National Junior College
2016 – 2017 H2 Further Mathematics
Topic F7: Further Complex Numbers (Assignment 1)

Name: ______________________________ **Suggested Duration: 65 min**

1. Find the fourth roots of $-8+\left(8\sqrt{3}\right)\mathrm{i}$ exactly in the form $r\mathrm{e}^{\mathrm{i}\theta}$. [3]

   Hence find the roots of the equation $z^8+16z^4+256=0$ exactly in the form $r\mathrm{e}^{\mathrm{i}\theta}$. [3]

2. (i) Without using a calculator, show that
$$\left(1-\sqrt{3}\mathrm{i}\right)^{-12}=\frac{1}{4096}.$$ [2]

   (ii) Hence solve the equation
$$z^6\left(1-\sqrt{3}\mathrm{i}\right)^{12}-1=0,$$
giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. Show the roots on an Argand diagram. [4]

   (iii) Show that $\left(z-r\mathrm{e}^{\mathrm{i}\theta}\right)\left(z-r\mathrm{e}^{-\mathrm{i}\theta}\right)=z^2-2rz\cos\theta+r^2$. [2]

   (iv) Use your answers in parts (ii) and (iii) to express $z^6\left(1-\sqrt{3}\mathrm{i}\right)^{12}-1$ as the product of three quadratic factors with real coefficients, giving each factor in non-trigonometrical form. [3]

   (2014/PJC/P1/Q8)

3. (i) Use the formula for the sum of a geometric series to show that
$$\sum_{k=1}^{n}(z+z^2+\ldots+z^k)=\frac{nz}{1-z}-\frac{z^2}{(1-z)^2}(1-z^n),\quad z\neq 1.$$ [3]

   (ii) Given that $z=\cos\theta+\mathrm{i}\sin\theta$, show that
$$\frac{z}{1-z}=\frac{\mathrm{i}}{2\sin\frac{\theta}{2}}\left(\cos\frac{\theta}{2}+\mathrm{i}\sin\frac{\theta}{2}\right).$$ [2]

   Deduce that $$\sum_{k=1}^{n}(\sin\theta+\sin 2\theta+\ldots+\sin k\theta)=\frac{(n+1)\sin\theta-\sin(n+1)\theta}{4\sin^2\frac{\theta}{2}}.$$ [5]

   (2003/FM/TPJC/P1/Q6)

4. Use de Moivre's theorem to prove that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta .$$ [3]

Show that $\cos\frac{\pi}{10}$ is a root of the equation $16x^4 - 20x^2 + 5 = 0$, and obtain the other three roots in trigonometric form. [3]

Hence show that $\cos\frac{\pi}{10}\cos\frac{3\pi}{10} = \frac{\sqrt{5}}{4}$. [3]

(2002/FM/TJC/P1/Q10)