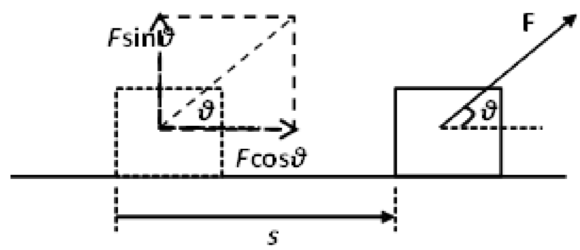
Work, Energy and Power

Work done by a force on a body is defined as the product of the **average** force acting on an object and the displacement moved in the direction of the force.

The general case:

The force (*F*) and displacement (*s*) may not point in the same direction.



Only the component of the force along the displacement does work.

The work done by *F* above is given by

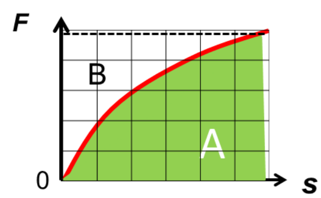
$$W = (F\cos\theta)\, s$$

Work done by a variable force

The work done by *F* can be obtained by finding the area under the *F-s* graph.

$$W = \int F\, ds$$

Or estimate the area under, e.g. by counting squares and multiplying by the area of each square.



W = area A for a F-s graph
W = area B for a s-F graph

The work done by a force can be either positive or negative

It is positive if the force acts in the **same direction** as the displacement.

It is negative if the force is in the **opposite direction** to the displacement.

When a force does positive work on a body, it increases the energy of the body.

When a force does negative work on a body, it removes energy from the body.

Different forms of Energy

(1) **Mechanical energy:** energy that is possessed by an object due to its motion or due to its position.

Mechanical energy = Kinetic energy + Gravitational potential energy + Elastic potential energy

(2) **Energy in a matter:** chemical energy, nuclear energy and internal energy.

(3) **Electromagnetic energy**: electric potential energy, magnetic potential energy, and radiant energy

Principle of Conservation of Energy

Energy cannot be created nor destroyed but it may be transformed (converted) from one form to another.

In general,

$$W = \int F\,dx$$

If F is a conservative force, this work does not change the total energy of the system.

This work transforms the energy of the system between kinetic energy and potential energy only. Thus any kinetic energy K gained by such a system must equal to its loss in potential energy U and vice versa. That is,

$W = dK = -dU$ [d*K*: change in *K*; d*U*: change in *U*]

$\int F\,dx = -dU$

$$\boxed{F = -\frac{dU}{dx}}$$

- $F = -\frac{dU}{dx}$ is interpreted as:
  - The magnitude of the *F* at a point at displacement *x* is given by the gradient of the *U*-*x* graph at that point.
  - The negative sign mean that *F* acts in the direction in which *potential energy* would decrease when an object is moved by the force *F*.

- When applying the equation $F = -\frac{dU}{dx}$, U refers to the type of potential energy that is of the same nature as the force F. That is,

F	U
Elastic force	Elastic Potential Energy
Gravitational force	Gravitational Potential Energy
Electric force	Electric Potential Energy

Power is the rate of doing work with respect to time (or work done per unit time) or the rate of transfer of energy with respect to time.

Average Power

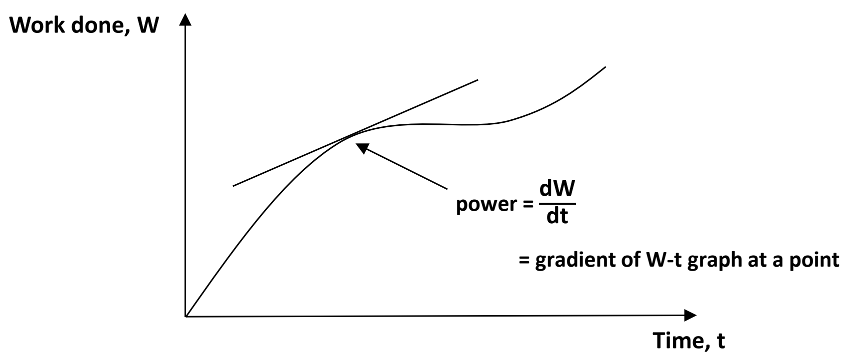
$$\text{Average power} = \frac{\text{Total work done}}{\text{time taken for this amount of work to be done}}$$

$$\langle P \rangle = \frac{W}{t}$$

Instantaneous Power

The rate at which work is done might not be constant.

On a graph of work done against time, instantaneous power $\left(\frac{dW}{dt}\right)$ is the gradient of the graph at a particular instant, as shown below.



Mathematically, the instantaneous power is expressed as:

$$P = \frac{dW}{dt} = \frac{d(F.s)}{dt}$$

If the force F remains constant with time,

$$P = F\frac{d(s)}{dt} = Fv$$

$$\boldsymbol{Efficiency = \frac{useful\ energy\ output}{total\ energy\ input} x100\%}$$

or

$$\boldsymbol{Efficiency = \frac{useful\ power\ output}{total\ power\ input} x100\%}$$