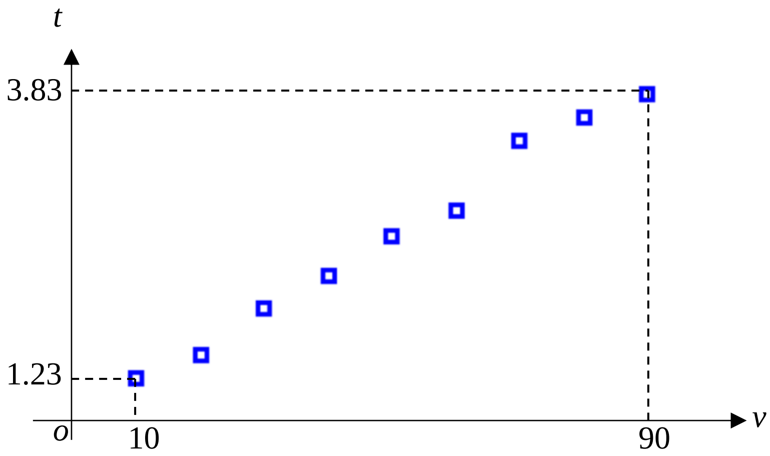
Q10 Solution

(i)	Unbiased estimate of population mean, $\bar{x} = \frac{\sum(x-5)}{120} + 5$ $= \frac{54}{120} + 5$ $= 5.45$ (exact value)
	Unbiased estimate of population variance, $s^2 = \frac{1}{120-1}\left[\sum(x-5)^2 - \frac{\left[\sum(x-5)\right]^2}{120}\right]$ $= \frac{1}{120-1}\left[162 - \frac{54^2}{120}\right]$ $= \frac{81}{70}$ $= 1.157142857$ ≈ 1.16 (3 s.f.)
(ii)	$\mathrm{H}_0 : \mu = 5.65$ $\mathrm{H}_1 : \mu \neq 5.65$
	Under H_0, since n = 120 is sufficiently large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(5.65, \frac{1.1571}{120}\right)$ approximately
	Test statistics, $z = \frac{\bar{x} - \mu_0}{\sqrt{\frac{s^2}{n}}} = \frac{5.45 - 5.65}{\sqrt{\frac{1.1571}{120}}} = -2.0367$ Using GC, p-value = 0.041676 (5 s.f.) = 0.0417 (3 s.f.) Or test statistics $z = -2.0367 <$ critical value $z = -1.95996$
	Since p-value = 0.0417 < 0.05, we reject H_0. Hence, we conclude at 5% significance level that there is sufficient evidence that the mean time spent by students participating in sports activities is not 5.65 hours.
	The p-value of 0.0417 means the probability of obtaining a sample mean at least as extreme as the given sample of 120 students whose mean time participating in sports activities is 5.45 hours, assuming that the population mean time is 5.65 hours, is 0.0417.
(iii)	$\mathrm{H}_0 : \mu = 5.65$ $\mathrm{H}_1 : \mu < 5.65$
	Under H_0, since n = 100 is sufficiently large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(5.65, \frac{1.1571}{100}\right)$ approximately
	Test statistics, $z = \frac{\bar{x} - \mu_0}{\sqrt{\frac{s^2}{n}}} = \frac{\bar{x} - 5.65}{\sqrt{\frac{1.1571}{100}}}$

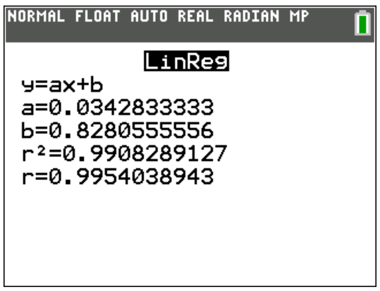
	Teachers' claim not justified $\Rightarrow$ do not reject H_0 at 2% sig. level $\Rightarrow$ test statistic > – 2.0537 $\Rightarrow \dfrac{\bar{x}-5.65}{\sqrt{\dfrac{1.1571}{100}}} > -2.0537$ $\Rightarrow \bar{x} > 5.4291$ $\therefore\ \left\{\bar{x} \in \mathbb{R} : \bar{x} > 5.43\right\}$ 
	<u>Alternative Method (no need to standardize)</u> $\bar{x} > \text{invNorm}\left(0.02,\ 5.65,\ \sqrt{\dfrac{1.1571}{100}}\right)$ $\Rightarrow \bar{x} > 5.4291$

Q11 Solution

(i)



(ii)



$r = 0.995$ (3s.f.)

There is a strong positive linear relationship between t and v hence $|r|$ is close to 1.

(r is close to 1 is accepted too)
As t increases, v also increases proportionately.

(iii)

$t = 0.8280555556 + 0.0342833333v$
$t = 0.828 + 0.0343v$

The gradient of the line is 0.0343 which means that when v increases by 1 km/h, t will increase by 0.0343 seconds.

(iv)

Using $t = 0.82806 + 0.034283v$,

When $v = 110 \Rightarrow t = 0.82806 + 0.034283(110) \approx 4.60$ seconds

Since $v = 110$ is not within the given range $10 \le v \le 90$, this is an extrapolation hence the estimate is not reliable.

(v)

$t = (0.828 + 0.5) + 0.0343v$
$t = 1.328 + 0.0343v$

Q12 **Solution**

(i)	Let X be the random variable denoting mass of one packet of cement in kg. Then $X \sim N\left(\mu, \sigma^2\right)$ $\mathrm{P}\left(X > 49.8\right) = 0.6$ $\mathrm{P}\left(Z > \frac{49.8-\mu}{\sigma}\right) = 0.6$ $\qquad Z \sim \mathrm{N}(0,1)$ $\frac{49.8-\mu}{\sigma} = -0.2533471011$ $\mu - 0.2533471011\sigma = 49.8 --------- (1)$ $\mathrm{P}\left(X < 49.1\right) = 0.15$ $\mathrm{P}\left(Z < \frac{49.1-\mu}{\sigma}\right) = 0.15$ $\frac{49.1-\mu}{\sigma} = -1.03643338$ $\mu - 1.03643338\sigma = 49.1 ------- (2)$ Using GC, solving (1) and (2), $\mu = 50.026 = 50.0$ $\sigma = 0.89390 = 0.894$
(ii)	Let Y be the random variable denoting mass of one brick in kg. Then $Y \sim \mathrm{N}(3.88, 0.1^2)$ $\mathrm{P}\left(Y > 4.07\right) = 0.0287$
(iii)	$Y_1 + Y_2 + ... + Y_{12} \sim N\left(12 \times 3.88, 12 \times 0.1^2\right)$ $\mathrm{P}\left(3.85(12) < Y_1 + Y_2 + ... + Y_{12} < 3.92(12)\right) = 0.768$ (3 sf)
	Alternative method Since Y is normally distributed, $\overline{Y} \sim N\left(3.88, \frac{0.1^2}{12}\right)$ $\mathrm{P}\left(3.85 < \overline{Y} < 3.92\right) = 0.768$
(iv)	$1.15Y \sim \mathrm{N}\left(1.15 \times 3.88, 1.15^2 \times 0.1^2\right)$ $1.15Y \sim \mathrm{N}\left(4.462, 132.25\right)$ $\mathrm{P}\left(1.15Y < 4.22\right) = 0.0177$
	Alternative method Using $Y \sim \mathrm{N}(3.88, 0.1^2)$ $\mathrm{P}\left(1.15Y < 4.22\right) = \mathrm{P}\left(Y < \frac{4.22}{1.15}\right) = 0.0177$

(v)	Let $F = X_1 + X_2 + 1.15Y_1 + 1.15Y_2 + ... + 1.15Y_{25}$ $F \sim \mathrm{N}\left(2\left(50.0\right) + 25\left(4.462\right),\ 2(0.894^2) + 25\left(132.25\right)\right)$ $F \sim N\left(211.602, 1.9291\right)$ $\mathrm{P}\left(F > m\right) = 0.8$ m = 210.4 (1 dp)
(vi)	Assumption is that mass of bricks and packets of cement are independent normal distributions.