

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

3

- 1 A cylinder has a radius of $(\sqrt{2}-1)$ cm and a volume of $(12+3\sqrt{2})\pi$ cm³.

Find, **without using a calculator**, the exact value of its height, h cm, in the form $a+b\sqrt{2}$, where a and b are integers.

[3]

[Turn over

4

- 2 Suppose that x and y are non-zero real numbers such that $\frac{x}{3} = y^2$ and $\frac{x}{9} = 9y$.

Find the value of $x - y$.

[3]

5

- 3 Express $y = -2x^2 - 12x + 1$ in the form $y = a - 2(x + b)^2$ and hence state the maximum value of y , and its corresponding value of x . [4]

[Turn over

6

- 4 A curve is such that $\frac{dy}{dx} = \frac{2}{x^2} + \frac{3}{(7-2x)}$ and $P(3, 1)$ is a point on the curve.

Find the equation of the curve.

[4]

7

5 Express $\frac{2x-1}{x^2(x+1)}$ in partial fractions.

[6]

[Turn over]

8

- 6 The function f is defined by $f(x) = 3x^3 - 5cx^2 + kc^2x + 4c^3$, where c and k are non-zero constants. $f(x)$ leaves a remainder of $-32c^3$ when divided by $x + 2c$.

(a) Find the value of k .

[2]

- (b) Using the value of k , determine whether $x^2 - cx - 2c^2$ is a factor of $f(x)$.
Justify your answer.

[3]

9

- 7 The function f is defined by $f(x) = 4 \cos ax + b$ for $0 \leq x \leq \pi$, where a and b are constants. The period of f is $\frac{2\pi}{3}$ and the function has a maximum value of 2.

(i) State the amplitude of f . [1]

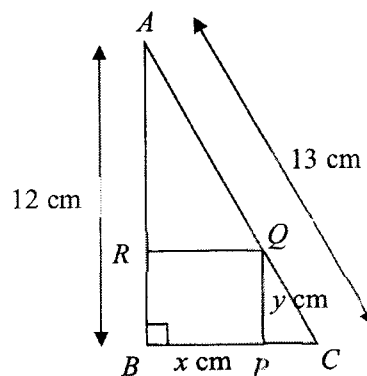
(ii) Write down the value of a and of b . [2]

(iii) Sketch the graph of $y = f(x)$ for $0 \leq x \leq \pi$, indicating clearly the x -coordinates, in terms of π , of the points where the graph crosses the x -axis. [4]

[Turn over

10

- 8 In $\triangle ABC$, $\angle ABC = 90^\circ$, $AB = 12$ cm and $AC = 13$ cm. The rectangle $BPQR$ is such that its vertices P , Q and R lie on BC , CA and AB respectively.



It is given that $BP = x$ cm and $PQ = y$ cm.

- (a) Show that $y = \frac{60 - 12x}{5}$.

[2]

- (b) Show that the area, A cm², of the rectangle $BPQR$ is given by $A = 12x - \frac{12x^2}{5}$.

[1]

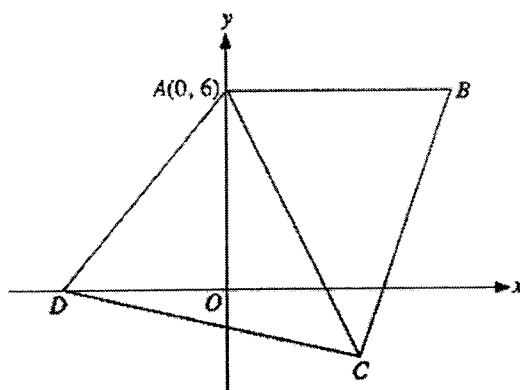
11

- 8 (c) Given that x can vary, find the stationary value of A and determine its nature. [4]

[Turn over

12

- 9 The diagram shows a quadrilateral $ABCD$ in which A is $(0, 6)$ and AB is parallel to the x -axis. D is a point on the x -axis such that the equation of DC is $x + 5y = -6$. AC is perpendicular to the line $2y - x = 7$.



(a) Find,

(i) the equation of AC ,

[2]

(ii) the coordinates of C .

[2]

13

- 9 (b) Given that the area of $\triangle ACD$ is 1.5 times that of $\triangle ABC$, find the coordinates of B . [3]

- (c) Showing your working clearly, explain whether $ABCD$ is a kite. [2]

[Turn over

14

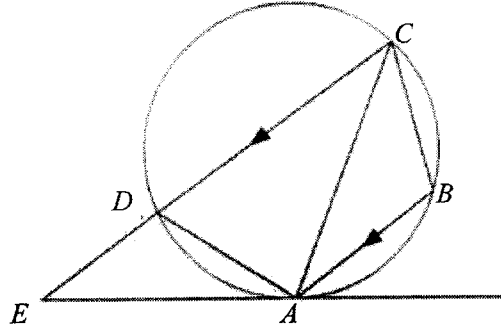
10 (a) Prove the identity $\frac{1 - \sin^4 x}{\sin^2 x} = \cot^2 x + \cos^2 x$. [4]

15

- 10 (b) Hence solve the equation $\cot^2 2x + \cos^2 2x = 0$ for $0^\circ < x < 180^\circ$. [4]

[Turn over

- 11 The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of a circle. The point E lies on the extended line CD such that AE is a tangent to the circle at A . CD and AB are parallel lines.



- (a) Explain why angle $CBA = \text{angle } EDA$. [2]

- (b) Show that triangles DAE and BAC are similar. [2]

- 11 (c) Given that $AD = DE$, explain why the line AC bisects the angle BCD . [3]

[Turn over

18

- 12 (a) Given that $2^{x-2} \times 3^{x+2} = 6^{2x}$, show that $x = \frac{\lg 9 - \lg 4}{\lg 6}$ [4]

19

12 (b) Solve $\log_9 y - 2 = \log_3 2y$.

[4]

[Turn over

- 13 (a) Water leaks from a container at a rate of $150 \text{ cm}^3/\text{s}$. The volume, $V \text{ cm}^3$, of the water in the container, when the height of water is $h \text{ cm}$, is given by

$$V = 10\pi + \frac{4\pi h^3}{9}. \text{ When } V = 334\pi \text{ cm}^3, \text{ find the}$$

- (i) value of h , [2]

- (ii) rate of change of h at this instant, correct to 3 significant figures. [3]

21

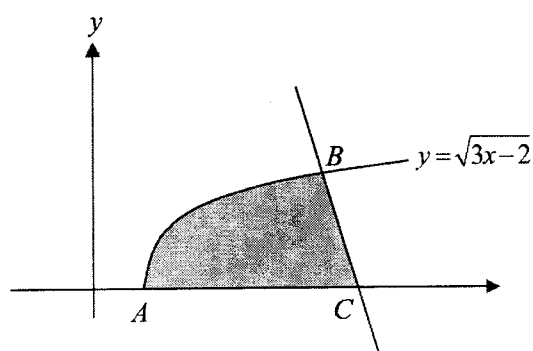
- 13 (b) The stopping distance, s m, of a car moving at v km/h can be modelled by the formula $\frac{s}{v} = \frac{1}{6} + \frac{v}{50}$.

(i) Find the rate at which s is changing with respect to v when $v = 60$. [3]

(ii) Explain the meaning of your answer to part (i). [1]

[Turn over

14



The diagram shows part of the curve $y = \sqrt{3x-2}$. The normal to the curve at B meets the x -axis at C . Given that the x -coordinate of B is 9, find

- (a) the coordinates of A and of C ,

[5]

23

14 (b) the area of the shaded region.

[5]

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Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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3

- 1 By using an appropriate substitution, solve the equation $e^{\frac{1}{2}x} = 2 + 24e^{-\frac{1}{2}x}$. [5]

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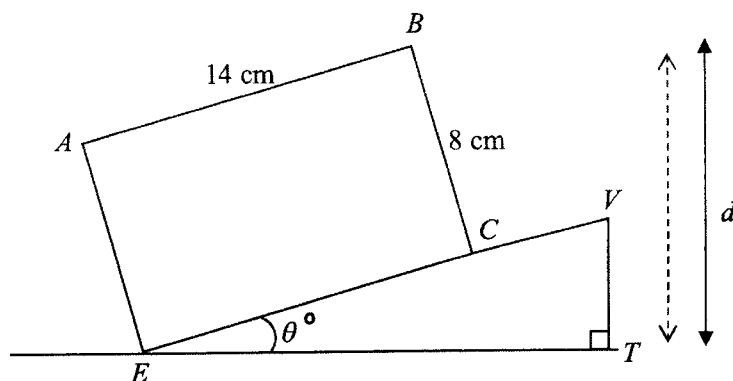
2 (a) Integrate $\left(\frac{6}{e^{3-x}}\right)^2$ with respect to x . [3]

(b) Explain, with working, why the equation $2x^3 + 3x^2 + 2x + 8 = 0$ has only 1 real root. Hence find this root. [5]

- 3 (a) Given that $y = x^2\sqrt{2x-1}$, show that $\frac{dy}{dx} = \frac{x(5x-2)}{\sqrt{2x-1}}$. Determine whether $y = x^2\sqrt{2x-1}$ is always an increasing function. [5]

- (b) Hence evaluate $\int_1^5 \frac{5x^2 - 2x + 1}{\sqrt{2x-1}} dx$. [4]

- 4 The diagram shows the front view of a rectangular block $ABCE$, with dimensions 14 cm by 8 cm, placed on a ramp, VE , tilted at an acute angle of θ° and $\angle VTE = 90^\circ$. The ramp is placed on a horizontal surface ET and d is the perpendicular distance from B to ET .



- (a) Show that $d = 14 \sin \theta + 8 \cos \theta$. [2]

- (b) Express d in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$. [3]

7

(c) State the maximum value of d and find the corresponding value of θ . [2]

(d) Find the smallest value θ such that $d = 13$. [2]

- 5 (a) Explain by completing the square that $x^2 - 2x + 3$ is always positive for all real values of x . [2]
- (b) Hence, find the range of values of p if the inequality $\frac{3x^2 + px + 3}{x^2 - 2x + 3} > 0$ is satisfied for all real values of x . [4]

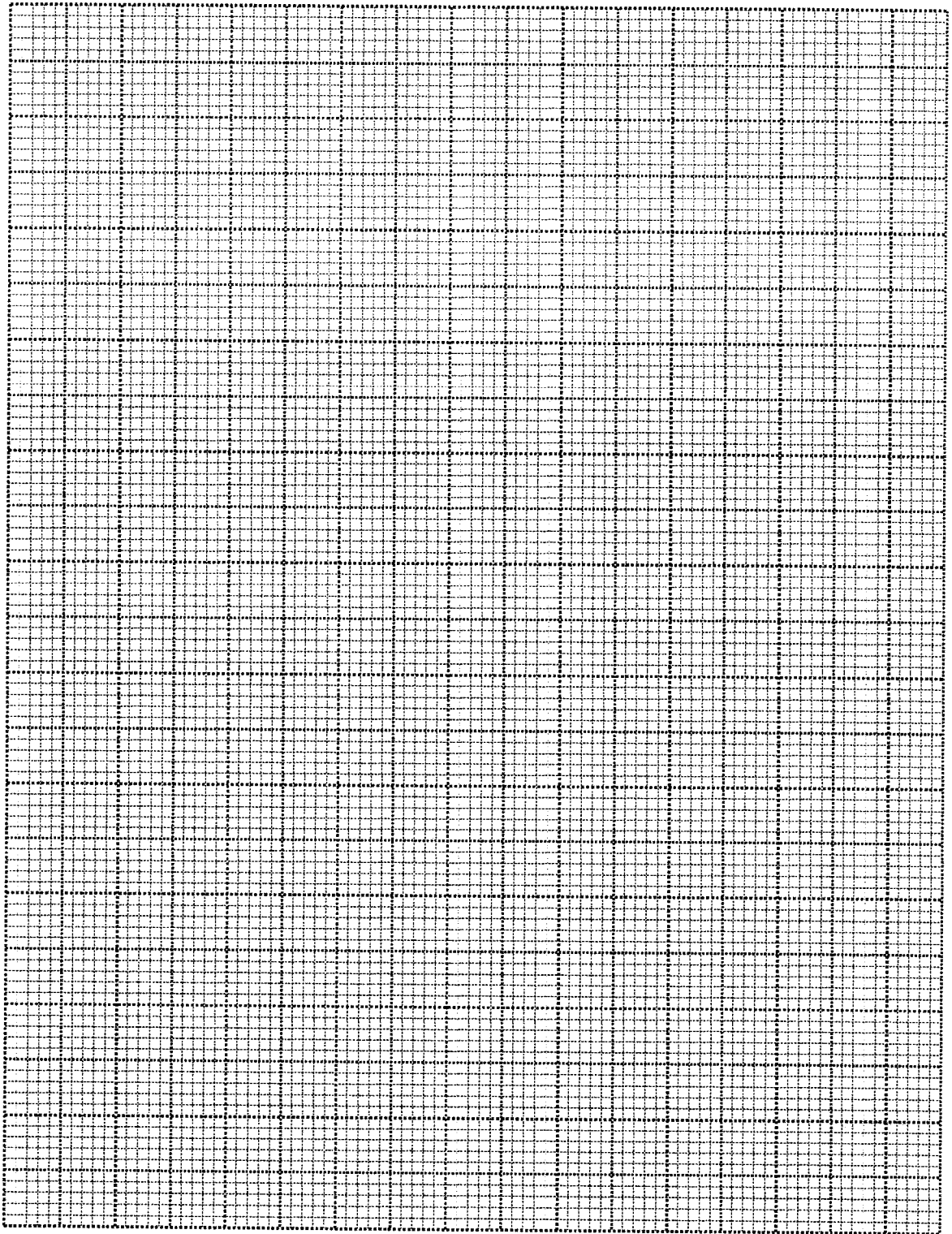
- 6 (a) In the binomial expansion of $\left(x + \frac{k}{x}\right)^5$, where k is a positive integer, the coefficients of $\frac{1}{x^3}$ and $\frac{1}{x}$ are the same. Find the value of k . [5]
- (b) Without expanding all the terms, explain why there is no constant term in the expansion of $(1 - 3x^2)\left(x + \frac{k}{x}\right)^5$. [3]

- 7 (a) The variables x and y , are related by the equation $\frac{3y^2}{k} + \frac{2x}{h} = 6$,
 where h and k are constants. When the graph of y^2 against x is plotted, a straight line is obtained. Given that the intercept on the y^2 axis is 8 and that the gradient of the line is -6 , calculate the value of h and of k . [3]

- (b) A glass of hot liquid is put on a table to cool. The temperature of the liquid, $T^\circ\text{C}$, after n minutes is given by $T = 25 + pe^{-kn}$, where p and k are constants. The table below shows the measured values of T and n .

n (minutes)	10	20	30	40	50
T ($^\circ\text{C}$)	79.6	58.1	45.1	37.2	32.4

- (i) Using the grid on page 11, plot $\ln(T - 25)$ against n and draw a straight line graph. [3]



12

Use your graph to estimate

(ii) value of p and of k ,

[4]

(iii) the temperature of the liquid after 35 minutes,

[2]

13

- (iv) the number of minutes it takes for the temperature of the liquid to drop by 75% of its initial value. [2]

- 8 The velocity, $v \text{ ms}^{-1}$, of a particle, moving in a straight line, t seconds after motion has begun, is given by $v = 6t^2 + kt + 12$, where k is a constant. The particle passes a fixed point O with an acceleration of -6 ms^{-2} when $t = 1$.

(a) Show that $k = -18$. [2]

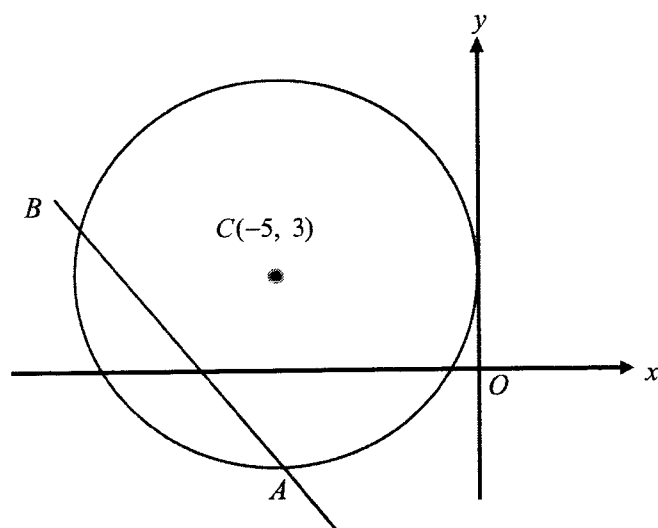
(b) Hence, find
(i) the minimum velocity achieved by the particle, [3]

15

- (ii) the total distance travelled during the first 4 seconds. [5]

16

9



- (a) The coordinates of the centre of a circle C_1 is $C(-5, 3)$. If the y -axis is a tangent of the circle, find the equation of the circle. [1]

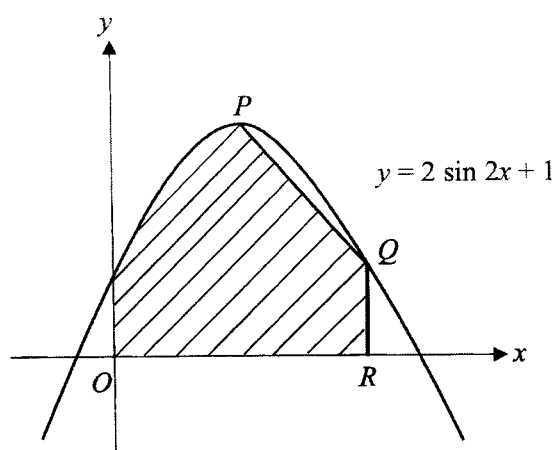
A straight line cuts the circle at points A and B such that AB is parallel to $y + 3x - 11 = 0$ and that AC is parallel to the y -axis.

- (b) Find
- (i) the equation of the line AB and the equation of the perpendicular bisector of AB , [5]

- (ii) hence, find the coordinates of B . [3]

- (c) Circle C_2 is obtained by reflecting circle C_1 in the line AB . Find the equation of circle C_2 . [3]

- 10 The diagram shows part of the curve $y = 2 \sin 2x + 1$. P is the maximum point of the curve and Q is the point on the curve at which the gradient of the tangent is -4 .



- (a) Find the coordinates of P and of Q .

[5]

19

- (b) Find the area of the shaded region bounded by the curve, the axes, line PQ and vertical line QR . Leave your answer in the exact form. [4]

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20

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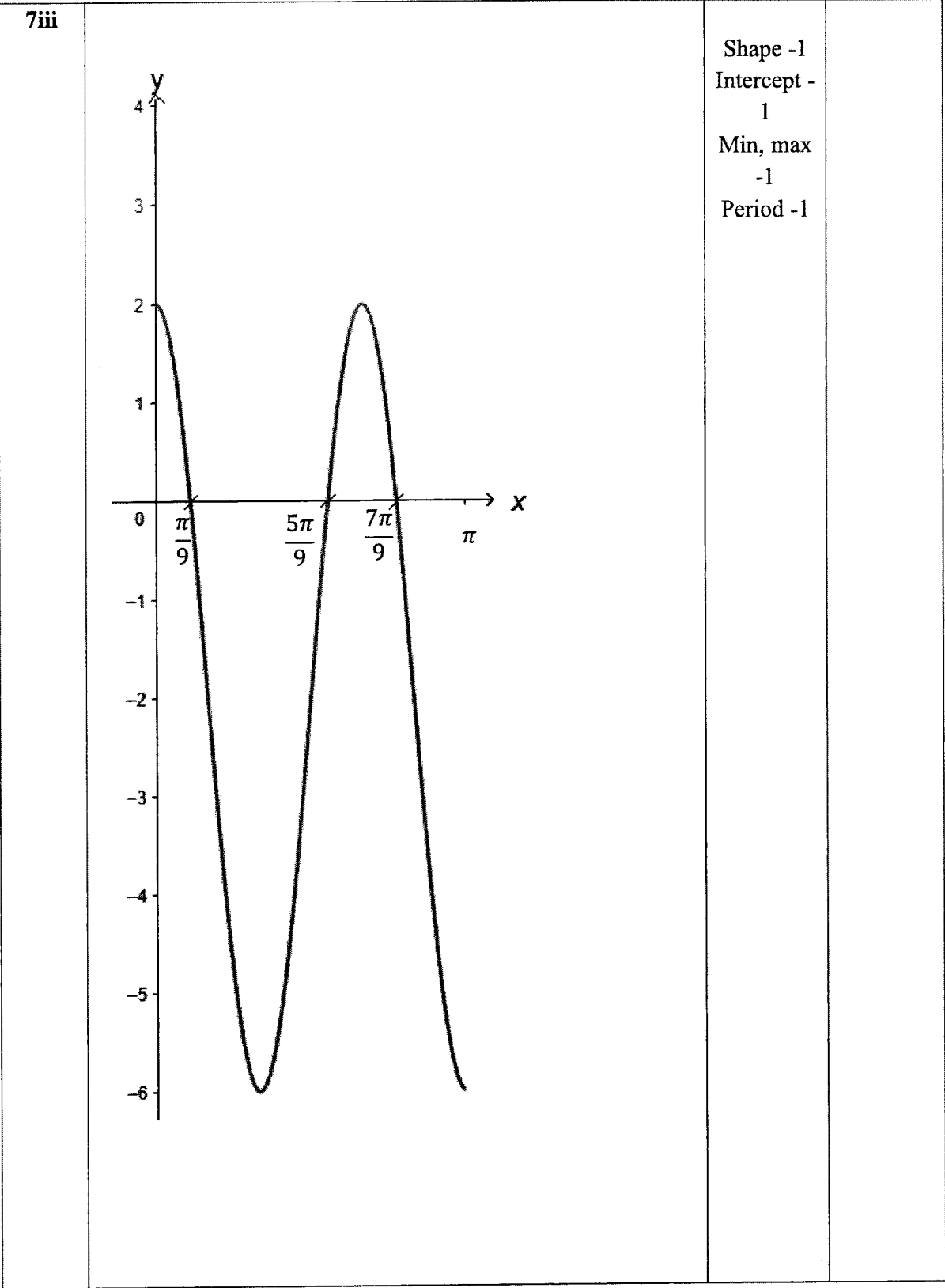
FHSS Prelim AM P1 2022 Marking Scheme

Qn	Solution	Mark allocation	Remarks
1	Volume of a cylinder, $V = \pi r^2 h$ $(12 + 3\sqrt{2})\pi = \pi(\sqrt{2} - 1)^2 h$ $h = \frac{12 + 3\sqrt{2}}{(\sqrt{2} - 1)^2}$ $h = \frac{12 + 3\sqrt{2}}{3 - 2\sqrt{2}}$ $h = \frac{(12 + 3\sqrt{2})(3 + 2\sqrt{2})}{(3 - 2\sqrt{2})(3 + 2\sqrt{2})}$ $h = \frac{(48 + 33\sqrt{2})}{9 - 8}$ $h = 48 + 33\sqrt{2}$	M1 M1 A1	
2	$\frac{x}{3} = y^2 \quad \text{--- (1)}$ $\frac{x}{9} = 9y \quad \text{--- (2)}$ From (2), $x = 81y$ From (1), $x = 3y^2$ $3y^2 = 81y$ $3y^2 - 81y = 0$ $3y(y - 27) = 0$ $y = 0 \quad \text{or} \quad y = 27$ (reject) $x = 2187$ $x - y = 2187 - 27$ $= 2160$	M1 M1 A1	

3	$y = -2x^2 - 12x + 1$ $= -2\left(x^2 + 6x - \frac{1}{2}\right)$ $= -2\left[x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 - \frac{1}{2}\right]$ $= -2\left[(x+3)^2 - 9 - \frac{1}{2}\right]$ $= -2\left[(x+3)^2 - 9\frac{1}{2}\right]$ $= 19 - 2(x+3)^2$ Maximum value of y is 19. The maximum value occurs when $x = -3$.	M1 M1 A1 A1	
4	$y = \int \frac{2}{x^2} + \frac{3}{(7-2x)} dx$ $y = -\frac{2}{x} + \frac{3}{-2} \ln(7-2x) + c$ Substitute $P(3, 1)$, $1 = -\frac{2}{3} + \frac{3}{-2} \ln(1) + c$ $c = \frac{5}{3}$ Equation of curve is $y = -\frac{2}{x} - \frac{3}{2} \ln(7-2x) + \frac{5}{3}$	M2 M1 A1	
5	$\frac{2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$ $= \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}$ $= \frac{Ax^2 + Cx^2 + Ax + Bx + B}{x^2(x+1)}$	M1 M1 M1	

Comparing coefficient of: $x^0 : -1 = B$ $x^1 :$ $2 = A + B$ $2 + 1 = A$ $3 = A$ $x^2 :$ $0 = A + C$ $0 = 3 + C$ $-3 = C$ $\therefore \frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{(x+1)}$ <u>Alternative method.</u> $\frac{2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$ $= \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}$ $2x-1 = Ax(x+1) + B(x+1) + Cx^2$ Let $x = -1$, $2(-1) - 1 = C(-1)^2$ $C = -3$ Let $x = 0$, $-1 = B(1)$ $B = -1$ Let $x = 1$, $2 - 1 = A(1)(2) + (-1)(2) + (-3)(1)^2$ $A = 3$	M1 M1 A1 M1 M1 M1 M1	
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	$\therefore \frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{(x+1)}$	A1	
6a	$f(-2c) = -32c^3$ $3(-2c)^3 - 5c(-2c)^2 + kc^2(-2c) + 4c^3 = -32c^3$ $-24c^3 - 20c^3 - 2kc^3 + 4c^3 = -32c^3$ $-2kc^3 = 8c^3$ $k = -4$	M1 A1	Allow long division method
6b	$f(x) = 3x^3 - 5cx^2 - 4c^2x + 4c^3$ $\begin{array}{r} 3x-2c \\ x^2-cx-2c^2 \overline{) 3x^3-5cx^2-4c^2x+4c^3} \\ \underline{3x^3-3cx^2-6c^2x} \\ -2cx^2+2c^2x+4c^3 \\ \underline{-2cx^2+2c^2x+4c^3} \\ 0 \end{array}$ Since remainder is 0, $x^2 - cx - 2c^2$ is a factor of $f(x)$. <u>Alternative method</u> $x^2 - cx - 2c^2 = (x+c)(x-2c)$ $f(-1) = 0$ $f(2c) = 0$	M1 M1 A1 M1 M1	
7i	Amplitude = 4	B1	
7ii	$\frac{2\pi}{3} = \frac{2\pi}{a}$ $a = 3$ $b = -2$	B1 B1	



8a	In right-angled $\triangle ABC$, by Pythagoras' Theorem, $AC^2 = AB^2 + BC^2$ $13^2 = 12^2 + BC^2$ $BC^2 = 13^2 - 12^2$ $= 25$ $BC = \pm 5$ Since $BC > 0$, $BC = 5$. By similar triangles $\triangle ABC$ and $\triangle QPC$, $\frac{y}{12} = \frac{5-x}{5}$ $y = \frac{12(5-x)}{5}$ $= \frac{60-12x}{5} \text{ (shown)}$	M1	
8b	$A = xy$ $= x \left(\frac{60-12x}{5} \right)$ $= x \left(12 - \frac{12x}{5} \right)$ $= 12x - \frac{12x^2}{5} \text{ (shown)}$	B1	
8c	$A = 12x - \frac{12x^2}{5}$ $\frac{dA}{dx} = 12 - \frac{24x}{5}$ For stationary values, $\frac{dA}{dx} = 0$. $12 - \frac{24x}{5} = 0$ $60 - 24x = 0$ $24x = 60$ $x = 2.5$	M1	M1

	When $x = 2.5$, $A = 12(2.5) - \frac{12(2.5)^2}{5}$ $= 15$ $\frac{dA}{dx} = 12 - \frac{24x}{5}$ $\frac{d^2A}{dx^2} = -\frac{24}{5} < 0$ By the second derivative test, A is a maximum. Hence, the stationary value of A is 15 cm^2 and it is a maximum.	A1	
9ai	$m_{AC} = -2$ Equation of AC is $y = -2x + 6$	M1 A1	
9aia	$y = -2x + 6 \text{ --- (1)}$ $x + 5y = -6 \text{ --- (2)}$ Sub (1) into (2) $x + 5(-2x + 6) = -6$ $-9x = -36$ $x = 4$ $y = -2$ $C(4, -2)$	M1 A1	

9b	$D = (-6, 0)$ $\text{Area of } \triangle ACD = \frac{1}{2} \begin{vmatrix} 0 & -6 & 4 & 0 \\ 6 & 0 & -2 & 6 \end{vmatrix}$ $= \frac{1}{2} (12 + 24 - (-36))$ $= 36 \text{ units}^2$ $\text{Area of } \triangle ABC = \frac{36}{1.5}$ $= 24 \text{ units}^2$ $\text{Let } B = (k, 6)$ $\frac{1}{2} \begin{vmatrix} 0 & 4 & k & 0 \\ 6 & -2 & 6 & 6 \end{vmatrix} = 24$ $\frac{1}{2} (24 + 6k - 24 + 2k) = 24$ $k = 6$ $B = (6, 6)$ <u>Alternative method,</u> $\frac{1}{2} (AB)(8) = 24$ $AB = 6$ $B(6, 6)$	M1	
9c	$ AB = 6 \text{ units}$ $ AD = \sqrt{72} \text{ units}$ $ CD = \sqrt{(4+6)^2 + (-2)^2}$ $= \sqrt{104} \text{ units}$ Since $AB \neq AD \neq CD$, $ABCD$ is not a kite.	M1 A1	

10a	$\frac{1 - \sin^4 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \sin^2 x$ $= \operatorname{cosec}^2 x - (1 - \cos^2 x)$ $= \cot^2 x + 1 - 1 + \cos^2 x$ $= \cot^2 x + \cos^2 x \text{ (Proven)}$	M1 M1 M1 A1	
10b	$\cot^2 2x + \cos^2 2x = 0$ $\frac{1 - \sin^4 2x}{\sin^2 2x} = 0$ $1 - \sin^4 2x = 0$ $\sin^4 2x = 1$ $\sin 2x = 1 \text{ or } \sin 2x = -1$ basic angle = 90° $2x = 90^\circ, 270^\circ$ $x = 45^\circ, 135^\circ$	M1 M1 M1 A1	
11a	$\angle CBA = 180^\circ - \angle ADC \text{ (opp. } \angle \text{s of cyclic quadrilateral)}$ $\angle ADE = 180^\circ - \angle ADC \text{ (adj. } \angle \text{s on a str. line)}$ $\therefore \angle CBA = \angle ADE$	M1 A1	
11b	In triangles DAE and BAC, $\angle CBA = \angle ADE \text{ (shown in (a))}$ $\angle DAE = \angle ACD \text{ (tangent-chord theorem)}$ $= \angle CAB \text{ (alt. } \angle \text{)}$ Hence, triangles BAE and DAC are similar.	M1 A1	
11c	$AD = DE$, implying that triangles DAE and BAC are similar isosceles triangles. $\angle ACB = \angle CAB \text{ (alt. } \angle \text{s)}$ $= \angle DCA$ Hence, the line AC bisects the angle BCD.	M1 M1 A1	

12a	$2^{x-2} \times 3^{x+2} = 6^{2x}$ $\frac{2^x \times 3^x \times 3^2}{2^2} = 6^{2x}$ $\frac{6^x \times 9}{4} = 6^{2x}$ $6^x = \frac{9}{4}$ $x = \frac{\lg\left(\frac{9}{4}\right)}{\lg 6}$ $= \frac{\lg 9 - \lg 4}{\lg 6} \text{ (shown)}$	M1 M1 M1 A1	
12b	$\frac{\log_3 y}{\log_3 9} - 2 = \log_3 2y$ $\frac{1}{2} \log_3 y - 2 = \log_3 2y$ $\frac{1}{2} \log_3 y - \log_3 2y = 2$ $\log_3 y - 2 \log_3 2y = 4$ $\log_3 y - \log_3 (2y)^2 = 4$ $\log_3 \frac{y}{4y^2} = 4$ $\frac{1}{4y} = 3^4 \text{ (since } y > 0)$ $y = \frac{1}{324}$	M1 M1 M1 A1	
13ai	When $V = 334\pi \text{ cm}^3$, $10\pi + \frac{4\pi h^3}{9} = 334\pi$ $\frac{4\pi h^3}{9} = 324\pi$ $h^3 = 729$ $h = 9$ Hence, the value of h is 9.	M1 A1	

13aii	$V = 10\pi + \frac{4\pi h^3}{9}$ $\frac{dV}{dh} = 3 \left(\frac{4\pi h^2}{9} \right)$ $= \frac{4\pi h^2}{3}$ Since $\frac{dV}{dt} = -150 \text{ cm}^3/\text{s}$, when $h=9$, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $-150 = \frac{4\pi(9)^2}{3} \times \frac{dh}{dt}$ $-150 = 108\pi \times \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{25}{18\pi}$ $= -0.442 \text{ (correct to 3 sig. fig.)}$ Hence, the rate of change of h is -0.442 cm/s.	M1	
13bi	$\frac{s}{v} = \frac{1}{6} + \frac{v}{50}$ $s = \frac{1}{6}v + \frac{v^2}{50}$ $\frac{ds}{dv} = \frac{1}{6} + \frac{2v}{50}$ $\left. \frac{ds}{dv} \right _{v=60} = \frac{1}{6} + \frac{2(60)}{50}$ $= \frac{77}{30} \frac{\text{m}}{\text{km/h}}$	M1 M1 A1	
13bii	$\frac{ds}{dv}$ is the rate of change of the stopping distance with respect to the speed of the car. When the car is travelling at 60 km/h, for every 1km/h increase in its speed, its stopping distance increases by approximately 2.57 m.	B1	
14a	To find the coordinates of A ,		

	When $y = 0$, $0 = \sqrt{3x - 2}$ $x = \frac{2}{3}$ Therefore, $A(\frac{2}{3}, 0)$. Coordinates of $B(9, 5)$ To find the coordinates of C, $\frac{d}{dx}(\sqrt{3x - 2}) = \frac{3}{2\sqrt{3x - 2}}$ At $x = 9$, $\text{Grad. of tangent} = \frac{3}{10}$ $\text{Gradient of normal} = -\frac{10}{3}$ Let $C = (c, 0)$, $\frac{5 - 0}{9 - c} = -\frac{10}{3}$ $15 = -90 + 10c$ Therefore $C = (10.5, 0)$.	M1 A1 M1 M1 A1	
--	--	---	--

14b	Area of shaded region = $\int_{\frac{2}{3}}^9 \sqrt{3x-2} \, dx + \text{Area of } \Delta$ $= \left[\frac{(3x-2)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(3)} \right]_{\frac{2}{3}}^9 + \left(\frac{1}{2} \times \frac{3}{2} \times 5 \right)$ $= \left[\frac{25^{\frac{3}{2}}}{\frac{9}{2}} - \frac{0^{\frac{3}{2}}}{\frac{9}{2}} \right] + \frac{15}{4}$ $\approx 31.5 \text{ units}^2$	M1	
		M2	
		M1	
		A1	

$$1. \quad e^{\frac{1}{2}x} = 2 + 24e^{-\frac{1}{2}x}$$

$$e^{\frac{1}{2}x} = 2 + \frac{24}{e^{\frac{1}{2}x}}$$

$$\text{let } u = e^{\frac{1}{2}x}$$

$$u = 2 + \frac{24}{u} \quad m1$$

$$u^2 = 2u + 24$$

$$u^2 - 2u - 24 = 0$$

$$(u-6)(u+4) = 0 \quad m1$$

$$u-6 = 0 \quad \text{or} \quad u+4 = 0$$

$$u = 6 \quad \text{or} \quad u = -4 \quad m1$$

$$e^{\frac{1}{2}x} = 6 \quad \text{or} \quad e^{\frac{1}{2}x} = -4$$

$$\frac{1}{2}x = \ln 6 \quad m1$$

$$\frac{1}{2}x = \ln(-4)$$

(rej.)

$$x = 2\ln 6$$

A1

$$\begin{aligned}
 2a \quad & \int \left(\frac{6}{e^{3-x}} \right)^2 dx \\
 = & \int \frac{36}{e^{6-2x}} \\
 = & 36 \int e^{2x-6} dx \\
 = & \frac{36 e^{2x-6}}{2} + C \\
 = & 18 e^{2x-6} +
 \end{aligned}$$

2b let $p(x) = 2x^3 + 3x^2 + 2x + 8$

$$p(-2) = 2(-2)^3 + 3(-2)^2 + 2(-2) + 8 \quad m1$$

$$= 0$$

$x+2$ is a factor

$$\begin{array}{r}
 \quad 2x^2 - x + 4 \\
 x+2 \overline{) 2x^3 + 3x^2 + 2x + 8} \\
 \underline{-(2x^3 + 4x^2)} \\
 -x^2 + 2x \quad m1 \\
 \underline{-(-x^2 - 2x)} \\
 4x + 8 \\
 \underline{-(4x + 8)} \\
 0
 \end{array}$$

$$p(x) = (x+2)(2x^2 - x + 4)$$

$$p(x) = 0$$

$$(x+2)(2x^2 - x + 4) = 0$$

$$x+2 = 0 \quad 2x^2 - x + 4 = 0 \quad m1$$

$$x = -2$$

A1

$$b^2 - 4ac < 0$$

since $b^2 - 4ac < 0$, $2x^2 - x + 4 = 0$

has no real solutions

$\therefore p(x) = 0$ has only

one real root A1

3a

$$y = x^2 \sqrt{2x-1}$$

$$u = x^2 \quad v = (2x-1)^{\frac{1}{2}}$$

$$\begin{aligned} \frac{du}{dx} &= 2x & \frac{dv}{dx} &= \frac{1}{2} (2x-1)^{-\frac{1}{2}} (2) \\ & & &= (2x-1)^{-\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{x^2}{\sqrt{2x-1}} + 2x \sqrt{2x-1} \\ &= \frac{x^2 + 2x \sqrt{2x-1} (\sqrt{2x-1})}{\sqrt{2x-1}} \\ &= \frac{x^2 + 2x (2x-1)}{\sqrt{2x-1}} \\ &= \frac{x^2 + 4x^2 - 2x}{\sqrt{2x-1}} \\ &= \frac{5x^2 - 2x}{\sqrt{2x-1}} \\ &= \frac{x(5x-2)}{\sqrt{2x-1}} \end{aligned}$$

$$2x-1 > 0$$

$$x > \frac{1}{2} \quad | \quad B1$$

$$5x-2 > \frac{1}{2}$$

$$\therefore 5x-2 > 0$$

$$x(5x-2) > 0$$

$$\frac{dy}{dx} > 0$$

It is an increasing function.

3b

$$\int_1^5 \frac{5x^2 - 2x + 1}{\sqrt{2x-1}} dx$$

$$= \int_1^5 \frac{5x^2 - 2x}{\sqrt{2x-1}} dx + \int_1^5 \frac{1}{\sqrt{2x-1}} dx \quad m_1$$

$$= \left(x^2 \sqrt{2x-1} \right)_1^5 + \left[\frac{(2x-1)^{\frac{1}{2}}}{\frac{1}{2} \times 2} \right]_1^5$$

$$= \left[5^2 \sqrt{2(5)-1} - 1^2 \sqrt{2(1)-1} \right] + \left[(2x-1)^{\frac{1}{2}} \right]_1^5$$

$$= 74 + \left[2(5)-1 \right]^{\frac{1}{2}} - \left[2(1)-1 \right]^{\frac{1}{2}} \quad m_1$$

$$= 74 + 2$$

$$= 76$$

4.a.

$$\sin \theta = \frac{x}{14}$$

$$x = 14 \sin \theta$$

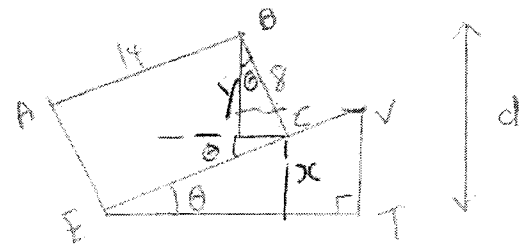
m1

$$\cos \theta =$$

$$y = 8 \cos$$

$$d = x + y$$

$$d = 14 \sin \theta + 8 \quad \theta \text{ A1}$$



$$180 - (90 + 90 - \theta)$$

4b.

$$= \sqrt{14^2 + 8^2}$$

m1

$$= \sqrt{260}$$

$$\tan \alpha = \frac{8}{14}$$

m1

=

$$d = + 29.7^\circ \text{ A1}$$

4c

$$\text{maximum } d = 2\sqrt{65} \quad B1$$

$$= 16.1$$

$$(\theta + 29.7448^\circ) = 90^\circ$$

$$\text{corresponding } \theta = 60.2552^\circ$$

$$= 60.3^\circ (1dp) \quad B1$$

4d.

$$13 = 2\sqrt{65} \sin(\theta + 29.7^\circ)$$

$$\sin(\theta + 29.7448^\circ) = \frac{13}{2\sqrt{65}} \quad M1$$

$$\theta + 29.7448^\circ = 53.7288$$

$$\text{smallest value of } \theta = 23.9800 (4dp)$$

$$= 24.0 (1dp) \quad A1$$

5a

$$x^2 - 2x + 3$$

$$= x^2 - 2x + \left(\frac{-2}{2}\right)^2 + 3 - \left(\frac{-2}{2}\right)^2 \quad m_1$$

$$= (x-1)^2 + 2$$

$$\text{Since } (x-1)^2 \geq 0$$

$$(x-1)^2 + 2 \geq 2$$

$\therefore x^2 - 2x + 3$ is always
positive for all real values
of x .

B1

5b

Since $x^2 - 2x + 3$ is always positive,

$$3x^2 + px + 3 > 0 \quad \text{m1}$$

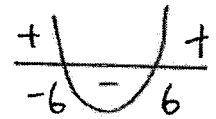
$$\therefore b^2 - 4ac < 0$$

$$(p)^2 - 4(3)(3) < 0 \quad \text{m1}$$

$$p^2 - 36 < 0$$

$$(p - 6)(p + 6) < 0 \quad \text{m1}$$

$$-6 < p < 6 \quad \text{A1}$$



$$\begin{aligned}
 \text{6a. } T_{r+1} &= \binom{n}{r} a^{n-r} b^r \\
 &= \binom{5}{r} (x)^{5-r} \left(\frac{k}{x}\right)^r \quad m1 \\
 &= \binom{5}{r} x^{5-r} k^r x^{-r} \\
 &= \binom{5}{r} k^r x^{5-2r}
 \end{aligned}$$

$$\begin{aligned}
 \text{For } \frac{1}{x^3}, \quad x^{5-2r} &= x^{-3} \quad m1 \\
 5-2r &= -3 \\
 2r &= 8 \\
 r &= 4
 \end{aligned}$$

$$\begin{aligned}
 \text{coefficient of } \frac{1}{x^3} &= \binom{5}{4} k^4 \\
 &= 5k^4
 \end{aligned}$$

$$\begin{aligned}
 \text{For } \frac{1}{x}, \quad x^{5-2r} &= x^{-1} \quad m1 \\
 5-2r &= -1 \\
 2r &= 6 \\
 r &= 3
 \end{aligned}$$

$$\begin{aligned}
 \text{coefficient of } \frac{1}{x} &= \binom{5}{3} k^3 \\
 &= 10k^3
 \end{aligned}$$

$$5k^4 = 10k^3$$

$$5k^4 - 10k^3 = 0 \quad m1$$

$$5k^3 (k - 2) = 0$$

$$k = 0 \quad \text{or} \quad k - 2 = 0$$

(rej)

$$k = 2 \quad A1$$

6b. $T_{r+1} = \binom{5}{r} k^r x^{5-2r}$

For constant term, $5-2r = 0$

$$\frac{2r}{r} = \frac{5}{2}$$

coefficient of $\frac{1}{x^2}$ $5-2r = -2 \quad m1$

$$2r = 7$$

$$r = \frac{7}{2}$$

Since r is not a positive integer,

there is no constant term in

the expansion of $(1-3x^2)\left(x + \frac{k}{x}\right)^5$

$$7a \quad \frac{3y^2}{k} + \frac{2x}{h} = 6$$

$$\frac{3y^2}{k} = -\frac{2x}{h} + 6$$

$$3y^2 = -\frac{2k}{h}x + 6k$$

$$y^2 = -\frac{2k}{3h}x + 2k \quad m1$$

$$Y = y^2, \quad X = x, \quad m = -\frac{2k}{3h}, \quad c = 2k$$

By comparison

$$2k = 8$$

$$k = 4$$

$$-6 = \frac{-2(4)}{3h}$$

$$3h = \frac{4}{3}$$

$$h = \frac{4}{9}$$

$$\therefore k = 4, \quad h = \frac{4}{9}$$

7b1

$$T = 25 + p \cdot e^{-kn}$$

$$T - 25 = p \cdot e^{-kn}$$

$$\ln(T - 25) = \ln p \cdot e^{-kn}$$

$$= \ln p + \ln e^{-kn}$$

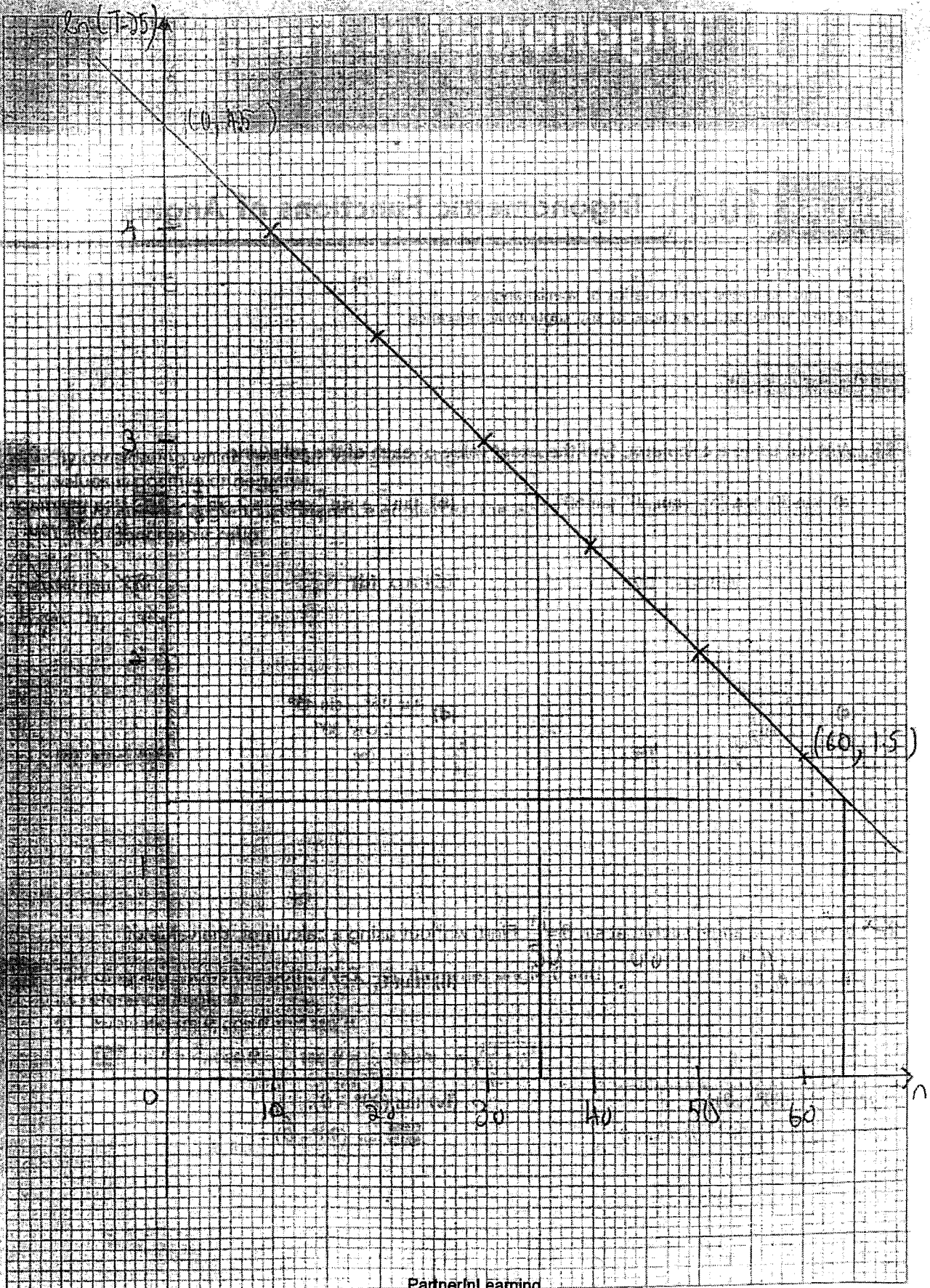
$$\ln(T - 25) = -kn + \ln p$$

$$Y = \ln(T - 25), X = n, m = -k, c = \ln p$$

n	10	20	30	40	50
$\ln(T - 25)$	4.0	3.5	3.0	2.5	2.0

Plot $\ln(T - 25)$ against n B1

Qn 7b1



7bii

$$-k \approx \frac{1.5 - 4.5}{60} \quad m_1$$

$$k \approx -0.05 \quad A_1$$

$$\ln \approx 4.5$$

$$p \approx e^{4.5} \quad m_1$$

$$\approx 90.017 \text{ (5s.f.)}$$

$$\approx 90.0 \text{ (3s.f.)} \quad A_1$$

7biii

$$n = 35$$

$$\ln(T-25) \approx 2.75 \quad m_1$$

$$T \approx 40.642^\circ\text{C} \text{ (5s.f.)}$$

$$\approx 40.6^\circ\text{C} \text{ (3s.f.)} \quad A_1$$

The temperature is 40.6°C after
35 minutes.

7biv

$$T = 25 + 90 e^{-0.05n}$$

$$n=0, \quad T = 115$$

$$25\% \text{ of } T \approx 28.75 \quad m_1$$

$$\ln(28.75 - 25) \approx$$

$$\text{At } 1.3 \quad n \approx 64 \text{ minutes}$$

It takes 64 minutes for the temperature of the liquid to drop by 75% of its initial value.

9a.

Equation of circle, C_1

$$(x+5)^2 + (y-3)^2 = 5^2$$

$$(x+5)^2 + (y-3)^2 =$$

9bi

$$y = -3x + 11$$

$$A(-5, -2)$$

$$y = -3x + c$$

Sub $A(-5, -2)$ into eqn

$$-2 = -3(-5) + c$$

$$c = -17$$

∴

$$y = -3x - 17 \quad A1$$

$$m_1 \times m_2 = -1$$

$$-3 \times m_2 = -1 \quad m_1$$

$$m_2 = \frac{1}{3}$$

$$y = \frac{1}{3}x + c$$

Sub $C(-5, 3)$ into eqn

$$3 = \frac{1}{3}(-5) + c$$

$$c = \frac{14}{3}$$

Equation of perpendicular bisector

$$y = \frac{1}{3}x + \frac{14}{3}$$

$$\begin{aligned} \text{q6ii} \quad y &= -3x - 17 & \text{--- (1)} \\ y &= \frac{1}{3}x + \frac{14}{3} & \text{--- (2)} \end{aligned}$$

$$\textcircled{1} = \textcircled{2} \quad -3x - 17 = \frac{1}{3}x + \frac{14}{3} \quad || \times 3$$

$$3\frac{1}{3}x = -21\frac{2}{3}$$

$$x = -\frac{13}{2}$$

$$\text{Sub } x = -\frac{13}{2} \quad \textcircled{1} : y = -3\left(-\frac{13}{2}\right) - 17 = \frac{5}{2}$$

$$\therefore \text{midpoint of AB} = \left(-\frac{13}{2}, \frac{5}{2}\right)$$

$$\text{Let coordinates of B } (x_1, y_1)$$

$$\frac{x_1 + (-5)}{2} = -\frac{13}{2}$$

$$\frac{y_1 + (-2)}{2} = \frac{5}{2}$$

$$x_1 = -8$$

$$y_1 = 7$$

$$\therefore \text{coordinates of B } (-8, 7)$$

9c let coordinates of centre of C_2 be (x_2, y_2)

$$\frac{x_2 + (-5)}{2} = -\frac{13}{2} \quad m_1 \quad \frac{y_2 + (3)}{2} = \frac{5}{2} \quad m_1$$

$$x_2 = -8$$

$$y_2 = 2$$

coordinates of centre of C_2 $(-8, 2)$

Equation of circle, C_2

$$(x+8)^2 + (y-2)^2 = 5^2$$

$$(x+8)^2 + (y-2)^2 = 25 \quad \text{A1}$$

10

$$y = 2 \sin 2x + 1$$

$$\frac{dy}{dx} = 4 \cos 2x$$

$$4 \cos 2x = 0$$

$$\cos 2x = 0$$

$$2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}$$

$$x = \frac{\pi}{4}, \quad y = 2 \sin 2\left(\frac{\pi}{4}\right) + 1$$

$$= 3$$

Coordinates of P $\left(\frac{\pi}{4}, 3\right)$

$$4 \cos 2x = -4$$

$$\cos 2x = -1$$

$$2x = \pi$$

$$x =$$

$$y = 2 \sin 2\left(\frac{\pi}{2}\right) + 1$$

$$= 1$$

Coordinates of Q $\left(\frac{\pi}{2}, 1\right)$

$$\frac{d^2y}{dx^2} = -8 \sin 2x$$

$$\text{At } x = \frac{\pi}{4}$$

$$\frac{d^2y}{dx^2} = -8 \sin 2\left(\frac{\pi}{4}\right)$$

$$= -8 < 0$$

$\therefore$ P is maximum point

Area of shaded region

$$= \int_0^{\frac{\pi}{4}} (2\sin 2x + 1) dx + \left[\frac{1}{2} \times (3+1) \times \left(\frac{\pi}{2} - \frac{\pi}{4} \right) \right] \quad \text{M1}$$

$$= \left(\frac{-2\cos 2x}{2} + x \right) \Big|_0^{\frac{\pi}{4}} + \frac{\pi}{2}$$

$$= \left(-\cos 2x + x \right) \Big|_0^{\frac{\pi}{4}} + \frac{\pi}{2}$$

$$= -(-1) + \frac{\pi}{2}$$

$$= 1 + \frac{3\pi}{4} \quad \text{A1}$$