Name:		Centre/Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination 2023
Year 6

MATHEMATICS 9758/02

Paper 2 **19 September 2023**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number, name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	8	9	10	13	5	8	6	7	10	12	12	100

Section A: Pure Mathematics [40 marks]

1 **(a)** Given that f is a continuous and increasing function, explain with the aid of a sketch, why $\frac{1}{n}\sum_{r=0}^{n-1} \mathrm{f}\left(\frac{r}{n}\right)$ is less than $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$. [3]

(b) Find a similar expression that is greater than $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$. [1]

(c) It is given that $\mathrm{f}(x) = x^2 + 1$ and $n = 10$.

(i) Using the expression in part **(a)** and your expression in part **(b)**, find the lower and upper bounds of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$. [2]

(ii) Comparing your answers from part **(c)(i)** with the calculated value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, comment which limit is a better estimate. [2]

2 **(a)** A sequence of real numbers $u_1, u_2, u_3 \ldots$ is such that

$$u_{n+1} = 3 - 0.5u_n, \quad \text{for } n \geq 1.$$

It is given that $u_1 = c$ where c is a constant.

(i) Describe how the sequence behaves when

(A) $c = 5$, [1]

(B) $c = 2$. [1]

(ii) Find the value of c for which $2u_3 = -5u_2$. [3]

(b) Another sequence is defined by $v_1 = p$, $v_2 = 2$, where p is a constant, and

$$v_{n+2} = 2v_n + v_{n+1} - 1, \quad \text{for } n \geq 1.$$

(i) If $v_3 + 1 = 2u_2 - u_1$, find a relationship between c and p. [2]

(ii) Find the value of p for which $v_5 = 77$. [2]

3

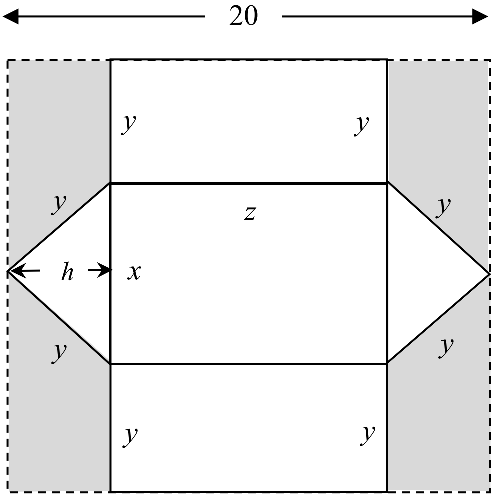


Fig. 1

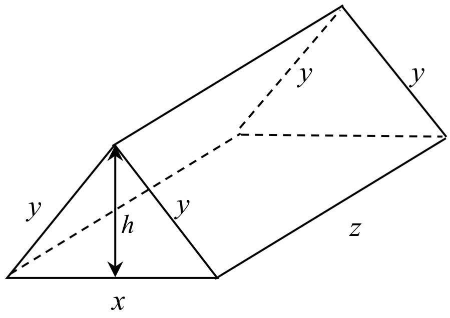


Fig. 2

Fig. 1 shows the net of a triangular prism cut from a square cardboard of side length 20 cm. The net consists of one rectangle z cm by x cm, two rectangles z cm by y cm and two isosceles triangles each with base x cm and perpendicular height h cm. When the shaded area of the cardboard is removed, the net is folded to form a triangular prism with a rectangular base as shown in Fig. 2.

(a) Show that $y = 5 + \dfrac{h^2}{20}$ and find x in terms of h. [3]

(b) Show that the volume of the prism $V = \dfrac{1}{10}\left(h^4 - 10h^3 - 100h^2 + 1000h\right)$. Use differentiation to find the maximum volume of the prism. [7]

4 A tetrahedron has four vertices O, A, B and C where O is the origin. It is given that the coordinates of B are (1, 4, 6) and the line AB has equation $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + a\mathbf{j} + \mathbf{k})$, where λ is a parameter. It is also given that the coordinates of C are (4, 4, 9) and the line AC has equation $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} + b\mathbf{j} + 2\mathbf{k})$, where μ is a parameter.

(a) Show that $a + b = 0$. [3]

(b) Using a suitable cross product to find the normal to plane ABC, find the cartesian equation of the plane if its normal is parallel to $-3\mathbf{i} + \mathbf{j} + 3\mathbf{k}$. [4]

(c) Hence find the coordinates of A and the acute angle between lines AB and AC. [3]

(d) By considering the distance of plane ABC from the origin, find the equation of the planes which are $\sqrt{19}$ units from plane ABC. [3]

Section B: Probability and Statistics [60 marks]

5 Tim plays a game with 3 outcomes having the score of 2, 5 and 6. The corresponding probabilities are k, $\frac{2}{7}$ and $\frac{4}{7}$ respectively, where k is a constant.

(a) State the value of k. [1]

Tim pays \$$m$ in total to play the game twice. The outcome of each game is independent. He receives an amount corresponding to twice the absolute difference of the two scores from each game.

(b) If Tim is expected make a profit, determine the range of values of m. [3]

(c) For $m = 2.3$, explain why Tim may not profit for every 2 such games that he plays. [1]

6 A student needs to create a 7-character alpha-numeric passcode (that is, at least one letter and one digit must be included). She uses lower case letters from her name "venessa see" and digits 1 to 9.

(a) Find the number of different passcodes if all the letters and numbers are distinct such that

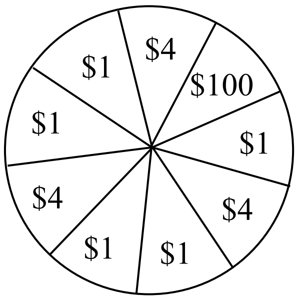
(i) they begin with a letter and end with a digit, [2]

(ii) all the letters and digits are to be separated. [3]

(b) Find the number of different passcodes if there are 3 identical letters, and all other letters and digits are distinct. [3]

7 In a supermarket lucky draw, a participant will first spin a wheel, followed by throwing a fair six-sided die.

The wheel has 9 sections and an arrow which has an equal chance of coming to rest over any of the 9 sections. The wheel has one section labelled "\$100", three sections labelled "\$4" and five sections labelled "\$1" (see diagram).



The fair six-sided die has two faces labelled "0.02", three faces labelled "0.5" and one face labelled "2".

The amount of money that the participant wins is calculated by taking the multiplication of the outcomes of the wheel and die.

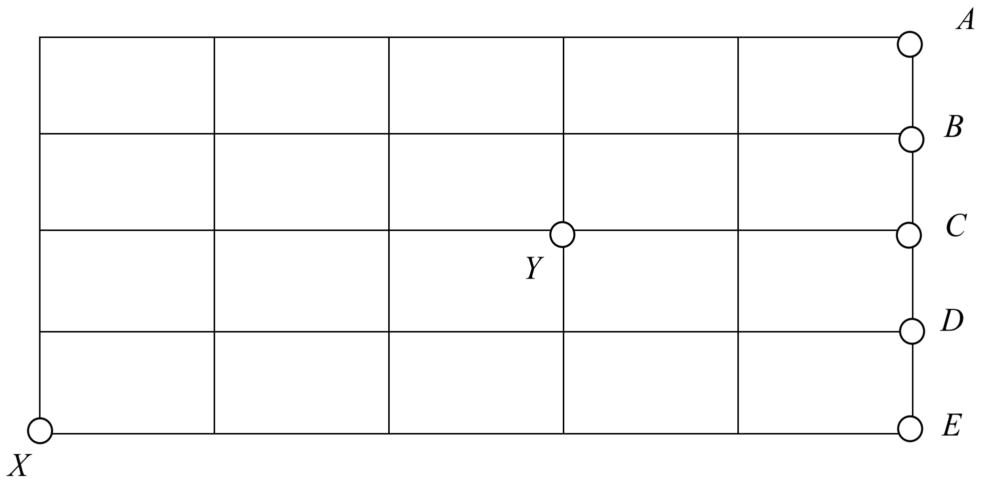
(a) Find the probability that a participant wins \$2. [2]
(b) If a participant wins \$2, find the probability that the wheel came to rest over "\$100". [1]

(c) A participant plays three consecutive lucky draws. Find the probability that a participant wins \$6 in total, such that the die shows a different face value each time. [2]

(d) Another participant plays one lucky draw. He is now given an option to either end the game by taking the amount based on the section that the wheel came to rest over, or to proceed to throw the die. Explain, with justification, which option should he take. [1]

8 One round of a game is played by randomly moving the counter from the starting point X to one of the ending points A to E along the grid lines as shown in the diagram below.

- The counter can only either move up or to the right along the grid lines.
- At every movement after X, the player randomly moves the counter to the right with probability p or up with probability q, $p+q=1$. The moves taken at each junction are independent.
- For the player to win a round, the counter must pass through Y and end up in point B.



(a) Show that the probability of winning a round is $30p^5q^3$. [3]

It is given that $p=\frac{4}{5}$. Wang plays one game which comprises 15 rounds. Each round is independent of one another. Let the random variable W be the number of rounds that Wang wins.

(b) Find the probability that Wang wins at least one-third of the 15 rounds. [1]

(c) Wang plays 40 such games independently. Estimate the probability that the mean number of games where he is successful in winning a round is at most one. [3]

9 In this question you should state the parameters of any distributions that you use.

In a supermarket, the masses in grams of Fuji apples have the distribution $\mathrm{N}\left(205, 9^2\right)$ and the masses in grams of Gala apples have the distribution $\mathrm{N}\left(180, 6^2\right)$.

(a) Find the probability that a randomly chosen Fuji apple has a greater mass than a randomly chosen Gala apple. [2]

(b) Three Fuji apples are selected at random. Find the probability that two Fuji apples have mass exceeding 203 grams each and one Fuji apple has mass less than 185 grams. [2]

(c) The supermarket also sells assorted packets of apples containing a mixture of a total of 10 Fuji and Gala apples. If there are n Fuji apples in an assorted packet, find the least value of n such that there is a probability of at least 0.5 that the mass of an assorted packet exceeds nine times the mass of a Fuji apple by more than 28 g. [5]

(d) State an assumption needed for your calculations in parts **(a)** and **(c)**. [1]

10 The average waiting time (in minutes) for a customer to be served at a fast-food restaurant is k minutes. The manager claims that with subsequent increased staff strength, the restaurant can now better serve the customers and the average waiting time has improved. The population variance of waiting time is known to be 0.22 minutes^2.

The observed waiting times (in minutes) of a random sample of 9 customers after the increased staff strength are:

3.6, 2.5, 2.4, 3.5, 2.8, 2.9, 3.1, 3.7, 3.0

(a) Find the unbiased estimates of the population mean and variance of the waiting time. [2]

(b) Using the value $k = 3.3$.

(i) Carry out a one-tailed test at 5% level of significance to determine whether the manager's claim is valid, stating clearly the p-value and any assumption(s) you have made. [5]

(ii) Explain the meaning of p-value in the context of the question. [1]

(c) Using the same assumption in part **(b)(i)**, a one-tailed test was carried out at 10 % and it was found that the manager's claim was justified. Find the range of possible values of k. [4]

11 **(a)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A) and (B) below. In each case, your diagram should include 6 points, approximately equally spaced with respective to x, and with all x- and y-values positive. The letters a, b, c and d represent constants.

(A) $y = a + b\sqrt{x}$, where a is positive and b is negative,

(B) $y = c + \dfrac{d}{x}$, where c is positive and d is negative. [2]

(b) The data below shows the diameter h, in cm, of coconuts of 9 different ages, in k months.

Age in month (k)	1	2	3.5	4.5	5	7.5	9	10.5	12
Diameter in cm (h)	6	9.5	13	15	16.5	17.5	18.5	19	19.5

(i) Calculate the value of the product moment correlation coefficient r between k and h. State, with a reason, whether the value of r would be different if the diameter is measured in inches instead. [2]

(ii) Give a sketch of the scatter diagram for the data. [1]

(iii) It is desired to predict the diameter of coconuts in the long run. Explain why neither a linear nor a quadratic model is likely to be appropriate. [2]

It is suggested that diameter h can be modelled by the formula $h = m + n\ln k$.

(iv) Find the equation of the regression line for the suggested model, and the value of the product moment correlation coefficient between $\ln k$ and h. [2]

(v) Estimate, to 1 decimal place, the age of a coconut with a diameter of 13.7 cm, and comment on the reliability of your answer. [2]

(vi) Given that 1 inch = 2.54 cm, state the equation of regression line if the diameter is measured in inches instead of cm. [1]