Name:		Centre/Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination 2023
Year 6

MATHEMATICS

9758/01

Paper 1

13 September 2023

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number, name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	4	6	6	7	9	9	10	11	12	12	14	100

1 For $a > 4$, show that $\int_0^a \left|2x^2 - 3x - 2\right| \, dx = \frac{1}{k}(4a^3 - 9a^2 - 12a + 56)$ where k is an integer to be found. [4]

2 The curve C has equation $y = \frac{x^2 - 3x + 3}{1 - x}$.

(a) Sketch C, stating the coordinates of any turning points and points of intersection with the axes, and the equations of any asymptotes. [3]

(b) Determine the set of exact values of k such that C and the line $y = kx$ has two points of intersection for all real values of x. [3]

3 The equation $z^3 - 2z + k = 0$, where k is a real constant, has a root $z = 1 + a\mathrm{i}$, where a is a positive real constant.

(a) Deduce another root in terms of a. [1]

(b) Find the values of a and k, showing your working. [4]

(c) Hence find the area of the triangle formed by all the roots on an Argand diagram. [1]

4 **(a)** Find the set of values of α for which the expression $\frac{2 - \mathrm{i}\sin 2\alpha}{1 + 2\mathrm{i}\sin 2\alpha}$ is purely real. [3]

(b) If z and w are different non-zero complex numbers and $|w| = 1$, find $\left|\frac{3(w - z)}{1 - z^* w}\right|$. [4]

5 With respect to an origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively where $\mathbf{a}$ and $\mathbf{b}$ are non-parallel. It is given that B lies on the line segment AC such that $\overrightarrow{BC} = 3\mathbf{b} - \mu\mathbf{a}$.

(a) Find the value of μ. Hence find $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

(b) Q is a point on line segment OC where $\overrightarrow{OQ} = t\,\overrightarrow{OC}$. The line segment AQ meets line segment OB at point P. Given that $AP : PQ = \lambda : 1$, deduce the value of λt. [4]

(c) By using your result in part **(b)**, find the ratio $OP : PB$ when $\lambda = 5$. [2]

6 In this question, you may use expansions from the List of Formulae (MF26).

(a) Given that $f(x) = e^{\sin 3x}$, find the Maclaurin expansion of $f(x)$ in ascending powers of x, up to and including the term x^2. Show that this expansion is independent of x^3. [4]

(b) Use your expansion from part **(a)** and integration to find an approximate expression for $\int \frac{e^{\sin 3x}}{x^2} \, dx$. Hence obtain an approximate value for $\int_{0.1}^{0.2} \left(\frac{2}{x}\right)^2 e^{\sin 3x} \, dx$, correct to 4 decimal places. [3]

(c) Use your calculator to find $\int_{0.1}^{0.2} \left(\frac{2}{x}\right)^2 e^{\sin 3x} \, dx$, correct to 4 decimal places. [1]

(d) A student compares the answers to parts **(b)** and **(c)** and concludes that the approximation is accurate. Give a reason to support his conclusion. [1]

7 The curve C has parametric equations

$$x = t^2 + 2, \quad y = t^3, \text{ where } t \in \mathbb{R}.$$

(a) Sketch C. [1]

(b) Find the equation of the tangent to C at the point $(6, 8)$. [2]

(c) The tangent to C at the point $(6, 8)$ meets the curve C again at point P. Find the coordinates of point P. [3]

(d) The normal to C at the point $(m^2 + 2, m^3)$ meets the x- and y-axes at the points Q and R respectively. Given that the point F is the mid-point of QR, find the coordinates of F in terms of m. [4]

8 **(a)** The sum of the first n terms of a series is given by $S_n = 3n(n+2)$. Show that this series follows an arithmetic progression. [3]

(b) The second, seventh and mth term of the series in part **(a)** are the first three consecutive terms of a geometric series, such that its nth term is given by v_n. Find the value of m and explain whether the sum to infinity of the geometric series exists. [3]

(c) The nth term of another geometric series is given by $w_n = e^{5+nx(x+1)}$, where x is a constant. Find the range of values of x such that this geometric series converges. [3]

(d) Using $x = -0.5$ in part **(c)** and given that the sum of the first n terms of the geometric series in part **(b)** first exceeds the nth term of the geometric series in part **(c)**, i.e., $\left(\sum_{r=1}^{n} v_r\right) > w_n$, determine the least value of n. [2]

9 [In this question, all measurements of length are in metres.]

A monument is to be constructed which comprises two parts – the main structure and the base. The main structure is in the form of the solid obtained when the region R bounded by the curves $y = 3\sin\left(2x^2\right)$ and $y = x^2$, and the line $x = \sqrt{\frac{\pi}{8}}$ is rotated by 2π radians about the y-axis. (See Figure 1.)

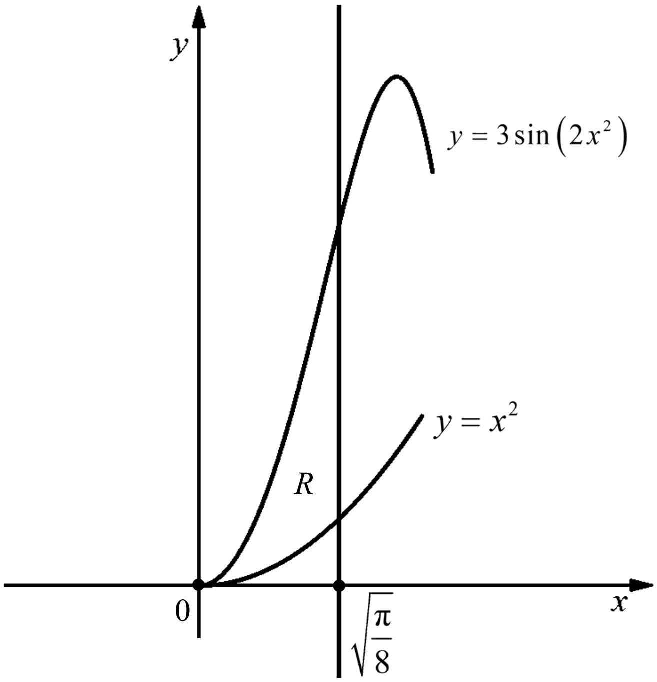


Figure 1

(a) Show that $\int_0^p \sin^{-1}\frac{t}{3}\,\mathrm{d}t = p\sin^{-1}\frac{p}{3} + \sqrt{9-p^2} - 3$, where p is a real constant. [3]

(b) Using the result from part **(a)**, find the exact volume of the main structure. [4]

The main structure is to be mounted on top of a base which is a solid of uniform thickness. The horizontal cross-sectional area of the base is in the shape of region Q, bounded by the curve $y=\frac{8\sqrt{x}}{1+x^3}$ and the curve $y=4x^{\frac{7}{2}}$. (See Figure 2.)

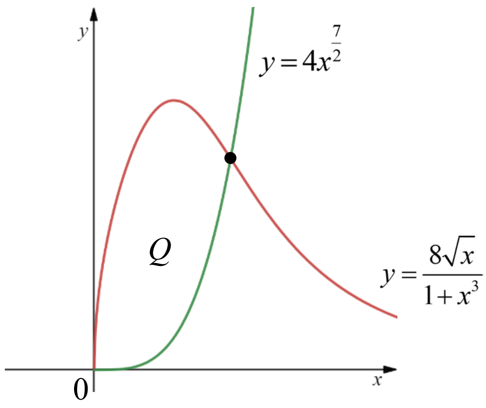


Figure 2

(c) Find the area of region Q, giving your answer correct to 1 decimal place. [2]

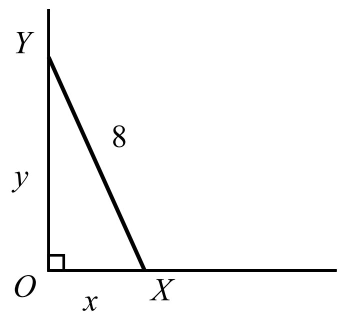
The main structure is to be made of granite while the base is to be made of marble. An area of region Q will eventually be in contact with the ground. Owing to the softness of the ground on which this monument will be placed on, a special foundation needs to be built if its total weight per square metre on the ground beneath exceeds 20 kN/m^2.

Some relevant information:

- Density of granite: 1463.46 kg/m^3
- Density of marble: 2550 kg/m^3
- 1000 kg is equivalent to 9.81 kN (kN = kilo-Newtons)

(d) The engineer in-charge of building this monument claims that as long as the thickness of the base does not exceed 70 cm, there will be no need to build the special foundation. How accurate is the engineer's claim? Justify your answer. [3]

10 Jim places an 8 m rod XY against a vertical wall, as shown in the diagram below. The points O, X and Y are coplanar, with O being the point where the vertical wall and the flat ground meet. At time t seconds, the ends X and Y are x and y meters from O respectively.



Jim makes a conjecture that as the rod slips, y decreases at a rate proportional to y.

(a) Based on Jim's conjecture, show that $\dfrac{dx}{dt} = \dfrac{k(64-x^2)}{x}$, where k is a positive constant. [4]

It is given that X is 4 meters away from O initially, and that $k = 3$.

(b) Find an expression for x in terms of t. Hence, find the time taken for Y to be 3 meters from O from the instance the rod starts sliding from its initial position, giving your answer to 1 decimal place. [5]

(c) Sketch the graph of x against t. [2]

(d) Comment, with justification, on whether Jim's conjecture is appropriate for the behaviour of the rod. [1]

11 The function f is given by

$$\mathrm{f} : x \mapsto a + \frac{3a}{(x+3)(x-1)}, \text{ where } a > 0.$$

(a) Using calculus, find the x-coordinate of the stationary point of $y = \mathrm{f}(x)$. [2]

(b) Sketch the graph of $y = \mathrm{f}(x)$, clearly stating the coordinates of the stationary point and equations of any asymptotes. [3]

(c) State the range of values of x for which the graph of $y = \mathrm{f}(x)$ is increasing and concave downwards. [1]

(d) Sketch the graph of $y = \mathrm{f}'(x)$, clearly stating the coordinates of the points where the graph crosses the axes and equations of any asymptotes. Leave your answers in terms of a where appropriate. [3]

It is given that $\mathrm{g}(x) = \mathrm{f}(x)$ for $x \geq 2$. Suppose the composite function gg exists.

(e) Deduce, with a reason, the condition on a. [3]

(f) Find the exact range of gg for $a = 5$. [2]