# H2 Mathematics (9758)
# Chapter 6 3D Vector Geometry
# Extra Practice Questions Solutions

| Qn 1 | |
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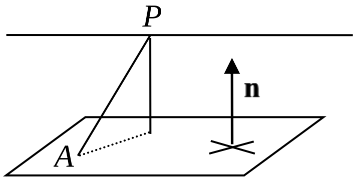
| | $l: \mathbf{r}=\begin{pmatrix}2\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\4\end{pmatrix}, \lambda\in\mathbb{R}$, and $\pi: \mathbf{r}.\begin{pmatrix}1\\5\\1\end{pmatrix}=5$<br>$\begin{pmatrix}1\\-1\\4\end{pmatrix}\bullet\begin{pmatrix}1\\5\\1\end{pmatrix}=1-5+4=0 \Rightarrow$ $l$ and $\pi$ are parallel.<br><br><br>$P(2,-2,3)$ is a point on $l$.<br>$A(0,1,0)$ is a point on $\pi$.<br>Shortest distance between $l$ and $\pi$<br>$=\left\lvert\overrightarrow{PA}\bullet\mathbf{n}\right\rvert=\left\lvert\left[\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{pmatrix}2\\-2\\3\end{pmatrix}\right]\bullet\frac{1}{\sqrt{27}}\begin{pmatrix}1\\5\\1\end{pmatrix}\right\rvert=\frac{1}{\sqrt{27}}\left\lvert\begin{pmatrix}-2\\3\\-3\end{pmatrix}\bullet\begin{pmatrix}1\\5\\1\end{pmatrix}\right\rvert=\frac{10}{\sqrt{27}}=\frac{10}{9}\sqrt{3}$ units |

| Qn 2 | 2007/IJC/I/5 |
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| (i) | $\overrightarrow{OA}=\begin{pmatrix}1\\1\\1\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}4\\3\\2\end{pmatrix}, \overrightarrow{OC}=\begin{pmatrix}-7\\-2\\-1\end{pmatrix}$<br>Find any 2 of the 3 vectors: $\overrightarrow{AB}=\begin{pmatrix}3\\2\\1\end{pmatrix}, \overrightarrow{BC}=\begin{pmatrix}-11\\-5\\-3\end{pmatrix}, \overrightarrow{AC}=\begin{pmatrix}-8\\-3\\-2\end{pmatrix}$<br>Since $\overrightarrow{AB}$ not parallel to $\overrightarrow{BC}$ (or equivalent), therefore *A*, *B* & *C* not collinear. |
| (ii) | $\begin{pmatrix}3\\2\\1\end{pmatrix}\times\begin{pmatrix}11\\5\\3\end{pmatrix}$ or $\begin{pmatrix}3\\2\\1\end{pmatrix}\times\begin{pmatrix}8\\3\\2\end{pmatrix}$ or equivalent, Vector $\perp$ to plane *ABC* $=\begin{pmatrix}1\\2\\-7\end{pmatrix}$<br>(or $-\begin{pmatrix}1\\2\\-7\end{pmatrix}$) |

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| **(iii)** | $\overrightarrow{OP}=\begin{pmatrix}2\\4\\7\end{pmatrix}$ and $\overrightarrow{OQ}=\begin{pmatrix}4\\4\\6\end{pmatrix} \Rightarrow \overrightarrow{PQ}=\begin{pmatrix}2\\0\\-1\end{pmatrix}$ <br> **<u>Method 1:</u>** <br> Length of projection $=\frac{\lvert\overrightarrow{PQ}\times \underset{\sim}{n}\rvert}{\lvert\underset{\sim}{n}\rvert}=\frac{\left\lvert\begin{pmatrix}2\\0\\-1\end{pmatrix}\times\begin{pmatrix}1\\2\\-7\end{pmatrix}\right\rvert}{\sqrt{1+4+49}}=\frac{\left\lvert\begin{pmatrix}2\\13\\4\end{pmatrix}\right\rvert}{\sqrt{54}}$ <br> $=\frac{\sqrt{4+169+16}}{\sqrt{54}}=\sqrt{\frac{7}{2}}$ *or* $\frac{\sqrt{14}}{2}$ <br> **<u>Method 2:</u>** Length of projection of $\overrightarrow{PQ}$ onto $\underset{\sim}{n}$ $=\frac{\lvert\overrightarrow{PQ}\bullet \underset{\sim}{n}\rvert}{\lvert\underset{\sim}{n}\rvert}=\frac{\left\lvert\begin{pmatrix}2\\0\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-7\end{pmatrix}\right\rvert}{\sqrt{1+4+49}}=\frac{9}{\sqrt{54}}$ <br> Using Pythagoras' Theorem, length of projection of $\overrightarrow{PQ}$ onto plane <br> $=\sqrt{5-\frac{81}{54}}=\sqrt{\frac{7}{2}}$ *or* $\frac{\sqrt{14}}{2}$ |

| **Qn 3** | **2007/PJC/I/6** |
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| **(i)** | $l_1:\mathbf{r}=\begin{pmatrix}p\\3\\7\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}, \lambda\in\mathbb{R}$ <br> Given $q=1$ and $p=4$, <br> Since $C$ lies on $l_1$, $\overrightarrow{OC}=\begin{pmatrix}4\\3\\7\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}$; <br> $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}4\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}$ <br> $\overrightarrow{AC}\perp l_2 \Rightarrow \begin{pmatrix}4+\lambda\\\lambda\\4\end{pmatrix}\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}=0$ <br> $\Rightarrow \lambda=-12$ <br> $\overrightarrow{OC}=-8\mathbf{i}-9\mathbf{j}+7\mathbf{k}$ |
| **(ii)** | $\overrightarrow{AB}=q\mathbf{i}+2\mathbf{k}$ <br> Given acute angle between $l_1$ and $l_2$ is $60°$, <br> $\cos 60° = \frac{\left\lvert\begin{pmatrix}1\\1\\0\end{pmatrix}\bullet\begin{pmatrix}q\\0\\2\end{pmatrix}\right\rvert}{\sqrt{2}\sqrt{q^2+4}}$ |

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| | $\sqrt{2}\sqrt{q^2+4}=2q \;\Rightarrow q=\pm 2$ |

| **Qn 4** | **2008/JJC/I/3** |
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| **(i)** | $l_1:\mathbf{r}=\begin{pmatrix}1\\3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\7\end{pmatrix},\quad \lambda\in\mathbb{R}$<br>since $P$ lies on $l_1$,<br>$\begin{pmatrix}a\\1\\16\end{pmatrix}=\begin{pmatrix}1\\3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\7\end{pmatrix}$ for some suitable $\lambda\in\mathbb{R}$<br>$\Rightarrow 3-\lambda=1 \quad \Rightarrow \lambda=2$<br>$\Rightarrow a=1+\lambda=3\,(\text{proven})$ |
| **(ii)** | since $Q$ lies on $l_2$,<br>$\overrightarrow{OQ}=\begin{pmatrix}1\\3\\2\end{pmatrix}+\mu\begin{pmatrix}0\\-4\\3\end{pmatrix}$ for some suitable $\mu\in\mathbb{R}$<br>$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=\begin{pmatrix}-2\\2\\-14\end{pmatrix}+\mu\begin{pmatrix}0\\-4\\3\end{pmatrix}=\begin{pmatrix}-2\\2-4\mu\\-14+3\mu\end{pmatrix}$<br>since $PQ\perp l_2$<br>$\begin{pmatrix}-2\\2-4\mu\\-14+3\mu\end{pmatrix}\bullet\begin{pmatrix}0\\-4\\3\end{pmatrix}=0$<br>$(0)+(-8+16\mu)+(-42+9\mu)=0$<br>$\Rightarrow \mu=2$<br>$\Rightarrow \overrightarrow{OQ}=\begin{pmatrix}1\\-5\\8\end{pmatrix}(\text{ans})$ |
| **(iii)** | $\left\lvert\begin{pmatrix}1\\-1\\7\end{pmatrix}\bullet\begin{pmatrix}0\\-4\\3\end{pmatrix}\right\rvert=\sqrt{1+1+49}\sqrt{0+16+9}\cos\theta$<br>$\cos\theta=\dfrac{25}{5\sqrt{51}}\Rightarrow\theta=45.6^\circ$ (to 1 d.p.) |

| **Qn 5** | **2008/HCI/I/12** |
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| | Required Distance $= \frac{(\mathbf{a}-\mathbf{r}_1)\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert} = \frac{\mathbf{a}\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert} - \frac{\mathbf{r}_1\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert}$ <br> $= \begin{pmatrix}1\\7\\-10\end{pmatrix}\bullet\frac{1}{\sqrt{21}}\begin{pmatrix}1\\2\\-4\end{pmatrix} - \frac{13}{\sqrt{21}} = \frac{42}{\sqrt{21}} = 2\sqrt{21}$ <br><br> **<u>Alternatively</u>** <br> Take a point in $\prod_1$, say $C$ ( 13,0,0) which satisfies $\mathbf{r}\bullet\begin{pmatrix}1\\2\\-4\end{pmatrix}=13$ <br> $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}12\\-7\\10\end{pmatrix}$ <br> Length of projection of $\overrightarrow{AC}$ onto $\begin{pmatrix}1\\2\\-4\end{pmatrix}$ $= \left\lvert\frac{\overrightarrow{AC}\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert}\right\rvert = \left\lvert\begin{pmatrix}12\\-7\\10\end{pmatrix}\bullet\frac{1}{\sqrt{21}}\begin{pmatrix}1\\2\\-4\end{pmatrix}\right\rvert = 2\sqrt{21}$ <br><br> **<u>Alternatively: Finding foot of perpendicular first (Long Method)</u>** <br> Vector equation of a line through $A$ and parallel to $\begin{pmatrix}1\\2\\-4\end{pmatrix}$: <br> $\mathbf{r}=\begin{pmatrix}1\\7\\-10\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-4\end{pmatrix},\ \lambda\in\mathbb{R}$ <br> Let $C$ be the foot of the perpendicular from $A$ to $\prod_1$ <br> Then $\overrightarrow{OC}=\begin{pmatrix}1+\lambda\\7+2\lambda\\-10-4\lambda\end{pmatrix}$ <br> $C$ lies on $\prod_1$: $\begin{pmatrix}1+\lambda\\7+2\lambda\\-10-4\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-4\end{pmatrix}=13 \Rightarrow \lambda=-2$ <br> $\overrightarrow{OC}=\begin{pmatrix}1+\lambda\\7+2\lambda\\-10-4\lambda\end{pmatrix}=\begin{pmatrix}-1\\3\\-2\end{pmatrix}$; $\overrightarrow{AC}=\begin{pmatrix}1+\lambda-1\\7+2\lambda-7\\-10-4\lambda+10\end{pmatrix}$ |

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| | $\Rightarrow$ Required distance $= AC = \sqrt{84} = 2\sqrt{21}$ |
| **(ii)** | Since $d$ is positive, the angle between $\left(\mathbf{a}-\mathbf{r}_1\right)$ & $\mathbf{n}$ is acute<br>$\overrightarrow{OB} = \overrightarrow{OA} - 2\,d\,\dfrac{\mathbf{n}}{\lvert\mathbf{n}\rvert} = \begin{pmatrix}1\\7\\-10\end{pmatrix} - 2\left(2\sqrt{21}\right)\dfrac{1}{\sqrt{21}}\begin{pmatrix}1\\2\\-4\end{pmatrix} = \begin{pmatrix}-3\\-1\\6\end{pmatrix}$<br>**<u>Alternatively: Finding foot of perpendicular first (Long Method)</u>**<br>Vector equation of a line through $A$ and parallel to $\begin{pmatrix}1\\2\\-4\end{pmatrix}$:<br>$\mathbf{r} = \begin{pmatrix}1\\7\\-10\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-4\end{pmatrix}, \lambda \in \mathbb{R}$<br>Let $C$ be the foot of the perpendicular from $A$ to $\prod_1$<br>Then $\overrightarrow{OC} = \begin{pmatrix}1+\lambda\\7+2\lambda\\-10-4\lambda\end{pmatrix}$<br>$C$ lies on $\prod_1$: $\begin{pmatrix}1+\lambda\\7+2\lambda\\-10-4\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-4\end{pmatrix} = 13 \quad \Rightarrow \lambda = -2$<br>$\overrightarrow{OC} = \begin{pmatrix}1+\lambda\\7+2\lambda\\-10-4\lambda\end{pmatrix} = \begin{pmatrix}-1\\3\\-2\end{pmatrix}$<br>By ratio theorem : $\overrightarrow{OC} = \dfrac{1}{2}\left(\overrightarrow{OA}+\overrightarrow{OB}\right) \Rightarrow \overrightarrow{OB} = 2\overrightarrow{OC} - \overrightarrow{OA}$ $\therefore \overrightarrow{OB} = \begin{pmatrix}-3\\-1\\6\end{pmatrix}$ |
| **(iii)** | $\prod_1$ : $x + 2y - 4z = 13$ ……. (1)<br>$\prod_2$ : $x + 3y + 3z = -8$ ……. (2)<br>By G.C. solve equations (1) & (2)<br>The vector equation of the line of intersection is<br>$l : \mathbf{r} = \begin{pmatrix}55\\-21\\0\end{pmatrix} + \lambda\begin{pmatrix}18\\-7\\1\end{pmatrix}$ where $\lambda \in \mathbb{R}$ or $\mathbf{r} = \begin{pmatrix}1\\0\\-3\end{pmatrix} + \lambda\begin{pmatrix}18\\-7\\1\end{pmatrix}$ etc |
| **(iv)** | Since $B$ and $l$ lie on the image plane of $\prod_2$ so the equation of image plane is<br>$\mathbf{r} = \begin{pmatrix}-3\\-1\\6\end{pmatrix} + \alpha\begin{pmatrix}18\\-7\\1\end{pmatrix} + \beta\left[\begin{pmatrix}55\\-21\\0\end{pmatrix} - \begin{pmatrix}-3\\-1\\6\end{pmatrix}\right]$<br>$\mathbf{r} = \begin{pmatrix}-3\\-1\\6\end{pmatrix} + \alpha\begin{pmatrix}18\\-7\\1\end{pmatrix} + \beta\begin{pmatrix}29\\-10\\-3\end{pmatrix}$ where $\alpha$ and $\beta \in \mathbb{R}$ |

| **Qn 6** | **2010 DHS Prelim/P2/Q4** |
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| **(i)** | Let $\theta$ be acute angle between the 2 planes. <br> $\cos\theta = \left\|\frac{\mathbf{n}_1 \bullet \mathbf{n}_2}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|}\right\| = \frac{\left\|\begin{pmatrix}2\\4\\1\end{pmatrix}\bullet\begin{pmatrix}1\\3\\1\end{pmatrix}\right\|}{\sqrt{21}\sqrt{11}} = \frac{15}{\sqrt{21}\sqrt{11}}$ <br> $\therefore \theta = 9.3^\circ$. |
| **(ii)** | $2x+4y+z=10$ <br> $x+3y+z=8$ <br> Using GC, <br> Let $z=t\in\mathbb{R}$, <br> $\Rightarrow \quad x=-1+\frac{t}{2}, \quad y=3-\frac{t}{2},$ <br> $\therefore \quad l_1: \ \mathbf{r} = \begin{pmatrix}-1\\3\\0\end{pmatrix} + \alpha\begin{pmatrix}1\\-1\\2\end{pmatrix}, \quad \alpha = \frac{t}{2}\in\mathbb{R}$ |
| **(iii)** | Since the point with co-ordinates (6,m.5) lies on the first plane, <br> $\mathbf{a}\bullet\mathbf{d}_1 = D_1$ <br> $\Rightarrow \begin{pmatrix}6\\m\\5\end{pmatrix}\bullet\begin{pmatrix}2\\4\\1\end{pmatrix} = 10$ <br> $\Rightarrow 12+4m+5=10$ <br> $\Rightarrow m = -\frac{7}{4}$ |
| **(iv)** | $l_2$**:** $\mathbf{r} = \mathbf{a_2} + \beta\mathbf{d_2} = \begin{pmatrix}2\\m\\7\end{pmatrix} + \beta\begin{pmatrix}2\\0\\-1\end{pmatrix}, \qquad \beta\in\mathbb{R}.$ <br> $\mathbf{d_1}\bullet\mathbf{d_2} = \begin{pmatrix}1\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}2\\0\\-1\end{pmatrix} = 2-2=0 \qquad (\textit{independent of the value of } m)$ <br> Therefore lines $l_1$ and $l_2$ are perpendicular for all real values of *m*. |

| **Qn 7** | **2009/CJC/I/11** |
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| **(i)** | Let $\mathbf{n}_1$ and $\mathbf{n}_2$ be the normals of $p_1$ and $p_2$ respectively.<br>$\mathbf{n}_1 = \begin{pmatrix}1\\0\\-1\end{pmatrix}\times\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$<br>$\mathbf{n}_2 = \begin{pmatrix}2\\3\\0\end{pmatrix}\times\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}3\\-2\\-3\end{pmatrix}$<br>$\begin{pmatrix}1\\-1\\1\end{pmatrix}\times\begin{pmatrix}3\\-2\\-3\end{pmatrix}=\begin{pmatrix}5\\6\\1\end{pmatrix}$<br>Therefore $l$ is parallel to $5\mathbf{i} + 6\mathbf{j} + \mathbf{k}$ |
| **(ii)** | Acute angle bet. $p_1$ and $p_2 = \cos^{-1}\dfrac{\left\|\begin{pmatrix}1\\-1\\1\end{pmatrix}\bullet\begin{pmatrix}3\\-2\\-3\end{pmatrix}\right\|}{\sqrt{3}\sqrt{22}}$<br>$= 75.7^\circ \quad (1 \text{ d.p.})$ |
| **(iii)** | Perpendicular distance $= \dfrac{\left\|\left[\begin{pmatrix}4\\2\\2\end{pmatrix}-\begin{pmatrix}2\\4\\6\end{pmatrix}\right]\bullet\begin{pmatrix}3\\-2\\-3\end{pmatrix}\right\|}{\sqrt{22}}$<br>$= \dfrac{\left\|\begin{pmatrix}2\\-2\\-4\end{pmatrix}\bullet\begin{pmatrix}3\\-2\\-3\end{pmatrix}\right\|}{\sqrt{22}}$<br>$= \dfrac{\|-22\|}{\sqrt{22}} = \sqrt{22}$ |
| **(iv)** | $p_1$: $\mathbf{r} \cdot \begin{pmatrix}1\\-1\\1\end{pmatrix} = 4$ and $p_3$: $\mathbf{r} \cdot \begin{pmatrix}2\\-2\\2\end{pmatrix} = b$<br>Distance between $p_1$ and $p_3$ = $\dfrac{1}{\sqrt{3}}$<br>$\left\|\dfrac{4}{\sqrt{3}}-\dfrac{b}{2\sqrt{3}}\right\| = \dfrac{1}{\sqrt{3}}$<br>$\dfrac{4}{\sqrt{3}}-\dfrac{b}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}$ or $\dfrac{4}{\sqrt{3}}-\dfrac{b}{2\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$<br>$b$ = 6 or 10 |

| Qn 8 | 2009/MJC/I/9 |
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| **(i)** | Sub $\left(\alpha,\beta,0\right)$ into $\Pi_1$ and $\Pi_2$.<br>$\pi_1$ : $1(\alpha)+3(\beta)+a(0)=8$<br>$\pi_2$ : $3(\alpha)+1(\beta)+b(0)=0$<br><br>Using GC, $\alpha=-1,\beta=3$ |
| **(ii)** | $\begin{pmatrix}1\\3\\a\end{pmatrix}\times\begin{pmatrix}3\\1\\b\end{pmatrix}=\begin{pmatrix}3b-a\\3a-b\\-8\end{pmatrix}$<br>$l_1:\mathrm{r}=\begin{pmatrix}-1\\3\\0\end{pmatrix}+\lambda\begin{pmatrix}3b-a\\3a-b\\-8\end{pmatrix} \quad ,\lambda\in\mathbb{R}$ |
| **(iii)** | $\begin{pmatrix}-1\\3\\0\end{pmatrix}+\lambda\begin{pmatrix}3b-a\\3a-b\\-8\end{pmatrix}=\begin{pmatrix}5\\2\\2\end{pmatrix}+\mu\begin{pmatrix}4\\1\\0\end{pmatrix}$<br><br>$\Rightarrow\lambda=-\dfrac{1}{4}$<br>Sub $a=-b$ and $\lambda=-\dfrac{1}{4}$, we have<br>$-1-\dfrac{1}{4}\left(-4a\right)=5+4\mu$<br>$3-\dfrac{1}{4}\left(4a\right)=2+\mu$<br>$\therefore \quad a=2$<br>$\quad\quad b=-2$ |

| **Qn 9** | **2014 SRJC P2 Q1** |
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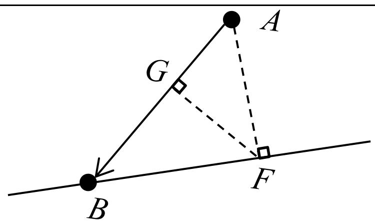
| **(i)** | Vector equation $l$ is $\mathbf{r}=\begin{pmatrix}4\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-4\\0\\3\end{pmatrix},\ \lambda\in\mathbb{R}$<br>A unit vector **c** parallel to $l=\frac{1}{5}\begin{pmatrix}-4\\0\\3\end{pmatrix}$ (OR $\frac{1}{5}\begin{pmatrix}4\\0\\-3\end{pmatrix}$)<br>$\left\|\overrightarrow{AB}\bullet\mathbf{c}\right\|=\left\|\left(\begin{pmatrix}0\\1\\3\end{pmatrix}-\begin{pmatrix}2\\1+\sqrt{5}\\-1\end{pmatrix}\right)\bullet\frac{1}{5}\begin{pmatrix}-4\\0\\3\end{pmatrix}\right\|$<br>$=\frac{1}{5}\left\|\begin{pmatrix}-2\\-\sqrt{5}\\4\end{pmatrix}\bullet\begin{pmatrix}-4\\0\\3\end{pmatrix}\right\|=4$<br>$\left\|\overrightarrow{AB}\bullet\mathbf{c}\right\|$ is the length of projection $\overrightarrow{AB}$ onto $l$ (or onto **c**) |
| **(ii)** | $\left\|\overrightarrow{AB}\right\|=\left\|\begin{pmatrix}-2\\-\sqrt{5}\\4\end{pmatrix}\right\|=5$<br>By Pythagoras' Theorem,<br>The shortest distance from $A$ to $l=\sqrt{5^2-4^2}=3$ |
| **(iii)** | $\Delta AGF$ and $\Delta BGF$ are similar triangles.<br>Since $AF$ corresponds to $BF$,<br>$\frac{\text{Area of }\Delta AGF}{\text{Area of }\Delta BGF}=\frac{3^2}{4^2}=\frac{9}{16}$<br> |
| **(iv)** | $\frac{\text{Area of }\Delta AGF}{\text{Area of }\Delta BGF}=\frac{\frac{1}{2}(AG)(GF)}{\frac{1}{2}(BG)(GF)}=\frac{AG}{BG}$<br>From above, $\frac{AG}{BG}=\frac{9}{16}$ |

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| | $\overrightarrow{OG}=\dfrac{9\overrightarrow{OB}+16\overrightarrow{OA}}{25}$ <br> $=\dfrac{1}{25}\left(9\begin{pmatrix}0\\1\\3\end{pmatrix}+16\begin{pmatrix}2\\1+\sqrt{5}\\-1\end{pmatrix}\right)$ <br> $=\dfrac{1}{25}\begin{pmatrix}32\\25+16\sqrt{5}\\11\end{pmatrix}$ |