# AK MATHEMATICS

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## Units

1. 1km = 1000 m
   1m = 100 cm
   1cm = 10 mm
2. 1h = 60min
   1min = 60s
3. 1years= 365 days= 52weeks
4. 1kg = 1000g
   1ton = 1000kg
5. 1litre = 1000ml = 1000cm$^3$
6. $1^\circ = 100$ cents

## Real numbers

1. Natural numbers
   [ 1,2,3,4,5 ... ]
2. Whole numbers
   [ 0,1,2,3,4,5 ... ]
3. Integers
   [ ... -3,-2,-1,0,1,2,3 ... ]
4. Rational numbers
   [can put into fractions]
   eg: $\frac{3}{2}, \frac{-3}{7}, 5, \frac{1}{4}, 0$
5. Irrational numbers
   [ CANNOT put into fractions ]
   eg: $\pi, \sqrt{2}, \sqrt{3}$ ...
   [To determine rational or not, press (then number into calculator and see)

## Prime Factorization

| | |
|---|---|
| 2 | 24 |
| 2 | 12 |
| 2 | 6 |
| 3 | 3 |
| | 1 |

$24 = 2 \times 3^3$

**Prime numbers**
[ 2,3,5,7,11,13...]
1. Can be ONLY divided by ITSELF
2. Or "1"

**Composite no.**
[ 4,6,8,9,10,12...]
1. Opposite of prime Numbers

[Note: "1" is not prime nor composite]

## LCM

**Method 1: Staircase**

| | |
|---|---|
| 2 | 24 , 42 |
| 3 | 12 , 21 |
| 2 | 4 , 7 |
| 2 | 2 , 7 |
| 7 | 1 , 7 |
| | 1 , 1 |

LCM= $2^3 \times 3 \times 7$
= 168
[Do till all became 1]

**Method 2: Prime factors**

If $24 = 2^3 \times 3$
$42 = 2 \times 3 \times 7$

HCF
1. Common base
2. Lower power
= $2 \times 3$

LCM
1. ALL Bases
2. Higher power
= $2^3 \times 3 \times 7$

## HCF

| | |
|---|---|
| 2 | 24 , 42 |
| 3 | 12 , 21 |
| | 4 , 7 |

HCF= $2 \times 3$
= 6
[Do till cannot]

## Pythagoras theorem

$a^2 + b^2 = c^2$
[c is the hypotenuse]
[hypo - longest side]

## Simple Interest

$$I = \frac{PRT}{100}$$

P - Principal (Original)
R - Interest rate (%)
T - Time (Years)

## Compound Interest

$$A = P\left(1 + \frac{r}{100}\right)^n$$

A - Total Amount
P - Principal (Original)
r - interest rate (%)
n - no of times compounded

$$A = P + I$$

[No. of times compounded]
"r" and "n" will be affected
1. Compounded monthly
   ( r/12 , n X12 )
2. Compounded quarterly
   ( r/4 , n X4 )
3. Compounded half-yearly
   ( r/2 , n X2 )

## Standard Form

$$A \times 10^n$$

where $1 < A < 10$
and n is an integer

**Common prefixes**

Tetra = $10^{12}$
Giga = $10^9$ [billion]
Mega = $10^6$ [million]
Kilo = $10^3$
Milli = $10^{-3}$
Micro = $10^{-6}$
Nano = $10^{-9}$
Pico = $10^{-12}$

## Quadratic Eqn

To solve $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[Rmb to use calculator to check]
Mode 3,3 or Mode 5,2,2

## Algebra

### Factorisation

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $ax + bx + ay + by$
   $= x(a + b) + y(a + b)$
   $= (x + y)(a + b)$
   [grouping method]

### Complete square

Express $-2x^2 + 4x + 3$ as $(x-a)^2 + b$

**Step 1:** factorize out -2 from $-2x^2$
$= -2(x^2 - 2x - \frac{3}{2})$

**Step 2:** complete square
$= -2[(x - \frac{2}{2})^2 - (\frac{2}{2})^2 - \frac{3}{2}]$

**Step 3:** press cal for green part
$= -2[(x - \frac{2}{2})^2 - \frac{5}{2}]$

**Step 4:** rainbow in the "-2"
$= -2(x - 1)^2 + 5$

For eqns like $x^2 - 4x + 1$ only require step 2 and 3

Completing square is used to find "turning point" of quadratic graph
Turning point here is ( 1, 5 )

### Proportions

**DIRECT**
y is directly proportional to x
$y = kx$

**INDIRECT**
y is indirectly proportional to x
$y = \frac{k}{x}$

### Inequality

*When to flip inequality sign?*
1. when flipping inequality
   If $5 > x$
   Then, $x < 5$
2. when multiply / divide inequality by negative number
   If $-5x > 10$
   Then $x < \frac{-10}{5}$

Combining inequality into number line

$x \geq 5$ Or $x < 10$

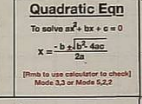
$5 \leq x < 10$

## Mensuration

| | Area | Volume |
|---|---|---|
| Cylinder | $2\pi r^2 + 2\pi rh$ | $\pi r^2 h$ |
| Pyramid | Area of 4△ + Base Area | $\frac{1}{3}$(base area) x h |
| Prism | (base area) x h | — |
| Cone | $\pi r^2 + \pi r^2$ | $\frac{1}{3}\pi r^2 h$ |
| Sphere | $4\pi r^2$ | $\frac{4}{3}\pi r^3$ |
| Trapezium | $\frac{1}{2}(a + b)h$ | — |

## Coordinate Geometry

1. Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$
2. Length = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
3. Equation of line : y=mx + c
   m : gradient of line
   c : y-intercept of line

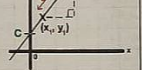



4. Special equation

## Construction

1. Perpendicular bisector of line AB
2. Angle bisector of ∠ABC

It's a must to show working lines !




## Algebraic Fraction

1. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$
2. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
3. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
4. $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$

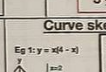
**Note**
1. only have to make same base if adding/subtracting fractions
2. For multiplication/division of algebraic fractions, DO NOT need to make same base !

## Indices

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^0 = 1$
5. $(a \times b)^m = a^m \times b^m$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
7. $a^{-1} = \frac{1}{a}$
8. $\left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1$
9. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

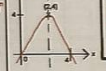
## Curve sketching

Eg 1: y = x(4 - x)




Steps
1. x-intercept , LET y=0
   x=0 or x=4
2. y-intercept, LET x=0
   y=0
3. TURNING POINT
   1.Find line of symmetry
   $x = \frac{0 + 4}{2} = 2$
   2. Sub back, y=4
   TP (2,4)

Eg 2: y = (x-2)$^2$ - 1




Steps
1. x-intercept, LET y=0
   $x-1$ or $x=3$
2. y-intercept, LET x=0
   y=3
3. TURNING POINT
   (2, -1)

## Number Patterns

**1st degree difference**
( increase is constant )

Eg: 2 , 5 , 7 , 10 , 13 ...
+3 +3 +3 +3

$T_n = a + (n-1)d$

a - first term (in, 2)
d - common difference (in, +3)

**2nd degree difference**
(increase not constant)

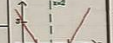
Eg: 4 , 5 , 8 , 13 , 20 ...
1st degree: +1 +3 +5 +7
2nd degree: +2 +2 +2
*only constant when "second degree"

General equation: $an^2 + bn + c$

Step 1: $2a = 2$ → $a = 1$
Step 2: $3(1) + b = 1$ → $b = -2$
Step 3: $a + b + c = 4$
$(1) + (-2) + c = 4$
$c = 5$
General eqn = $n^2 - 2n + 5$

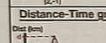
Solve for a, b, c and sub back into general equation

## Distance-Time graph



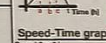

1. Speed = gradient
2. Total dist = $2 \times c$ (At time 1)
3. Resting between b=a and b=c
4. Ave speed= Total dist / Total time

## Speed-Time graph




1. Acceleration = gradient
2. Total dist = Area under graph

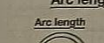
Convert to dist-time graph
Add up the distances found from above




Convert to acc-time graph
Acceleration at
b=5 b=10 b=15
a=0 a=0 a=-6

## Arc length & Sector area

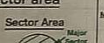
**Arc length**




Degree: $\frac{\theta}{360} \times 2\pi r$
Radian: $r\theta$

[degree formula is NOT given]
only radian is given

**Sector Area**




Degree: $\frac{\theta}{360} \times \pi r^2$
Radian: $\frac{1}{2} r^2 \theta$

**Segment Area**

Minor segment

$\frac{\text{Minor Sector}}{ABC} - \frac{\text{Area}\triangle}{ABC}$

Major segment

$\frac{\text{Major Sector}}{ABC} + \frac{\text{Area}\triangle}{ABC}$

## Graphs

$x^3$

$\frac{a}{x}$

$x^2$

$\frac{a}{x^2}$

## Congruency/Similarity

**Congruency**

1. SSS
2. SAS
3. ASA/ AAS
4. RHS

1. $\angle BAC = \angle YXZ = 90$ (A)
2. $AB = XY = 5cm$ (S)
3. $AC = XZ = 7cm$ (S)

Thus, $\triangle BAC \cong \triangle YXZ$ (SAS)

*Note that ORDER matters*
For S, Correct say AB = YX

**Similarity**

1. AA
2. SSS
3. SAS

COMMON SIMILAR TRIANGLES

Prove $\triangle ACB$ similar to $\triangle ECD$

1. $\angle ACB = \angle ECD$ (Vert opp $\angle$)
2. $\angle CAB = \angle CED$ (Alt $\angle$, AB//DE)

Thus, $\triangle ACB$ similar to $\triangle ECD$ (AA)

## Similar Figures

If A is geometrically similar to B

$$\frac{\text{Area A}}{\text{Area B}} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{\text{Volume A}}{\text{Volume B}} = \left(\frac{l_1}{l_2}\right)^3$$

If question use "mass/weight", treat is like volume

## Polygons

**Interior angle**

1. Sum of interior angles $= (n-2) \times 180°$
2. Each interior angle $= \frac{(n-2) \times 180°}{n}$

**Exterior angle**

1. Ext. angle $= 180° -$ interior angle
2. Ext. angle $= \frac{360°}{n}$

Important Facts

1. Int. $\angle$ + Ext. $\angle = 180°$
2. All Ext. $\angle$ add up to $360°$

## Angles

1. Angle sum in a triangle $\angle a + \angle b + \angle c = 180°$
2. Adjacent angles on a straight line $\angle a + \angle b = 180°$
3. Angles at a point $\angle a + \angle b + \angle c = 360°$
4. Alternate $\angle$ (AB//CD) $\angle a = \angle b$
5. Corresponding $\angle$ (AB//CD) $\angle a = \angle b$
6. Interior $\angle$ (AB//CD) $\angle a + \angle b = 180°$
7. Vertically Opposite $\angle$ $\angle a = \angle b$

## Trigonometry

**1. Toa Cah Soh**

$$\sin A = \frac{O}{H} = \frac{BC}{AB} \text{ (Soh)}$$
$$\cos A = \frac{A}{H} = \frac{AC}{AB} \text{ (Cah)}$$
$$\tan A = \frac{O}{A} = \frac{BC}{AC} \text{ (Toa)}$$

Only for right-angled triangles

**2. Sine-rule**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

For non right-angled triangles

**3. Cos-rule**

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**4. Area $\triangle$ ABC** $= \frac{1}{2} ab \sin C$

**5. Obtuse angle rules**

Must Memorise

$$\sin(180 - A) = \sin A$$
$$\cos(180 - A) = -\cos A$$
$$\tan(180 - A) = -\tan A$$

Acute Angle A

Obtuse angle (180 - A)

## Bearings

Bearing of A from B

1. Start from B
2. Draw North from B
3. Clockwise until hit line AB

If bearing is $<100°$, must put "0" infront lic If bearing $= 50°$, must write 050°

All north are parallel
1. Always use interior angles

## Angle of Elevation/Depression

Angle of elevation of B from A

Must draw right-angled $\triangle$

Angle depression

Angle elevation

Do Toa Cah Soh

$\angle$Elevation = $\angle$Depression (Alternate $\angle$)

Greater $\angle$ elevation = use shortest dist to object

## Shortest distance of A to BC

Method 1: Toa Cah Soh A ABX

Step1: Let AX be shortest dist of A to BC

Step2: Find AX by either:
1. Toa Cah Soh $\triangle ABX$ or $\triangle ACX$
2. Using area of $\triangle$

$\sin 30 = \frac{AX}{7cm}$

$AX = 7cm \times 30 = 3.5cm$

Method 2: Area of triangle

$\frac{1}{2}(15)(AX) = \frac{1}{2}(7)(15)\sin 30$

$\frac{1}{2} \times \text{base} \times \text{ht}$ $\frac{1}{2} ab \sin C$

## Matrices

1. *Addition/Subtraction*

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a \pm w & b \pm x \\ c \pm y & d \pm z \end{pmatrix}$$

2. *Multiplication (*)*

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{pmatrix}$$

Order of matrix multiplication

$$\underset{(2 \times 2)}{\begin{pmatrix} x & x \\ x & x \end{pmatrix}} \underset{(2 \times 2)}{\begin{pmatrix} x & x \\ x & x \end{pmatrix}} = \underset{(2 \times 2)}{\begin{pmatrix} x & x \\ x & x \end{pmatrix}}$$

3. *Identity Matrix*

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

4. *Null Matrix*

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

5. *Scalar Multiple*

$$2\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix}$$

6. *Square Matrix*

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\neq \begin{pmatrix} a^2 & b^2 \\ c^2 & d^2 \end{pmatrix}$$

(1 1 1) matrix
[ To sum values up ]

(0 0 0) matrix
[ To remove values]

## Vectors

1. Vectors have magnitude & direction

$\overrightarrow{AB} = \begin{pmatrix} +3 \\ +2 \end{pmatrix}$ x value +3, y value +2

2. Magnitude of AB

$$|\overrightarrow{AB}| = \left|\begin{pmatrix} 3 \\ 2 \end{pmatrix}\right| = \sqrt{(3)^2 + (2)^2}$$

3. Finding vectors (2methods)

Method 1: (when diagram given) [triangle law/ parallelogram law]

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -\overrightarrow{OA} + \overrightarrow{OB}$$
$$= -a + b$$

Method 2: (when no diagram given) [ second - first vector ]

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

4. Translation (just add up vectors)

Translate $\begin{pmatrix} a \\ b \end{pmatrix}$ by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$$= \begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$$

4. Equal vectors

1. Both must be parallel [AB // CD]
2. Both same length [AB : CD] 1 : 1

Hence: $\overrightarrow{AB} = \overrightarrow{CD}$

5. Prove parallel/ collinear

To prove parallel
$a = kb$
(k = scalar multiple)

To prove collinear
1. Prove parallel
2. Show that vectors share a common point

Example
Prove that A, B and C are collinear/ lie on the same line

$\overrightarrow{AB} = 3a + 2b$
$\overrightarrow{AC} = 6a + 4b$

Answer
1. $\overrightarrow{AC} = 2(3a + 2b)$
2. $\overrightarrow{AC} = 2\overrightarrow{AB}$

Hence, AC//AB
2. $\overrightarrow{AC}$ & $\overrightarrow{AB}$ share a common point A

Hence, A,B,C are collinear

## Statistics

1. Cumulative frequency

Can find 2 things
1. Median
2. IQR

Q1 - Lower Quartile (25%)
Q2 - Median (50%)
Q3 - Upper Quartile (75%)

IQRs Q3 - Q1
[IQR measures the spread of data same as standard deviation]

2. Box and Whiskers

Can find 2 things
1. Median
2. IQR

MIN Q1 Q2 Median Q3 MAX

IQR (Q3-Q1)

RANGE (MAX - MIN)

HIGHER MEDIAN
-Higher score, Performed better

LOWER IQR
-Less spread/ More consistent

**MEAN** (average)

$$= \frac{\Sigma fx}{\Sigma f}$$

**MEDIAN** (most central)

1. Median Position $= \frac{(n+1)}{2}$
2. Find Median

**MODE** (most common)

Class's maths results
55, 66, 67,67,67,68,69

Mode = 67 marks

**Standard deviation**

$$= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Calculator method (Casio fx-9860 PLUS)

1. Press "mode" 2. Press "STAT". 3. Press "1-VAR". 4. Enter all DATA 5. Press "AC"
6. Press "SHIFT, then 1" 7. Press 4, "Var" 8. Press 2 "Ø" for MEAN Press 3 "σx" for SD [use to check ans]

## Circles

1. Angles in opposite segments

Requirement:
-all 4 sides touch circumference
Result:
-$\angle a + \angle b = 180°$

2. Tangent $\perp$ radius

Requirement:
-perpendicular must pass through centre
Result:
-$\angle AMO = \angle BMO = 90°$

3. Tangents from exterior point

Requirement:
1. AM & BM must be tangents to circle
2. OB & OA must pass through centre

Result:
1. OBM ≡ OAM (RHS)
2. $\angle OBM = \angle OAM = 90$ (Tan $\perp$ rad)

4. Angles in the same segment

Requirement
-all 4 sides must touch circumference
Result:
$\angle BAC = \angle BDC$
$\angle ABC = \angle DBC$

5. Angle at centre = 2X angle at circumference [4 types]

6. Angles in a semicircle

Requirement
1. Hypotenuse must be diameter
2. AB 5 sides touch circumference

Result:
-angle opp diameter $= 90°$

7. Base angle of isos $\triangle$

Requirement:
- OA & OB must be the radius

Result:
$\angle OAB = \angle OBA$

## Sets & Venn diagram

1. $\varepsilon$ = universal set
2. $\emptyset$ or { } = empty set

A∩B=A intersect B

A∪B=A union B

Rules

1. A ⊂ B=A is a proper subset of B [A is inside of B]
2. A ⊆ B=A is a subset of B [A same as B]
3. A' = complement of set A
4. n(A) = no. of elements in set A
5. x∈A = x is an element in set A

## Sets sketching

Union (∪): sketch everything in both sets

Intersect (∩): take overlapped portion

Step1: sketch both sets in opposite directions

Step2: take only the overlapped portion as answer

## Area of triangle

1. Triangles are similar

$$A_1 = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{A_{ABC}}{A_{ADE}} = \left(\frac{3}{2}\right)^2$$

2. Triangles are not similar [COMMON HEIGHT METHOD]

$$\frac{A_{BPCR}}{A_{APRS}} = \frac{b_1}{b_2}$$

## Probability

Rules

1. Probability of event A = P(A) = $\frac{\text{no. of events A}}{\text{total events}}$
2. If A is an impossible event > P(A)=0
3. P(A) + P(not A) = 1
   Also , P(not A) = 1 - P(A) [complement]
4. P(A or B) = P(A) + P(B) [mutually exclusive]
5. P(A and B) = P(A) x P(B) [independent]

Tree diagram

Possibility diagram

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| H | H,1 | H,2 | H,3 | H,4 | H,5 |
| T | T,1 | T,2 | T,3 | T,4 | T,5 |

P(H and even) $= \frac{5}{10} = \frac{1}{4}$