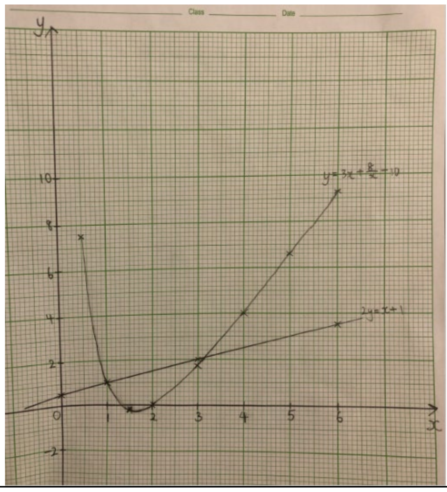
2022 4E5A Preliminary Examination Paper 2 Mark Scheme

1a	$\frac{3x-1}{3}<\frac{2x+1}{4}$ $12x-4<6x+3$ $12x-6x<3+4$ $x<1\frac{1}{6}$	M1 A1
1b	$\frac{7x}{(4-3x)^2}-\frac{2}{4-3x}$ $=\frac{7x-2(4-3x)}{(4-3x)^2}$ $=\frac{7x-8+6x}{(4-3x)^2}$ $=\frac{13x-8}{(4-3x)^2}$	M1 A1
1c	$\left(\frac{m^{10}}{49n^6}\right)^{-\frac{1}{2}}$ $=\left(\frac{49n^6}{m^{10}}\right)^{\frac{1}{2}}=\frac{7n^3}{m^5}$	M1 A1
1d	$\frac{50x^2-32}{5x^2-11x-12}$ $=\frac{2(25x^2-16)}{(5x+4)(x-3)}$ $=\frac{2(5x+4)(5x-4)}{(5x+4)(x-3)}$ $=\frac{2(5x-4)}{x-3}$	M1 M1 A1
1e	$4x-5y=16$ ------ (1) $6x+3y=10$ ------ (2) $(1)\times 3:\quad 12x-15y=48 \quad \to (3)$ $(2)\times 5:\quad 30x+15y=50 \quad \to (4)$ $(3)+(4):\quad 42x=98$ $x=2\frac{1}{3}$ *Substitute* $x=2\frac{1}{3}$ in (2), $6\left(2\frac{1}{3}\right)+3y=10$ $14+3y=10$ $y=-1\frac{1}{3}$	M1 A1 A1

2a	$A=\frac{1}{3}[\begin{pmatrix}8 & 15\\ 20 & 30\\ 6 & 5\end{pmatrix}+\begin{pmatrix}10 & 15\\ 20 & 35\\ 12 & 6\end{pmatrix}+\begin{pmatrix}6 & 8\\ 13 & 14\\ 9 & 5\end{pmatrix}]$ $A=\frac{1}{3}\begin{pmatrix}24 & 38\\ 53 & 79\\ 27 & 16\end{pmatrix}$ $A=\begin{pmatrix}8 & 12\frac{2}{3}\\ 17\frac{2}{3} & 26\frac{1}{3}\\ 9 & 5\frac{1}{3}\end{pmatrix}$	M1 A1
2b	The number of bus trips **per day** made by each type of bus of the 2 companies.	B1
2ci	$3BA$ exists because B is a 1x3 matrix and A is a 3x1 matrix	B1
2cii	The elements in this 1x2 matrix represents the total number of bus trips made by Company A and B respectively.	B1
2di	$\begin{pmatrix}12 & 25 & 50\end{pmatrix}\begin{pmatrix}8 & 15\\ 20 & 30\\ 6 & 5\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}$	B1 for $\begin{pmatrix}12 & 25 & 50\end{pmatrix}$ B1 for $\begin{pmatrix}1\\ 1\end{pmatrix}$
2dii	$[(8 trips\times 12 passengers)+$ $(20 trips\times 25 passengers)+$ $(6 trips\times 50 passengers)]\times \$22$ $=896\times 22$ $=\$19712$	M1 A1

<table>
<tr><td>3a</td><td>9.3</td><td>B1</td></tr>
<tr><td>3b</td><td>
</td><td>P1-plot all points
P1-join up points
P1-smooth curve</td></tr>
<tr><td>3c</td><td>$3x + \frac{8}{x} = 15$
$3x + \frac{8}{x} - 15 = 0$
$3x + \frac{8}{x} - 10 = 5$
Draw $y = 5$,
From the graph,
$x = 0.6$ or $x = 4.4$</td><td>B1, B1</td></tr>
<tr><td>3di</td><td><table><tr><td>x</td><td>0</td><td>3</td><td>6</td></tr><tr><td>y</td><td>0.5</td><td>2</td><td>3.5</td></tr></table>
Line $2y = x + 1$ drawn.</td><td>P1

P1</td></tr>
<tr><td>3dii</td><td>From the graph,
$x = 1$ or $x = 3.15$</td><td>B1, B1</td></tr>
<tr><td>3diii</td><td>$2\left(3x + \frac{8}{x} - 10\right) = x + 1$
$6x + \frac{16}{x} - 20 = x + 1$
$6x^2 + 16 - 20x = x^2 + x$
$5x^2 - 21x + 16 = 0$
$\therefore B = -21, C = 16$</td><td>M1

M1

A1</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

4ai	Bearing of A from D $= 360 - (180 - 126)$ $= 306°$	B1
4aii	$\angle ABD = 270 - 242$ $= 28°$ $\dfrac{AD}{\sin 28°} = \dfrac{480}{\sin 36°}$ $AD = 383.3821$ $= 383.4m$ (1 d.p)	M1 M1 A1
(b)	Let x be the angle of depression $\tan x = \dfrac{4800}{630}$ $x = 82.5226$ $= 82.5°$	M1 A1
(c)	$\angle APD = 180 - 36 - 36$ $= 108°$ $\dfrac{PD}{\sin 36°} = \dfrac{383.3821}{\sin 108°}$ $PD = 236.94317$ $= 237m$ (3 s.f)	M1 M1 A1

5a	Let x be the height of the cone that was removed. By similar triangles, $\frac{x}{x+6}=\frac{1}{5}$ $5x=x+6$ $4x=6$ $x=1.5$ Volume of paint bottle $=\pi(5^2)(20)+(\frac{1}{3}\pi\times5^2\times7.5)-(\frac{1}{3}\pi\times1^2\times1.5)$ $=1765.5755$ $=1765.6cm^3$ (shown)	M1 A1 M1 (volume of cylinder), ecf M1 (volume of frustrum), ecf A1 (answer shown correctly)
(b)	$\frac{1765.6-1500}{1765.6}\times100$ $=15.043$ $=15.0\%$ (3 s.f)	B1
(c)	Total mass $=30+1500(3)$ $=4530g$ $=4.53kg$	M1 A1
(d)	$\frac{V_L}{V_S}=\frac{3}{1}$ $\frac{l_L}{l_S}=\sqrt[3]{\frac{3}{1}}$ $\frac{A_L}{A_S}=\left(\frac{3}{1}\right)^{\frac{2}{3}}$ Base area of larger bottle $=\left(\frac{3}{1}\right)^{\frac{2}{3}}\times\pi(5)^2$ $=163.3694...$ $=163cm^2$ (3 s.f)	M1 A1

6ai	$\tan \angle DOB = \frac{3}{4}$ $\angle DOB = 36.8699$ $\angle AOD = 180 - 36.8699$ $= 143.1301$ Area of shaded region $= \frac{143.1301}{360} \times \pi (8)^2 - \frac{1}{2}(4)(8)\sin 143.1301$ $= 70.3389$ $= 70.3 cm^2$ [Accept finding area of smaller sector and using area of semicircle to subtract the area of sector and triangle]	M1 M1 A1
(a)(ii)	$AD = \frac{143.1301}{360} \times 2\pi(8)$ $= 19.98473$ $ED^2 = 4^2 + 8^2 - 2(4)(8)\cos 143.1301$ $ED = 11.45426$ Perimeter $= 4 + 11.45426 + 19.98473$ $= 35.438987$ $= 35.4 cm$	M1 M1 A1
(b)(i)	$\angle PRS = 51$ (alternate $\angle s$) $\angle POS = 51 \times 2$ ($\angle$at centre = 2 times $\angle$at circumference) $= 102°$	B1 B1
(b)(ii)	$\angle QPR = \angle QSR$ ($\angle$s in the same segment) $\angle PQS = \angle QSR$ (alt. $\angle$s) Since $\angle QPR = \angle PQS$, ΔPQT is an isosceles triangle	M1 A1
(b)(iii)	$\angle$ in semicircle is 90° *OR* Opposite angles in quadrialteral *OPUS* are supplementary Hence points *O*, *P*, *U*, *S* can lie on the circumference of a circle *OR* Yes it can.	M1 (either one o.e) A1

7a	Median position $=\frac{9+1}{2}=5$ Median for Group A = 51 minutes	B1
7b	IQR (Grp B) $=\frac{61+62}{2}-\frac{31+42}{2}$ $=61.5-36.5$ $=25\,\text{min}$	M1 (for either correct Q1 or Q3) A1
7c	Standard deviation $=\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$ $=\sqrt{\frac{28324}{9}-\left(\frac{476}{9}\right)^2}$ $=18.70498...$ $=18.7 \quad (3sf)$	B1
7d	There is an outlier of 96 mins in Grp B, thus the interquartile range is more appropriate because it is less sensitive to outliers.	B1

8i	$\overrightarrow{PQ} = \begin{pmatrix} -6 \\ 2 \end{pmatrix} - \begin{pmatrix} -8 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$	B1
(ii)	$\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $= 2\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\overrightarrow{PA} = \begin{pmatrix} -11 \\ 7 \end{pmatrix} - \begin{pmatrix} -8 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 9 \end{pmatrix}$ $= 3\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\therefore$ *PQ* is not parallel to *PA*. *A* does not lie on the line *PQ*.	M1 M1 (or vector *AQ*) A1
(iii)	$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$ $= \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ $PR = \sqrt{6^2 + 4^2}$ $= 7.21$ units [Accept finding coordinates of *R* first]	M1 A1
(iv)	$\overrightarrow{OR} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -6 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ $R = (-2, 2)$ $m_{PQ} = m_{RS}$ $= \frac{4}{2}$ $= 2$ $y = mx + c$ $2 = 2(-2) + c$ $c = 6$ $y = 2x + 6$	M1 (for finding *R*) M1 (either *m* or *c* correct) A1

9a)	Cost $=32\times 30+35\times 20=\$1660$ Revenue $=40\times(30+20)$ $=\$2000$ The shopkeeper made a gain.	 M1 (oe) A1
9bi	$\dfrac{800}{x}$	B1
9bii	$(\dfrac{800}{x}-2)(x+2)$	B1 (ecf)
9biii	$(\dfrac{800}{x}-2)(x+2)-800=99$ $800+\dfrac{1600}{x}-2x-4-800=99$ $\dfrac{1600}{x}-2x-103=0$ $-2x^2-103x+1600=0$ $2x^2+103x-1600=0\ \ (shown)$	M1 (ecf) M1 A1
9biv	$2x^2+103x-1600=0$ $x=\dfrac{-103\pm\sqrt{103^2-4(2)(-1600)}}{2(2)}$ $x=12.5\quad or\quad -64$	 M1 (oe) A1/A1
9bv	$\dfrac{800}{12.5}-2=62$	B1

<table>
<tr><td>10ai</td><td>$\frac{(1107.8+1066.3+1123.6+1259+1249.5+1281.6)}{6}$
$=\frac{7087.8}{6}=1181.3\quad kWh$</td><td>M1

A1</td></tr>
<tr><td>10aii)</td><td>$Amount\quad without\quad GST=1181.3\times \$0.2139$
$=\$252.68007$

$Amount\quad with\quad GST=\$252.68007\times 1.07$
$=\$270.3676749$

$=\$270.37\quad (3sf)$</td><td>M1 (ecf)

A1</td></tr>
<tr><td>10b)</td><td><table><tr><td></td><td></td><td>Total number solar panels</td></tr><tr><td>$9\div 1.65=5.4545...$
≈ 5</td><td>$4\div 1=4$</td><td>$5\times 4=20$</td></tr><tr><td>$9\div 1=9$</td><td>$4\div 1.65=2.4242...$
≈ 2</td><td>$9\times 2=18$</td></tr></table>
The max number of solar panels is 20.</td><td>B1</td></tr>
<tr><td>10c)</td><td>Amount of electricity saved per month
$=19kWh\times 20\quad solar\quad panels$
$=380kWh$

Average amount of electricity used after installation
$=1181.3-380$
$=801.3kWh$

Average cost of electricity per month (after installation)
$=801.3\times 0.2139\times 1.07$
$=\$183.3959349$

$=\$183.40\quad (2dp)$

Average cost of solar panels per month
$=\frac{2\times \$6250}{20\times 12}$
$=\$52.083333$

$=\$52.08\quad (2dp)$

Total average cost of electricity after installation of solar panels</td><td>Ecf for earlier parts

B1

B1

B1</td></tr>
</table>

	$= 183.40 + 52.08$ $= \$235.48 \quad (< \$270.37 \quad from \quad aii)$ Since the average amount paid by Mr Tan after installing the solar panels will be less than what he is currently paying, he should go ahead and install the solar panels.	B1 B1