PJC 2011 JC2 H2 Mathematics End of Year Paper 1 Solution

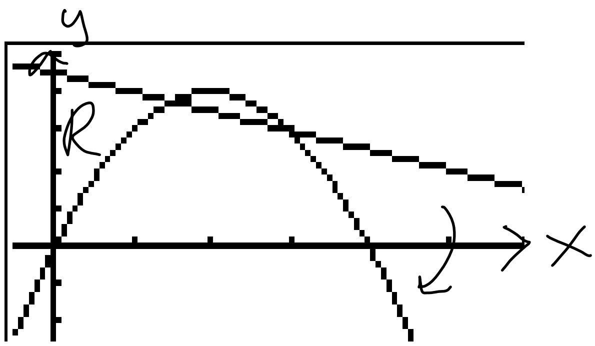
1(a)

$$\begin{aligned}\int x\tan^{-1}\left(2x^2\right)\mathrm{d}x &= \frac{1}{2}x^2\tan^{-1}\left(2x^2\right)-\frac{1}{2}\int x^2\left(\frac{4x}{1+4x^4}\right)\mathrm{d}x \\ &= \frac{1}{2}x^2\tan^{-1}\left(2x^2\right)-\frac{1}{8}\int\frac{16x^3}{1+4x^4}\,\mathrm{d}x \\ &= \frac{1}{2}x^2\tan^{-1}\left(2x^2\right)-\frac{1}{8}\ln\left(1+4x^4\right)+C\end{aligned}$$

$u=\tan^{-1}\left(2x^2\right)$ $\qquad \dfrac{\mathrm{d}v}{\mathrm{d}x}=x$

$\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{4x}{1+4x^4}$ $\qquad v=\dfrac{1}{2}x^2$

1(b)



To find point of intersection:

$y=4x-x^2 ---(1)$

$2y=9-x ----(2)$

Solving (1) & (2) by G.C.

$x=\dfrac{3}{2}$ or $x=3\,(\text{NA})$

Volume of R about x-axis

$$=\pi\int_0^{\frac{3}{2}}\left(\frac{9}{2}-\frac{x}{2}\right)^2\mathrm{d}x-\pi\int_0^{\frac{3}{2}}\left(4x-x^2\right)^2\mathrm{d}x=50.89\ \text{units}^3$$

2(i)

$$\begin{aligned}\frac{1}{\sqrt{4+x^2}} &= \frac{1}{\sqrt{4}}\left(1+\frac{x^2}{4}\right)^{-\frac{1}{2}} \\ &= \frac{1}{2}\left(1+\left(-\frac{1}{2}\right)\left(\frac{x^2}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(\frac{x^2}{4}\right)^2+\ldots\right) \\ &= \frac{1}{2}\left(1-\frac{x^2}{8}+\frac{3x^4}{128}-\ldots\right) \\ &\approx \frac{1}{2}-\frac{1}{16}x^2+\frac{3}{256}x^4\end{aligned}$$

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2(ii) $$\frac{x+1}{\sqrt{4+x^2}}=(x+1)\left(\frac{1}{\sqrt{4+x^2}}\right)$$
$$=(x+1)\left(\frac{1}{2}-\frac{1}{16}x^2+\frac{3}{256}x^4-\ldots\right)$$
$$\approx\frac{1}{2}+\frac{1}{2}x-\frac{1}{16}x^2-\frac{1}{16}x^3$$

2(iii) $$\left|\frac{x^2}{4}\right|<1$$
$$x^2<4$$
$$-2<x<2$$

3(i)

S_n

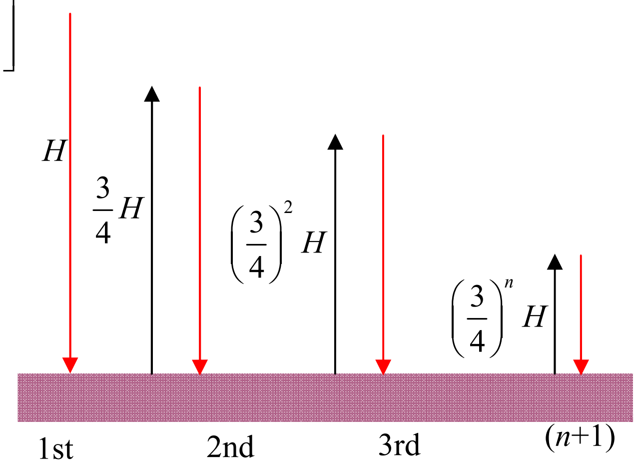
$$=4+2\left(\frac{3}{4}\right)4+2\left(\frac{3}{4}\right)^2 4+2\left(\frac{3}{4}\right)^3 4+\ldots\ldots+2\left(\frac{3}{4}\right)^n 4$$

$$=4+2(4)\left[\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+\ldots\ldots+\left(\frac{3}{4}\right)^n\right]$$

$$=4+8\left(\frac{3}{4}\right)\frac{\left[1-\left(\frac{3}{4}\right)^n\right]}{1-\left(\frac{3}{4}\right)}$$

$$=4+24\left[1-\left(\frac{3}{4}\right)^n\right]$$

$$=28-24\left(\frac{3}{4}\right)^n$$



3(ii)

$$S_n=28-24\left(\frac{3}{4}\right)^n>24$$

$$\frac{1}{6}>\left(\frac{3}{4}\right)^n$$

$$n>\frac{\ln\left(\frac{1}{6}\right)}{\ln\left(\frac{3}{4}\right)}$$

$n>6.23\Rightarrow n=7$

The ball must bounce at least 7 times for it to travel more than 24 m.

3(iii)

$n \to \infty, \quad \left(\frac{3}{4}\right)^n \to 0, \quad S_\infty = 28$

Since sum to infinity is 28, the ball will not travel more than 28m.

4(i)

$$\frac{1}{(r-3)(r-2)} \equiv \frac{A}{r-3} + \frac{B}{r-2}$$
$$1 = A(r-2) + B(r-3)$$
$$r = 2 : 1 = -B \quad \to \therefore B = -1$$
$$r = 3 : 1 = A \to A = 1$$
$$\therefore \frac{1}{(r-3)(r-2)} \equiv \frac{1}{(r-3)} - \frac{1}{(r-2)}$$
$$\sum_{r=4}^{N} \frac{1}{(r-3)(r-2)} = \sum_{r=4}^{N} \left(\frac{1}{(r-3)} - \frac{1}{(r-2)} \right)$$
$$= \left(\begin{array}{c} \frac{1}{1} - \cancel{\frac{1}{2}} \\ + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} \\ + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} \\ + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} \\ \ldots\ldots \\ + \cancel{\frac{1}{N-5}} - \cancel{\frac{1}{N-4}} \\ + \cancel{\frac{1}{N-4}} - \cancel{\frac{1}{N-3}} \\ + \cancel{\frac{1}{N-3}} - \frac{1}{N-2} \end{array} \right)$$
$$= 1 - \frac{1}{N-2}$$

4(ii)

$$\sum_{r=4}^{N} \frac{1}{(r-3)(r-2)}$$
$$= \frac{1}{(1)(2)} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \ldots + \frac{1}{(N-6)(N-5)} + \frac{1}{(N-5)(N-4)} + \frac{1}{(N-4)(N-3)} + \frac{1}{(N-3)(N-2)}$$

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$$\sum_{r=0}^{N-6}\frac{1}{(r+2)(r+1)}=\frac{1}{(2)(1)}+\frac{1}{(3)(2)}+\frac{1}{(4)(3)}+\ldots+\frac{1}{(N-5)(N-6)}+\frac{1}{(N-4)(N-5)}$$
$$=\sum_{r=4}^{N-2}\frac{1}{(r-3)(r-2)}$$
$$=1-\frac{1}{(N-2)-2}$$
$$=1-\frac{1}{N-4}$$

Alternatively,

$$\therefore\sum_{r=0}^{N-6}\frac{1}{(r+2)(r+1)}=\sum_{r=4}^{N}\frac{1}{(r-3)(r-2)}-\frac{1}{(N-4)(N-3)}-\frac{1}{(N-3)(N-2)}$$
$$=1-\frac{1}{N-2}-\frac{1}{(N-4)(N-3)}-\frac{1}{(N-3)(N-2)}$$
$$=1-\frac{1}{N-4}$$

4(iii)

$N\to\infty,\quad \frac{1}{N-2}\to 0\ \ \therefore\sum_{r=4}^{\infty}\frac{1}{(r-3)(r-2)}\to 1$ which is a finite value. Hence, the series converges.

$$\sum_{r=4}^{\infty}\frac{1}{(r-3)(r-2)}=\lim_{N\to\infty}\left(\sum_{r=4}^{N}\frac{1}{(r-3)(r-2)}\right)=\lim_{N\to\infty}\left(1-\frac{1}{N-2}\right)=1$$

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$$y=\mathrm{e}^x\cos^2 x$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\mathrm{e}^x\cos x\sin x+\mathrm{e}^x\cos^2 x$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\mathrm{e}^x\sin 2x+y$$
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^x 2\cos 2x-\mathrm{e}^x\sin 2x+\frac{\mathrm{d}y}{\mathrm{d}x}$$
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^x 2\cos 2x+\left(\frac{\mathrm{d}y}{\mathrm{d}x}-y\right)+\frac{\mathrm{d}y}{\mathrm{d}x}$$
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x}+y=-2\mathrm{e}^x\cos 2x\ \text{ (shown)}$$
$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}=4\mathrm{e}^x\sin 2x-2\mathrm{e}^x\cos 2x$$
$$x=0,\quad y=1,\quad \frac{\mathrm{d}y}{\mathrm{d}x}=1,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-1,\ \frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-5$$
$$y=\mathrm{e}^x\cos^2 x=1+x-\frac{x^2}{2!}-\frac{5x^3}{3!}+\ldots\ldots$$

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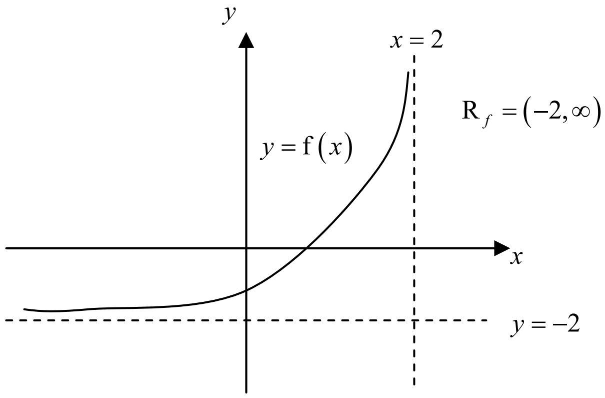
$$y = e^x \cos^2 x \approx 1 + x - \frac{x^2}{2} - \frac{5x^3}{6}$$

$$\begin{aligned}
&e^x \cos x \sin 2x \\
&= e^x \cos x (2\sin x \cos x) \\
&= 2\sin x \left(e^x \cos^2 x\right) \\
&= 2\left(x - \frac{x^3}{3!}\right)\left(1 + x - \frac{x^2}{2} - \frac{5x^3}{6}\right) \\
&= 2x + 2x^2 - x^3 - \frac{x^3}{3} + \ldots\ldots \\
&\approx 2x + 2x^2 - \frac{4x^3}{3}
\end{aligned}$$

6(i) $\quad \mathrm{f}(x) = -2 - \dfrac{3}{x-2}$



6(ii)
$$\begin{aligned}
&\text{let } y = \frac{1-2x}{x-2} \\
&xy - 2y = 1 - 2x \\
&x(y+2) = 1 + 2y \\
&x = \frac{1+2y}{y+2} \\
&\mathrm{f}^{-1} : x \mapsto \frac{1+2x}{x+2},\ x > -2
\end{aligned}$$

6(iii)

$R_g = (-\infty, \infty) \qquad D_f = (-\infty, 2)$

Since $R_g = (-\infty, \infty) \not\subset D_f = (-\infty, 2)$, fg does not exist

$R_f = (-2, \infty) \qquad D_g = (-3, \infty)$

Since $R_f = (-2, \infty) \subseteq D_g = (-3, \infty)$, gf exists

$$\begin{aligned}
\mathrm{gf}(x) &= \mathrm{g}\left(\frac{1-2x}{x-2}\right) \\
&= \ln\left(\frac{1-2x}{x-2} + 3\right) \\
&= \ln\left(\frac{1-2x+3x-6}{x-2}\right) = \ln\left(\frac{x-5}{x-2}\right), \quad x < 2
\end{aligned}$$

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7(i) $\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{3a}{t^4}$ $\qquad$ $\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{a}{t^2}$

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{t^2}{3}$

At $t=\frac{1}{2}$,

Gradient of tangent at $P=\frac{\left(\frac{1}{2}\right)^2}{3}=\frac{1}{12}$

Gradient of normal at $P=-12$

At P, $x=8a$ and $y=2a\rightarrow\left(8a,2a\right)$

Equation of tangent: $y-2a=\frac{1}{12}\left(x-8a\right)$

$y=\frac{1}{12}x+\frac{4}{3}a$

Equation of normal: $y-2a=-12\left(x-8a\right)$

$y=-12x+98a$

7(ii) $\frac{a}{t}=\frac{1}{12}\left(\frac{a}{t^3}\right)+\frac{4}{3}a$

$12t^2=1+16t^3$

$16t^3-12t^2+1=0$

By G.C, $\qquad t=\frac{1}{2}(\text{N.A.}),-\frac{1}{4}$

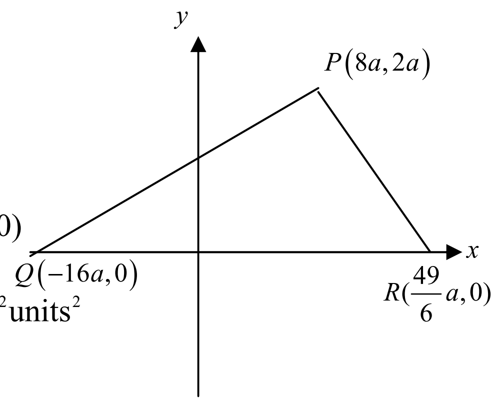
When $t=-\frac{1}{4},x=-64a,y=-4a$

Hence the tangent cuts the curve again at $\left(-64a,-4a\right)$

7(iii) At Q: $y=0$ $\qquad 0=\frac{1}{12}x+\frac{4}{3}a\ \rightarrow x=-16a\ \therefore Q\left(-16a,0\right)$

At R: $y=0$ $\qquad 0=-12x+98a\ \rightarrow\ x=\frac{49}{6}a\ \therefore R(\frac{49}{6}a,0)$

Area of triangle PQR $=\frac{1}{2}\left(\frac{49}{6}a-\left(-16a\right)\right)\left(2a\right)=\frac{145}{6}a^2\text{units}^2$



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8(i)

$\frac{dx}{dt} = k(1-2x)$

$\int \frac{1}{1-2x}\,dx = \int k\,dt$

$-\frac{1}{2}\ln|1-2x| = kt + C$

$\ln|1-2x| = -2kt + D$

$|1-2x| = e^{-2kt}e^{D}$

$1-2x = \pm e^{D}e^{-2kt}$

$1-2x = Ae^{-2kt}, \qquad A = \pm e^{D}$

$x = \frac{1}{2}\left(1 - Ae^{-2kt}\right)$

$t = 0, \quad x = 1$

$1 = \frac{1}{2}(1-A) \Rightarrow A = -1$

$t = 0, \quad \frac{dx}{dt} = -0.05$

$-0.05 = k(1-2)$

$k = 0.05$

$x = \frac{1}{2}\left(1 + e^{-0.1t}\right)$

8(ii)

$x = \frac{1}{2}\left(1 + e^{-0.1t}\right)$

$\frac{dx}{dt} = -\frac{1}{20}e^{-0.1t}$

since $e^{-0.1t} > 0$ for all t, $\frac{dx}{dt} = -\frac{1}{20}e^{-0.1t} < 0$ for all t, x is a decreasing function.

Amount of X is always decreasing.

Alternative

Since at $t = 0$, $\frac{dx}{dt} < 0$ and $\frac{d^2x}{dt^2} = -2k < 0$, x is a decreasing function.

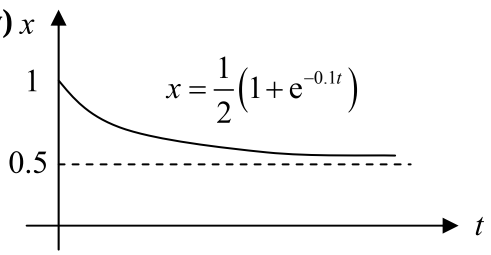
Amount of X is always decreasing.

8(iii)

When $t \to \infty, \quad e^{-0.1t} \to 0, \quad x \to \frac{1}{2}$

In the long run, X will not be used up and will stabilise at 0.5kg.

8(iv)



9(i)

$z = re^{i\theta}$ is a root, $z = re^{-i\theta}$ is another root.
A quadratic factor of $\mathrm{P}(z)$

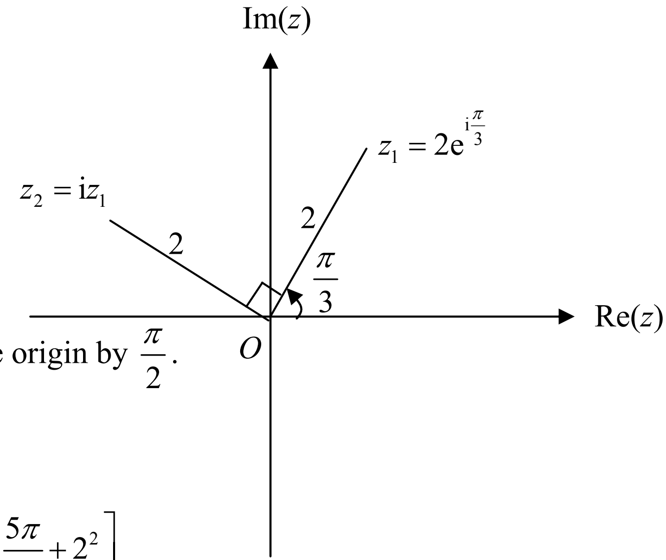
$= \left(z - re^{i\theta}\right)\left(z - re^{-i\theta}\right)$
$= z^2 - zre^{-i\theta} - zre^{i\theta} + r^2$
$= z^2 - zr\left(e^{i\theta} + e^{-i\theta}\right) + r^2$
$= z^2 - 2rz\cos\theta + r^2$ (shown)

Note :
$z + z^* = re^{i\theta} + re^{-i\theta}$
$= 2r\cos\theta = 2x = 2\,\mathrm{Re}(z)$
is a standard result that you may apply directly.

9(ii)

$z_2 = iz_1$
$|z_2| = |iz_1| = |i||z_1| = 2$
$\arg(z_2) = \arg(i) + \arg(z_1) = \dfrac{\pi}{2} + \dfrac{\pi}{3} = \dfrac{5\pi}{6}$

z_2 is an anti-clockwise rotation of z_1 about the origin by $\dfrac{\pi}{2}$.



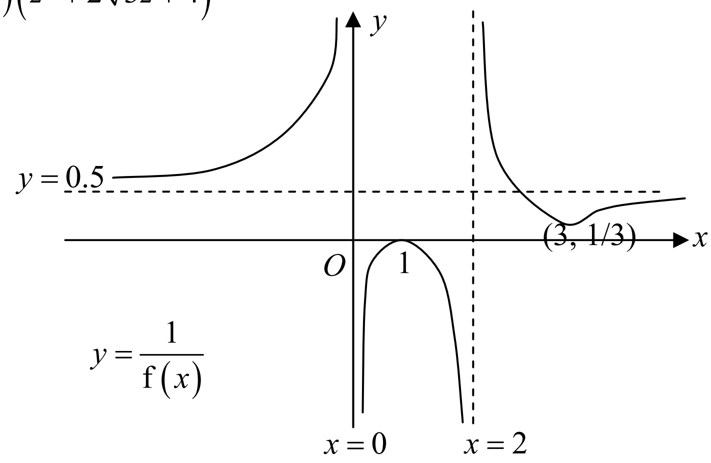
9(iii)

$\mathrm{P}(z) = \left[z^2 - 2(2)z\cos\dfrac{\pi}{3} + 2^2\right]\left[z^2 - 2(2)z\cos\dfrac{5\pi}{6} + 2^2\right]$

$= \left[z^2 - 4z\left(\dfrac{1}{2}\right) + 4\right]\left[z^2 - 4z\left(-\dfrac{\sqrt{3}}{2}\right) + 4\right]$

$= \left(z^2 - 2z + 4\right)\left(z^2 + 2\sqrt{3}z + 4\right)$

10(a)(i)



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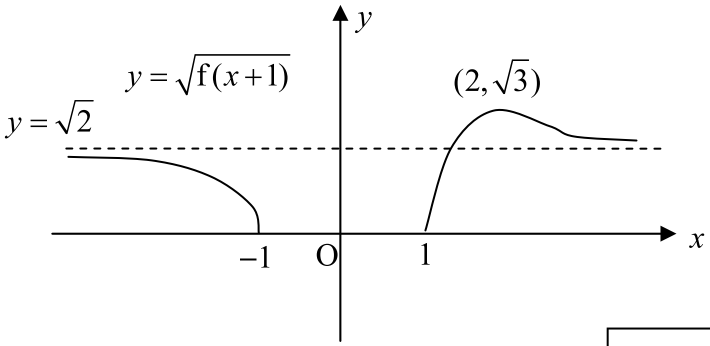
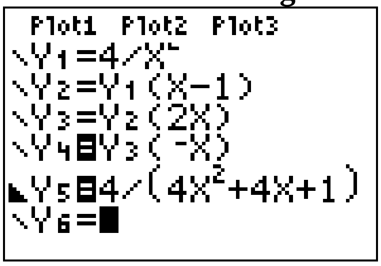
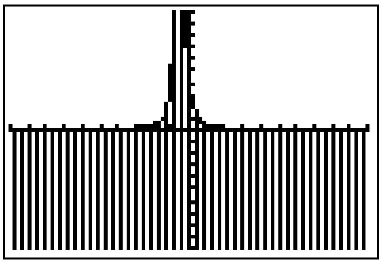
10(a)(ii)



Use GC for Checking





Y4 and Y5 should coincide if your equation is correct.

10(b) Let $\mathrm{h}(x)=\dfrac{4}{4x^2+4x+1}=\dfrac{4}{(2x+1)^2}$.

Before C, $y=\mathrm{h}(-x)=\dfrac{4}{\left[2(-x)+1\right]^2}=\dfrac{4}{(1-2x)^2}$

Let $\mathrm{p}(x)=\dfrac{4}{(1-2x)^2}$

Before B, $y=\mathrm{p}\left(\dfrac{x}{2}\right)=\dfrac{4}{\left[1-2\left(\dfrac{x}{2}\right)\right]^2}=\dfrac{4}{(1-x)^2}$

Let $\mathrm{g}(x)=\dfrac{4}{(1-x)^2}$

Before A, $y=\mathrm{f}(x)=\mathrm{g}(x+1)=\dfrac{4}{\left[1-(x+1)\right]^2}=\dfrac{4}{x^2}$

Note : The above method done without completing the square for the denominator is shown below.

Let $\mathrm{h}(x)=\dfrac{4}{4x^2+4x+1}$.

Before C, $y=\mathrm{h}(-x)=\dfrac{4}{4(-x)^2+4(-x)+1}=\dfrac{4}{4x^2-4x+1}$

Let $\mathrm{p}(x)=\dfrac{4}{4x^2-4x+1}$

Before B, $y=\mathrm{p}\left(\dfrac{x}{2}\right)=\dfrac{4}{4\left(\dfrac{x}{2}\right)^2-4\left(\dfrac{x}{2}\right)+1}=\dfrac{4}{x^2-2x+1}$

Let $\mathrm{g}(x)=\dfrac{4}{x^2-2x+1}$

Before A, $y=\mathrm{f}(x)=\mathrm{g}(x+1)=\dfrac{4}{(x+1)^2-2(x+1)+1}=\dfrac{4}{x^2}$

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11(i)

$$y=\frac{x^2-4x+k^2}{x-k}=x+k-4+\frac{2k^2-4k}{x-k}$$

vertical asymptote : $x=k$

oblique asymptote : $y=x+k-4$

11(ii)

$$\frac{\mathrm{d}y}{\mathrm{d}x}=1-\frac{\left(2k^2-4k\right)}{\left(x-k\right)^2}$$

At stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}=1-\frac{\left(2k^2-4k\right)}{\left(x-k\right)^2}=0$

$$\frac{\left(2k^2-4k\right)}{\left(x-k\right)^2}=1$$

$2k^2-4k=x^2-2kx+k^2$

$x^2-2kx+4k-k^2=0$

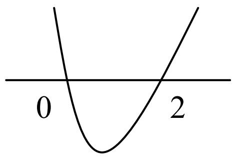
For C to have 2 stationary points,

$\left(-2k\right)^2-4\left(4k-k^2\right)>0$

$8k^2-16k>0$

$8k\left(k-2\right)>0$

$k<0$ or $k>2$



OR

$$\frac{\left(2k^2-4k\right)}{\left(x-k\right)^2}=1$$

$\left(x-k\right)^2=2k^2-4k$

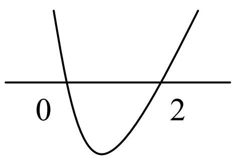
$\left(x-k\right)=\pm\sqrt{2k^2-4k}$

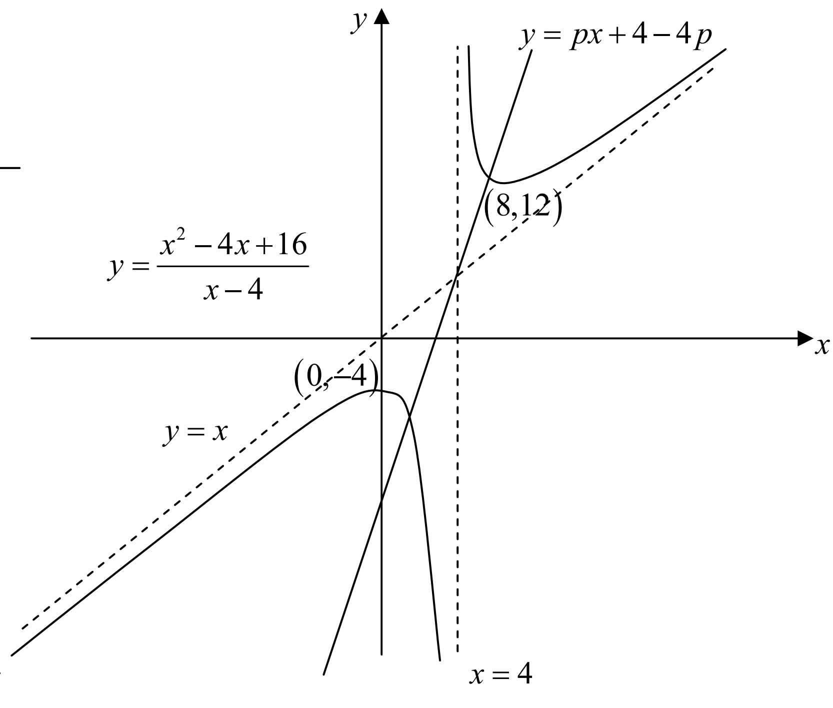
For C to have 2 stationary points

$2k^2-4k$

$2k\left(k-2\right)>0$

$k<0$ or $k>2$





11(iii)

$k-4=0 \Rightarrow k=4$

11(iv)

$$\left(x^2-4x+k^2\right)-\left(x-k\right)\left(px+4-4p\right)=0$$

$$x^2-4x+k^2=\left(x-k\right)\left(px+4-4p\right)$$

$$\frac{x^2-4x+k^2}{x-k}=px+4-4p$$

Add the graph of $y=px+4-4p$ which passes through the point $\left(4,4\right)$.

For $y=px+4-4p$ and $y=\dfrac{x^2-4x+k^2}{x-k}$ to intersect twice, $p>1$.

12(i) $\overrightarrow{BA}=\begin{pmatrix}0\\2\\-3\end{pmatrix}-\begin{pmatrix}3\\3\\-3\end{pmatrix}=\begin{pmatrix}-3\\-1\\0\end{pmatrix}$

$$\overrightarrow{CA}=\begin{pmatrix}0\\2\\-3\end{pmatrix}-\begin{pmatrix}-9\\2\\6\end{pmatrix}=\begin{pmatrix}9\\0\\-9\end{pmatrix}=9\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

$$n_1=\begin{pmatrix}-3\\-1\\0\end{pmatrix}\times\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}1\\-3\\1\end{pmatrix}$$

$$\therefore \Pi_1: \ \mathbf{r}.\begin{pmatrix}1\\-3\\1\end{pmatrix}=\begin{pmatrix}1\\-3\\1\end{pmatrix}.\begin{pmatrix}0\\2\\-3\end{pmatrix}=-9$$

$$\mathbf{r}.\begin{pmatrix}1\\-3\\1\end{pmatrix}=-9 \text{ (Shown)}$$

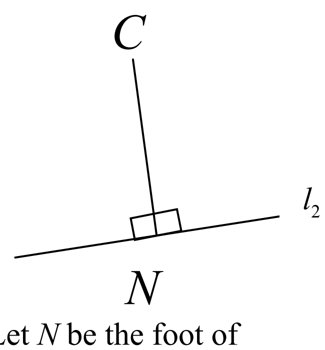
12(ii) $\overrightarrow{OC}=\begin{pmatrix}-9\\2\\6\end{pmatrix}$

Since pt N lies on line l

$$\therefore \overrightarrow{ON}=\begin{pmatrix}1\\1\\4\end{pmatrix}+\mu\begin{pmatrix}-1\\-2\\3\end{pmatrix}=\begin{pmatrix}1-\mu\\1-2\mu\\4+3\mu\end{pmatrix}$$

$$\overrightarrow{CN}=\begin{pmatrix}1-\mu\\1-2\mu\\4+3\mu\end{pmatrix}-\begin{pmatrix}-9\\2\\6\end{pmatrix}=\begin{pmatrix}10-\mu\\-1-2\mu\\-2+3\mu\end{pmatrix}$$

Since $\overrightarrow{CN}$ is perpendicular to l,



Let N be the foot of perpendicular from A to l

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$$\begin{pmatrix} 10-\mu \\ -1-2\mu \\ -2+3\mu \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} = 0$$

$$(-10+\mu)+(2+4\mu)+(-6+9\mu)=0$$

$$14\mu = 14$$

$$\mu = 1$$

$$\therefore \overrightarrow{ON} = \begin{pmatrix} 1-1 \\ 1-2(1) \\ 4+3(1) \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 7 \end{pmatrix}$$

12(iii) Convert Π_1 and Π_2 into Cartesian form:

$$4x-2y+z=-6$$

$$x-3y+z=-9$$

By using G.C.

$$x = -\frac{1}{10}\mu, y = 3+\frac{3}{10}\mu, z = \mu$$

$\therefore$ Eqn of line between π_1 *and* π_2 : $r = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 10 \end{pmatrix}, \lambda \in \mathbb{R}$

12(iv) n_3 is perpendicular to $\begin{pmatrix} -1 \\ 3 \\ 10 \end{pmatrix}$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 3 \\ 10 \end{pmatrix} = 0$$

$$-a+3b+10c = 0 ----(1)$$

Since line lies on Π_3, $\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$ must satisfy equation of Π_3

$$0a+3b+0c=3$$

$$b = 1 -----(2)$$

Since $\begin{pmatrix} 5 \\ -12 \\ 2 \end{pmatrix}$ lies in Π_3, $\begin{pmatrix} 5 \\ -12 \\ 2 \end{pmatrix}$ must satisfy equation of Π_3

$$5a-12b+2c=3 \quad -----(3)$$

By solving the 3 equations, $a=3, b=1, c=0$

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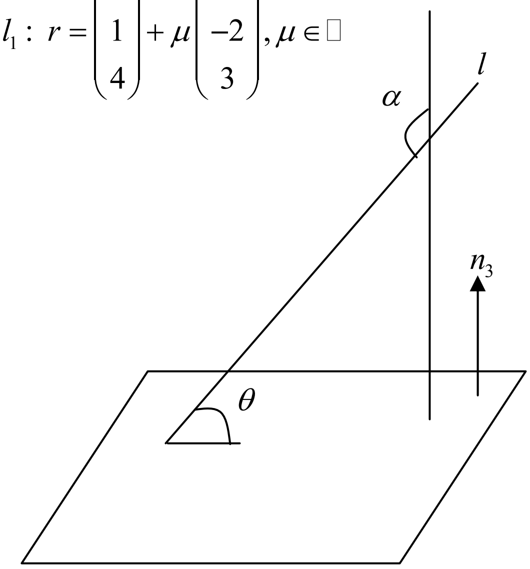
$\Pi_3 : r \cdot \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = 3$ $\qquad$ $l_1 :\ r = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}, \mu \in \mathbb{R}$



$$\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} = \sqrt{10}\sqrt{14}\cos\alpha$$

$$-5 = \sqrt{10}\sqrt{14}\cos\alpha$$

$$\cos\alpha = \frac{-5}{\sqrt{10}\sqrt{14}}$$

$$\alpha = 114.997^{\circ}$$

$$\therefore \theta = 114.997^{\circ} - 90^{\circ}$$

$$= 24.997^{\circ} = 25.0^{\circ} \text{ (1d.p.)}$$

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