**Jurong Pioneer Junior College**
**H2 Mathematics** **JC1-2024**
**Year-End Exam Solution**

**Q1**

**(a)**

$$\frac{d}{dx}\cos^{-1}\left(3x^2\right) = -\frac{6x}{\sqrt{1-\left(3x^2\right)^2}}$$

$$= -\frac{6x}{\sqrt{1-9x^4}}$$

**(b)**

$$\frac{d}{dx}\ln\left(\frac{\sqrt{2x+1}}{x^3}\right) = \frac{d}{dx}\left[\frac{1}{2}\ln\left(2x+1\right)-3\ln x\right]$$

$$= \frac{2}{2\left(2x+1\right)}-\frac{3}{x}$$

$$= \frac{1}{2x+1}-\frac{3}{x}$$

$$= \frac{-5x-3}{x\left(2x+1\right)}$$

**<u>Alternative (not recommended)</u>**

$$\frac{d}{dx}\ln\left(\frac{\sqrt{2x+1}}{x^3}\right) = \frac{x^3}{\sqrt{2x+1}}\cdot\frac{x^3\left(\frac{2}{2\sqrt{2x+1}}\right)-3x^2\sqrt{2x+1}}{x^6}$$

$$= \frac{x^5}{\sqrt{2x+1}}\cdot\frac{x-3\left(2x+1\right)}{x^6\sqrt{2x+1}}$$

$$= \frac{-5x-3}{x\left(2x+1\right)}$$

**Q2**

Let $V$ be the volume of the water and $h$ be the depth of the water at time $t$ minutes after the start.

Given $V=\frac{1}{3}\pi r^2 h$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=0.1\mathrm{m}^3/\min$.

Consider $\tan 30^\circ=\frac{r}{h}\ \Rightarrow r=\frac{\sqrt{3}}{3}h$

Consider $V=\frac{1}{3}\pi\left(\frac{\sqrt{3}}{3}h\right)^2 h=\frac{1}{9}\pi h^3$

Then $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{3}\pi h^2$

When $V=3\mathrm{m}^3$, $3=\frac{1}{9}\pi h^3\Rightarrow h^3=\frac{27}{\pi}\ \therefore h=\left(\frac{27}{\pi}\right)^{\frac{1}{3}}$

Consider $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$

$0.1=\frac{1}{3}\pi\left(\frac{27}{\pi}\right)^{\frac{2}{3}}\times\frac{\mathrm{d}h}{\mathrm{d}t}$

$\therefore\frac{\mathrm{d}h}{\mathrm{d}t}=0.0228$m/min (nearest to 3 s.f.) or $\frac{1}{30\pi^{\frac{1}{3}}}$ m/min

**Q3**

$\mathrm{f}(x)=\frac{a}{x^2}+bx+c$

At $(-1,4)$, $\mathrm{f}(-1)=4$

$\frac{a}{(-1)^2}+b(-1)+c=4$

$a-b+c=4$ ------ (1)

At $(2,-11)$, $\mathrm{f}(2)=-11$

$\frac{a}{(2)^2}+b(2)+c=-11$

$\frac{a}{4}+2b+c=-11$ ------ (2)

$y=\frac{1}{\mathrm{f}(x)}$ has a vertical asymptote with equation $x=1$ implies $C$ has an $x$-intercept at $x=1$.

$\mathrm{f}(1)=0$

$\frac{a}{(1)^2}+b(1)+c=0$

$a+b+c=0$ ------ (3)

Or

$\mathrm{f}(x)=\dfrac{a+bx^3+cx^2}{x^2}$

$\dfrac{1}{\mathrm{f}(x)}=\dfrac{x^2}{a+bx^3+cx^2}$

$y=\dfrac{1}{\mathrm{f}(x)}$ has a vertical asymptote with equation $x=1$,

$a+b(1)^3+c(1)^2=0$

$a+b+c=0$ ------- (3)

Solving (1), (2) & (3) using GC: $a=12, b=-2, c=-10$.

Hence, $\mathrm{f}(x)=\dfrac{12}{x^2}-2x-10$.

**Q4**

Consider $x=A(8x-8)+B$

By comparing coefficient of $x$ and constant term: $A=\dfrac{1}{8}$ , $B=1$.

$$\int_0^1 \frac{x}{4x^2-8x+5}\,\mathrm{d}x$$

$$= \frac{1}{8}\int_0^1 \frac{8x-8}{4x^2-8x+5}\,\mathrm{d}x + \int_0^1 \frac{1}{4x^2-8x+5}\,\mathrm{d}x$$

$$=\frac{1}{8}\Big[\ln\left|4x^2-8x+5\right|\Big]_0^1 + \int_0^1 \frac{1}{4\left[x^2-2x+(-1)^2-(-1)^2\right]+5}\,\mathrm{d}x$$

$$=\frac{1}{8}\left[\ln 1-\ln 5\right] + \int_0^1 \frac{1}{4(x-1)^2+1}\,\mathrm{d}x$$

$$= -\frac{1}{8}\ln 5 + \int_0^1 \frac{1}{4\left[(x-1)^2+\left(\frac{1}{2}\right)^2\right]}\,\mathrm{d}x$$

$$= -\frac{1}{8}\ln 5 + \frac{1}{4}\left[\frac{1}{\left(\frac{1}{2}\right)}\tan^{-1}\left(\frac{x-1}{\frac{1}{2}}\right)\right]_0^1$$

$$= -\frac{1}{8}\ln 5+\frac{1}{2}\left[\tan^{-1}(0)-\tan^{-1}(-2)\right]$$

$$=-\frac{1}{8}\ln 5-\frac{1}{2}\tan^{-1}(-2)$$

**Q5**

**(i)**

$y=\frac{x^2+ax+b}{x+1}$

$(2,0)$ is a turning point on $C$,

$0=2^2+2a+b$

$2a+b=-4$ Eqn (1)

$\frac{dy}{dx}=\frac{(2x+a)(x+1)-(x^2+ax+b)}{(x+1)^2}$

$=\frac{x^2+2x+a-b}{(x+1)^2}$

At $(2,0)$, $\frac{dy}{dx}=0$

$2^2+2(2)+a-b=0$

$a-b=-8$ Eqn (2)

(1) + (2)

$3a=-12$

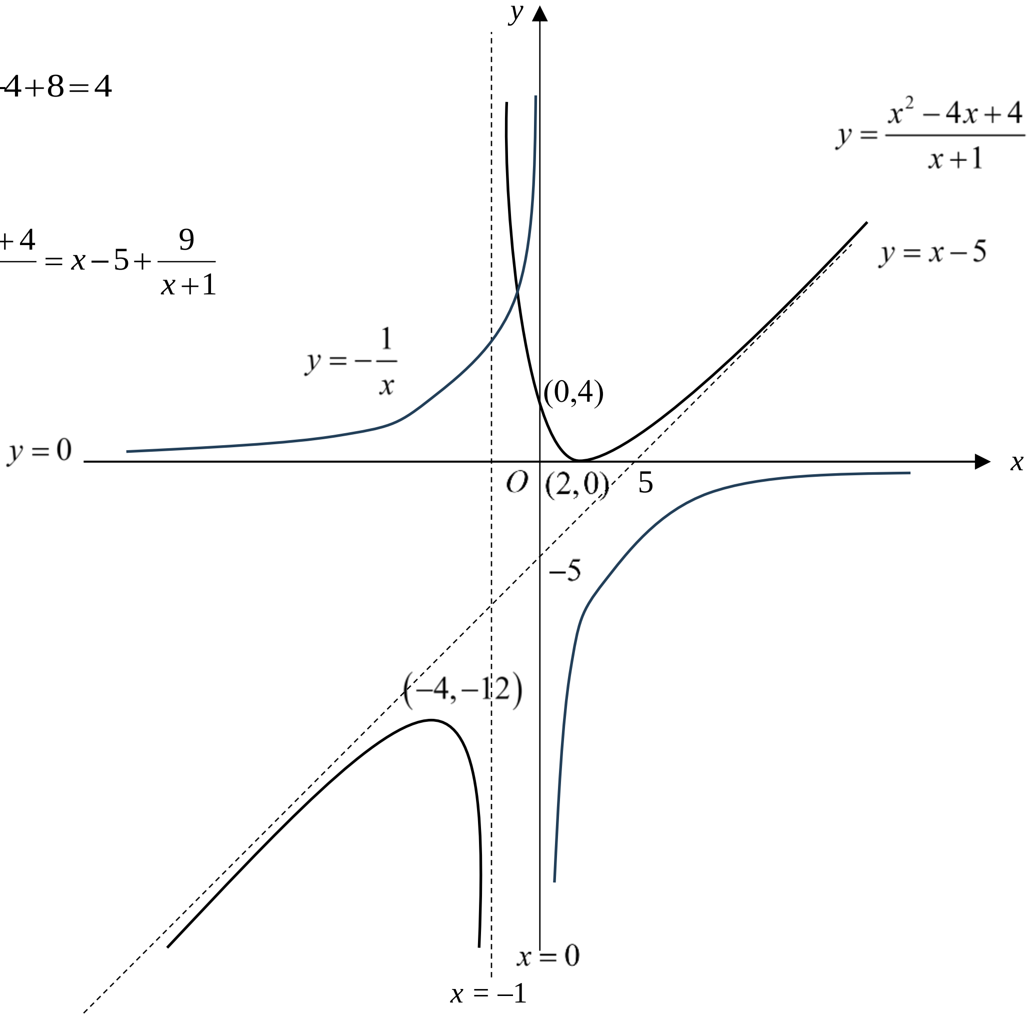
$a=-4$

From (2)

$b=a+8=-4+8=4$

**(ii)**

$y=\frac{x^2-4x+4}{x+1}=x-5+\frac{9}{x+1}$



**(iii)**

$$x^3+ax^2+(b+1)x+1=0$$
$$x^3+ax^2+bx=-x-1$$
$$x(x^2+ax+b)=-(x+1)$$
$$\frac{x^2+ax+b}{x+1}=-\frac{1}{x}$$

Since the graphs of $y=\dfrac{x^2+ax+b}{x+1}$ and $y=-\dfrac{1}{x}$ intersect only once, the equation $x^3+ax^2+(b+1)x+1=0$ has only one real root.

**Q6**

**(i)**

Cartesian equation of ellipse is $\dfrac{x^2}{a^2}+\dfrac{y^2}{(2a)^2}=1$

**(ii)**

$$\frac{x^2}{a^2}+\frac{y^2}{(2a)^2}=1$$
$$y^2=4a^2\left(1-\frac{x^2}{a^2}\right)$$
$$y^2=4(a^2-x^2)$$
$$y=\pm2\sqrt{a^2-x^2}$$

Since $x>0, y>0$ for coordinates of *P*,

Area of rectangle *PQRS*, $A=(2x)(2y)$

$=4xy$

$=4x(2\sqrt{a^2-x^2})$

$=8x\sqrt{a^2-x^2}$ (shown)

**(iii)**

$$\frac{\mathrm{d}A}{\mathrm{d}x}=8x\left(\frac{-2x}{2\sqrt{a^2-x^2}}\right)+8\sqrt{a^2-x^2}$$
$$=\frac{-8x^2+8(a^2-x^2)}{\sqrt{a^2-x^2}}$$
$$=\frac{8a^2-16x^2}{\sqrt{a^2-x^2}}$$

When *A* is a maximum, $\dfrac{\mathrm{d}A}{\mathrm{d}x}=0$.

**Alternatively**

$A^2=64x^2(a^2-x^2)=64(a^2x^2-x^4)$

$2A\dfrac{\mathrm{d}A}{\mathrm{d}x}=64(2a^2x-4x^3)$

Put $\dfrac{\mathrm{d}A}{\mathrm{d}x}=0,\quad 2x(a^2-2x^2)=0$

$x=0(NA)$ or $x=\dfrac{a}{\sqrt{2}}\,(x>0)$

$$\frac{8a^2-16x^2}{\sqrt{a^2-x^2}}=0$$

$$x^2=\frac{1}{2}a^2$$

$$x=\pm\frac{1}{\sqrt{2}}a$$

Since $x>0$, $x=\frac{1}{\sqrt{2}}a$

**(iv)**

$$A=8x\sqrt{a^2-x^2}$$

$$x=\frac{1}{\sqrt{2}}a, \qquad A=100$$

$$8\left(\frac{1}{\sqrt{2}}a\right)\sqrt{a^2-\left(\frac{1}{\sqrt{2}}a\right)^2}=100$$

$$8\left(\frac{1}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}a\right)=100$$

$$a^2=25$$

$$a=\pm 5$$

Since $a>0$, $a=5$

**Alternatively**

$$100^2=64\left(\frac{a^2}{2}\right)\left(a^2-\frac{a^2}{2}\right)$$

$$100^2=4^2a^4$$

$$a=5(a>0)$$

**Q7**

**(i)**

$$\frac{5-3x}{x^2+x-2}\geq -2$$

$$\frac{5-3x+2(x^2+x-2)}{x^2+x-2}\geq 0$$

$$\frac{2x^2-x+1}{(x+2)(x-1)}\geq 0$$

$$2x^2-x+1$$

$$=2\left(x^2-\frac{1}{2}x\right)+1$$

$$=2\left(x^2-\frac{1}{2}x+\left(-\frac{1}{4}\right)^2-\left(-\frac{1}{4}\right)^2\right)+1$$

$$=2\left(x-\frac{1}{4}\right)^2+\frac{7}{8}$$

Since $\left(x-\frac{1}{4}\right)^2\geq 0$, $2\left(x-\frac{1}{4}\right)^2+\frac{7}{8}>0$ for all real values of $x$.

**OR:**

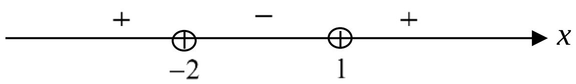
Consider $2x^2 - x + 1 = 0$

Discriminant = $(-1)^2 - 4(2)(1) = -7 < 0$ **and** coefficient of $x^2 = 2 > 0$

Hence $2x^2 - x + 1 > 0$ for all real values of $x$.

Thus, consider $\dfrac{1}{(x+2)(x-1)} \geq 0$.

$(x+2)(x-1) > 0$




Therefore $x < -2$ or $x > 1$

**(ii)**

$$\frac{-5-3x}{x^2-x-2} \geq 2$$

$$\frac{-(5+3x)}{x^2-x-2} \geq 2$$

$$\frac{5+3x}{x^2-x-2} \leq -2$$

$$\frac{5-3(-x)}{(-x)^2+(-x)-2} \leq -2$$ (Note that the inequality sign is different from that in (i))

Replace $x$ by $-x$:

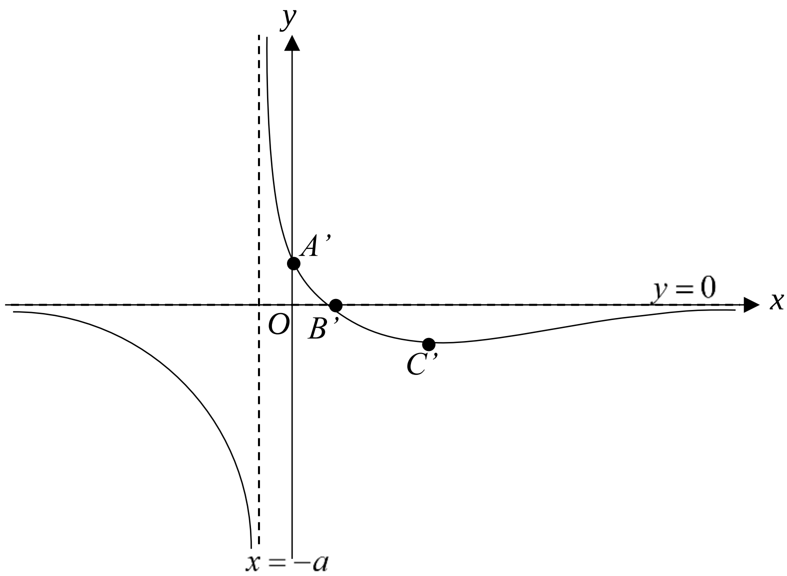
$-2 < -x < 1$

$-1 < x < 2$

**Q8**

**(a)(i)**

$y = \mathrm{f}(x-a) + a$

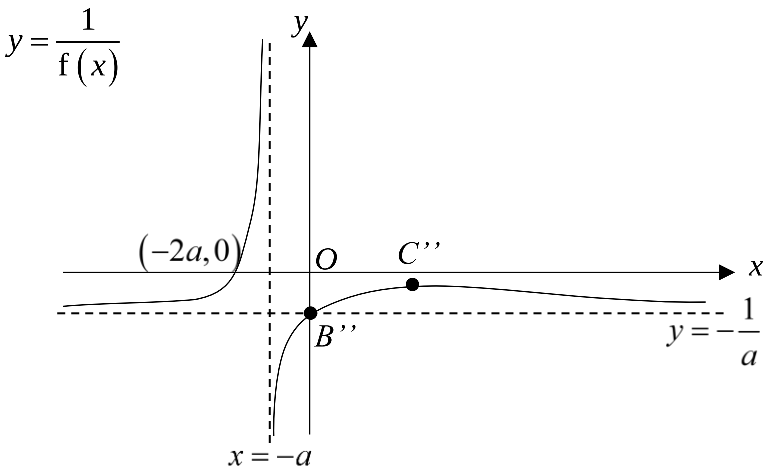



$A'(0, a)$

$B'(a, 0)$

$C'(3a, -a)$

**(a)(ii)**




$B''\left(0,-\frac{1}{a}\right)$

$C''\left(2a,-\frac{1}{2a}\right)$

**(b)**

$y=\mathrm{g}(x)=\sin x$

After **A**, $y=\mathrm{g}(x+\pi)=\sin(x+\pi)$

After **B**, $y=\mathrm{g}\left(\frac{x}{3}+\pi\right)=\sin\left(\frac{x}{3}+\pi\right)$

After **C**, $y=\mathrm{g}\left(\frac{x}{3}+\pi\right)+2=\sin\left(\frac{x}{3}+\pi\right)+2$

**Q9**

**(a)**

$$\begin{aligned}\int\frac{2x^2}{\sqrt{1-x^2}}\,\mathrm{d}x&=\int\frac{2\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cdot(\cos\theta)\,\mathrm{d}\theta\\&=\int\frac{2\sin^2\theta}{\sqrt{\cos^2\theta}}\cdot(\cos\theta)\,\mathrm{d}\theta\\&=\int 2\sin^2\theta\,\mathrm{d}\theta\\&=\int(1-\cos 2\theta)\,\mathrm{d}\theta\\&=\theta-\frac{1}{2}\sin 2\theta+c\\&=\sin^{-1}x-\cos\theta\sin\theta+c\\&=\sin^{-1}x-x\sqrt{1-x^2}+c\end{aligned}$$

$x=\sin\theta$

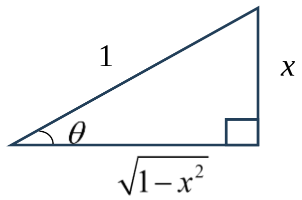
$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\cos\theta$

Note :

To obtain $\cos\theta$ in terms of $x$, use trigo identities or triangle.

$\cos^2\theta+\sin^2\theta=1$

Or



**(b)(i)**

$$\frac{d}{dx}\sin(\ln x) = \cos(\ln x)\left(\frac{1}{x}\right)$$

**(b)(ii)**

$$\int \sin(\ln x)\,dx = x\sin(\ln x) - \int x\left(\frac{1}{x}\cos(\ln x)\right)dx$$

$$= x\sin(\ln x) - \int \cos(\ln x)dx$$

$$= x\sin(\ln x) - \left[x\cos(\ln x) - \int -x\left(\frac{1}{x}\right)\sin(\ln x)dx\right]$$

$$= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)dx$$

$$2\int \sin(\ln x)dx = x\sin(\ln x) - x\cos(\ln x) + c_1$$

$$\therefore \int \sin(\ln x)dx = \frac{1}{2}x\left[\sin(\ln x) - \cos(\ln x)\right] + c$$

$u = \sin(\ln x)$ $\quad \frac{dv}{dx} = 1$

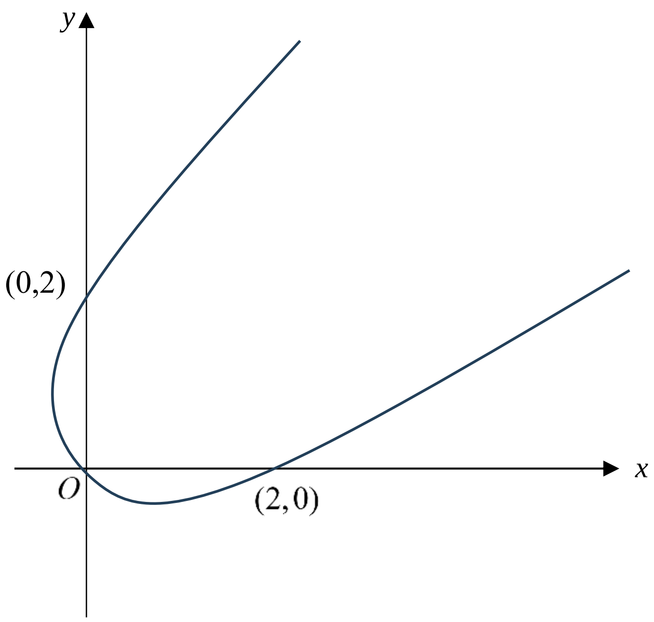
$\frac{du}{dx} = \cos(\ln x)\left(\frac{1}{x}\right)$ $\quad v = x$

$u = \cos(\ln x)$ $\quad \frac{dv}{dx} = 1$

$\frac{du}{dx} = -\sin(\ln x)\left(\frac{1}{x}\right)$ $\quad v = x$

**Q10**

**(i)**




Note :

Must include negative values of $t$ in the window settings.

When $x = 0,\quad t^2 + t = 0$

$t(t+1) = 0$

$t = 0$ or $-1$

$y = 0$ or $2$

When $y = 0,\quad t^2 - t = 0$

$t(t-1) = 0$

$t = 0$ or $1$

$x = 0$ or $2$

**(ii)**

$x = t^2 + t$ $\quad y = t^2 - t$

$\frac{dx}{dt} = 2t + 1$ $\quad \frac{dy}{dt} = 2t - 1$

$$\frac{dy}{dx} = \frac{2t-1}{2t+1}$$

Gradient of line $5y = 4x - 20$ is $\frac{4}{5}$.

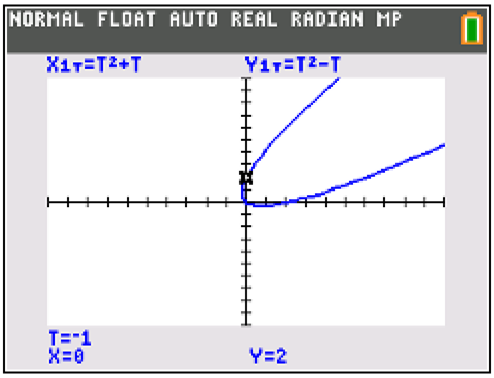
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2t-1}{2t+1} = \frac{4}{5}$$
$$10t - 5 = 8t + 4$$
$$t = \frac{9}{2}$$
$$x = \left(\frac{9}{2}\right)^2 + \frac{9}{2} = 24.75 \qquad y = \left(\frac{9}{2}\right)^2 - \frac{9}{2} = 15.75$$

Tangent to $C$ is parallel to $5y = 4x - 20$ at point $(24.75, 15.75)$

**(iii)**

From (i), at $(0,2)$, $t = -1$

(Can also obtain $t = -1$ from graph on GC)




$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(-1)-1}{2(-1)+1} = 3$$

Equation of tangent at $(0,2)$ is

$$y - 2 = 3(x - 0)$$
$$y = 3x + 2$$
$$t^2 - t = 3\left(t^2 + t\right) + 2$$
$$2t^2 + 4t + 2 = 0$$
$$t^2 + 2t + 1 = 0$$
$$(t+1)^2 = 0$$
$$t = -1$$

Since $t = -1$ is the only solution, the tangent at $(0,2)$ does not cut $C$ again.

**Q11**

**(i)**

Given $\overrightarrow{OA} = \mathbf{a}$ , $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \frac{1}{2}\mathbf{a}$.

By Ratio Theorem, $\overrightarrow{OD} = \frac{2\mathbf{c} + 3\mathbf{b}}{5} = \frac{1}{5}\left[2\left(\frac{1}{2}\mathbf{a}\right) + 3\mathbf{b}\right] = \frac{1}{5}(\mathbf{a} + 3\mathbf{b})$

**(ii)**

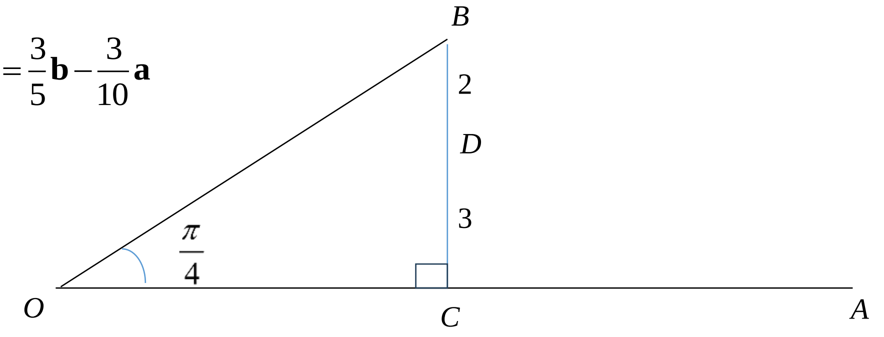
Area of triangle $OCD$

$$= \frac{1}{2}\left|\overrightarrow{OC} \times \overrightarrow{OD}\right|$$
$$= \frac{1}{2}\left|\frac{1}{2}\mathbf{a} \times \frac{1}{5}(\mathbf{a} + 3\mathbf{b})\right|$$
$$= \frac{1}{2}\left|\frac{1}{10}(\mathbf{a} \times \mathbf{a}) + \frac{3}{10}(\mathbf{a} \times \mathbf{b})\right|$$

$$=\frac{1}{2}\left|\frac{3}{10}(\mathbf{a}\times\mathbf{b})\right| \qquad \text{since } \mathbf{a}\times\mathbf{a}=\mathbf{0}$$

$$=\frac{3}{20}|\mathbf{a}\times\mathbf{b}| \quad \text{where } k=\frac{3}{20}$$

**(iii)**




$$\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}=\frac{1}{5}(\mathbf{a}+3\mathbf{b})-\frac{1}{2}\mathbf{a}=\frac{3}{5}\mathbf{b}-\frac{3}{10}\mathbf{a}$$

$$\overrightarrow{CD}.\overrightarrow{OA}=\left(\frac{3}{5}\mathbf{b}-\frac{3}{10}\mathbf{a}\right).\mathbf{a}$$

$$=\frac{3}{5}\mathbf{b}.\mathbf{a}-\frac{3}{10}\mathbf{a}.\mathbf{a}$$

$$=\frac{3}{5}|\mathbf{b}||\mathbf{a}|\cos\frac{\pi}{4}-\frac{3}{10}|\mathbf{a}|^2$$

$$=\frac{3}{5}\left(\sqrt{2}\right)(2)\left(\frac{\sqrt{2}}{2}\right)-\frac{3}{10}(2)^2=\frac{6}{5}-\frac{12}{10}=0$$

Since $\overrightarrow{CD}.\overrightarrow{OA}=0$, *CD* is perpendicular to *OA*. (Shown)

**Alternative Method**

$$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\frac{1}{2}\mathbf{a}-\mathbf{b}$$

$$\overrightarrow{BC}.\overrightarrow{OA}=\left(\frac{1}{2}\mathbf{a}-\mathbf{b}\right).\mathbf{a}$$

$$=\frac{1}{2}\mathbf{a}.\mathbf{a}-\mathbf{b}.\mathbf{a}$$

$$=\frac{1}{2}|\mathbf{a}|^2-|\mathbf{b}||\mathbf{a}|\cos\frac{\pi}{4}=\frac{1}{2}(2)^2-\left(\sqrt{2}\right)(2)\left(\frac{\sqrt{2}}{2}\right)=2-2=0$$

Since $\overrightarrow{BC}.\overrightarrow{OA}=0$, *BC* is perpendicular to *OA*.
Since *D* is on *BC*, $\overrightarrow{CD}$ parallel to $\overrightarrow{BC}$, *CD* is perpendicular to *OA*. (Shown)

**(iv)**

$|\mathbf{a}\square\mathbf{b}|$ is the length of projection of $\overrightarrow{OB}$ onto $\overrightarrow{OA}$ (or a line or vector parallel to $\overrightarrow{OA}$).

**Q12**

**(i)**

Given $l$ : $\mathbf{r} = \begin{pmatrix} 1 \\ -5 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ -6 \end{pmatrix}$ where $\lambda \in \mathbb{R}$

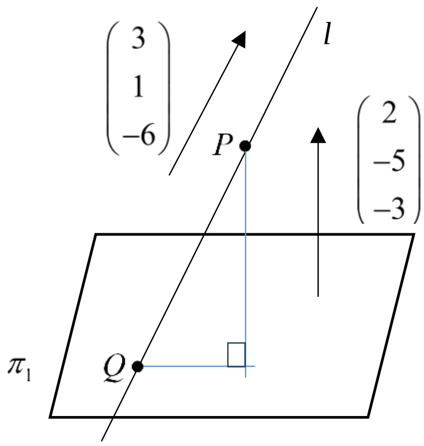
Given $\pi_1$: $\mathbf{r}. \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = 4$

Acute angle between $l$ and $\pi_1$

$$= 90^{\mathbf{o}} - \cos^{-1}\left( \frac{\left| \begin{pmatrix} 3 \\ 1 \\ -6 \end{pmatrix} . \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} \right|}{\sqrt{46}\sqrt{38}} \right)$$

$$= 90^{\circ} - \cos^{-1}\left( \frac{19}{\sqrt{46}\sqrt{38}} \right)$$

$= 27^{\circ}$ (nearest degree)




**(ii)**

$$\begin{pmatrix} 1+3\lambda \\ -5+\lambda \\ -5-6\lambda \end{pmatrix} . \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = 4$$

$2(1+3\lambda) - 5(-5+\lambda) - 3(-5-6\lambda) = 4$

$19\lambda = -38$

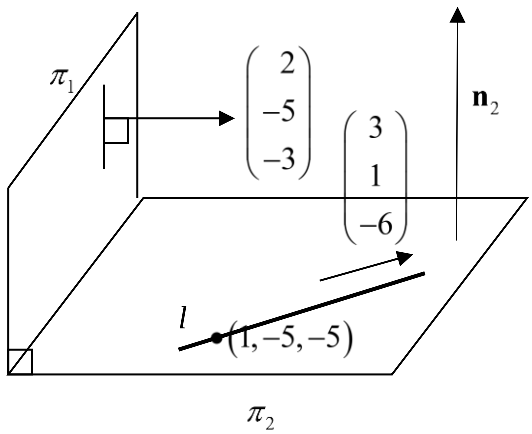
$\therefore \quad \lambda = -2$

When $\lambda = -2$, $\overrightarrow{OQ} = \begin{pmatrix} 1 \\ -5 \\ -5 \end{pmatrix} - 2\begin{pmatrix} 3 \\ 1 \\ -6 \end{pmatrix} = \begin{pmatrix} -5 \\ -7 \\ -7 \end{pmatrix}$.

The coordinates of $Q$ = $(-5, -7, 7)$

**(iii)**

To find normal to $\pi_2$: $\begin{pmatrix} 3 \\ 1 \\ -6 \end{pmatrix} \times \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} = \begin{pmatrix} -33 \\ -3 \\ -17 \end{pmatrix} = -\begin{pmatrix} 33 \\ 3 \\ 17 \end{pmatrix}$



$\mathbf{r}.\begin{pmatrix}33\\3\\17\end{pmatrix}=\begin{pmatrix}1\\-5\\-5\end{pmatrix}.\begin{pmatrix}33\\3\\17\end{pmatrix}$ or $\mathbf{r}.\begin{pmatrix}33\\3\\17\end{pmatrix}=\begin{pmatrix}-5\\-7\\7\end{pmatrix}.\begin{pmatrix}33\\3\\17\end{pmatrix}$

Thus $\mathbf{r}.\begin{pmatrix}33\\3\\17\end{pmatrix}=-67$

Cartesian equation of $\pi_2$: $33x+3y+17z=-67$

**(iv)**

Equation of $\pi_1$: $2x-5y-3z=4$
Equation of $\pi_2$: $33x+3y+17z=-67$
Using GC, let $z=\mu$, $x=-\frac{17}{9}-\frac{4}{9}\mu$, $y=-\frac{14}{9}-\frac{7}{9}\mu$

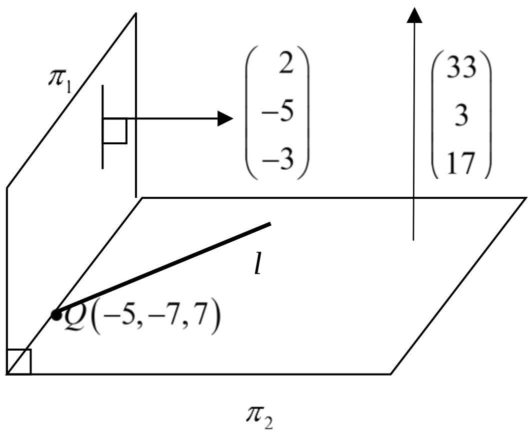
Vector equation of the line: $\mathbf{r}=\begin{pmatrix}-\frac{17}{9}\\-\frac{14}{9}\\0\end{pmatrix}+\mu\begin{pmatrix}-\frac{4}{9}\\-\frac{7}{9}\\1\end{pmatrix}=\begin{pmatrix}-\frac{17}{9}\\-\frac{14}{9}\\0\end{pmatrix}+t\begin{pmatrix}4\\7\\-9\end{pmatrix}$, where $t\in\mathbb{R}$ .

**Alternative Method:**

$\begin{pmatrix}2\\-5\\-3\end{pmatrix}\times\begin{pmatrix}33\\3\\17\end{pmatrix}=\begin{pmatrix}-76\\-133\\171\end{pmatrix}=-19\begin{pmatrix}4\\7\\-9\end{pmatrix}$

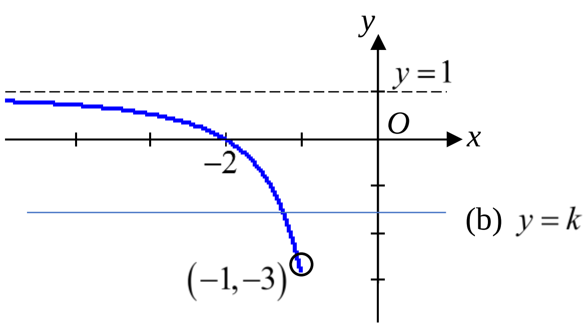
Deduced that point $Q$ lies on the intersection line as it lies on $\pi_1$ and $l$.

Vector equation of the line: $\mathbf{r}=\begin{pmatrix}-5\\-7\\7\end{pmatrix}+t\begin{pmatrix}4\\7\\-9\end{pmatrix}$, where $t\in\mathbb{R}$ .




**Q13**

**(i)**



**(ii)**

Any horizontal line $y = k$ cuts the graph of f at most once. Hence, f is one-one and $f^{-1}$ exists.

Let $y = 1 - \frac{4}{x^2}$

$x^2 = \frac{4}{1-y}$

$x = \pm\sqrt{\frac{4}{1-y}}$

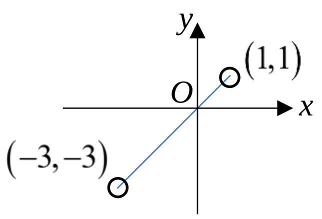
Since $x < -1$, $x = -\sqrt{\frac{4}{1-y}}$

$f^{-1}(x) = -\sqrt{\frac{4}{1-x}}$

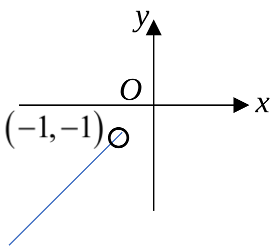
$D_{f^{-1}} = (-3,1)$

**(iii)**

$y = ff^{-1}(x) = x$, $D_{ff^{-1}} = D_{f^{-1}} = (-3,1)$




$y = f^{-1}f(x) = x$, $D_{f^{-1}f} = D_f = (-\infty,-1)$

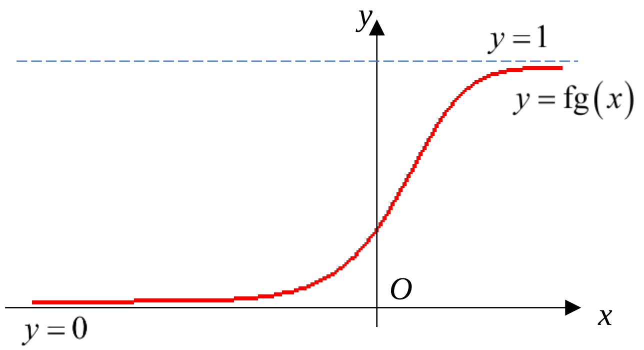



Hence, for $ff^{-1}(x) = f^{-1}f(x)$, we have $-3 < x < -1$.

**(iv)**

$R_g = (-\infty, -2)$
$D_f = (-\infty, -1)$
Since $R_g \subset D_f$, fg exists.

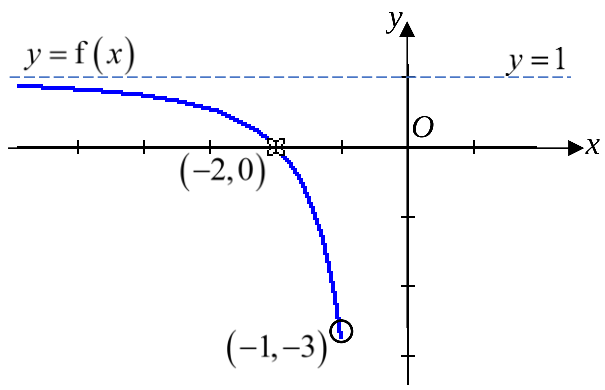
$$fg(x) = 1 - \frac{4}{(-e^{2x} - 2)^2}$$
$$= 1 - \frac{4}{(e^{2x} + 2)^2}$$




$R_{fg} = (0,1)$

**<u>Alternative</u>** to find $R_{fg}$:

$$\begin{array}{ccccc} D_g & \xrightarrow{g} & R_g & \xrightarrow{f} & R_{fg} \\ (-\infty, \infty) & & (-\infty, -2) & & (0,1) \end{array}$$




$R_{fg} = (0,1)$