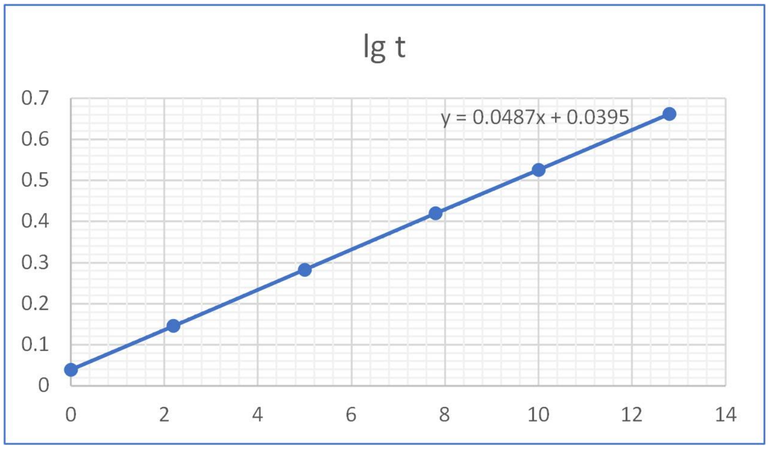
2024 AMKSS 4E5N Prelim AMP2 Marking Scheme

Qn	Solution	Marks
1	$100^x + 10^{x+1} - 24 = 0$ $10^{2x} + 10\left(10^x\right) - 24 = 0$ Let $y = 10^x$ $y^2 + 10y - 24 = 0$ $(y+12)(y-2) = 0$ $y = -12$ or $y = 2$ $10^x = -12$ (rejected) $10^x = 2$ $\lg 10^x = \lg 2$ $x = \lg 2$	B1 (correct quadratic) M1 (factorise or quadratic formula) A1 (reject $10^x = -12$, do not accept if reject $y = -12$) A1
2	Let $\mathrm{f}(x) = 3x^3 - 5x + 2$ $\mathrm{f}(1) = 3(1)^3 - 5(1) + 2 = 0$ By **factor theorem**, $x-1$ is a factor of $3x^3 - 5x + 2$. $3x^3 - 5x + 2 = (x-1)\left(3x^2 + Ax - 2\right)$ Comparing coefficient of *x*, $-5 = -2 - A$ $A = 3$ $(x-1)\left(3x^2 + 3x - 2\right) = 0$ $x = 1$ or $x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(3)(-2)}}{2(3)}$ $x = \dfrac{-3 \pm \sqrt{33}}{6}$	B1 M1 (or long division: must see $3x^2$) M1 (factors equate to 0) A1 (for $x = 1$) A1 (for exact values)
3(a)	$y = (x-1)\sqrt{4x+1}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (4x+1)^{\frac{1}{2}}(1) + (x-1)\dfrac{1}{2}(4x+1)^{-\frac{1}{2}}(4)$ $= (4x+1)^{-\frac{1}{2}}\left[(4x+1) + 2(x-1)\right]$ $= \dfrac{4x+1+2x-2}{\sqrt{4x+1}}$ $= \dfrac{6x-1}{\sqrt{4x+1}}$	M1 (for chain rule), M1 (product rule) M1 (factorise or combine to single fraction) A1

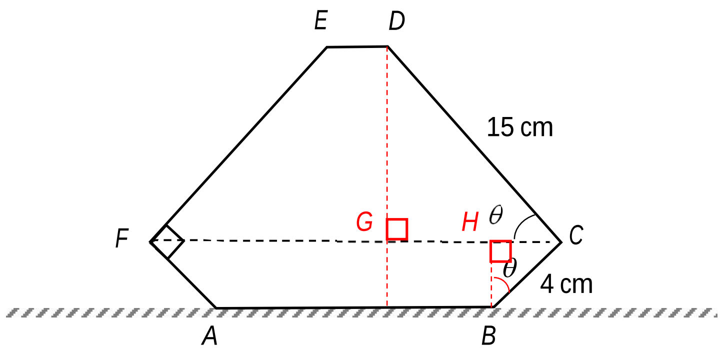
Qn	Solution	Marks
3(b)	$\int_2^6 \frac{6x-1}{\sqrt{4x+1}}\,\mathrm{d}x = \left[(x-1)\sqrt{4x+1}\right]_2^6$ $\int_2^6 \frac{6x}{\sqrt{4x+1}}\,\mathrm{d}x - \int_2^6 \frac{1}{\sqrt{4x+1}}\,\mathrm{d}x = \left[(x-1)\sqrt{4x+1}\right]_2^6$ $\int_2^6 \frac{6x}{\sqrt{4x+1}}\,\mathrm{d}x = \left[(x-1)\sqrt{4x+1}\right]_2^6 + \left[\frac{(4x+1)^{\frac{1}{2}}}{\frac{1}{2}(4)}\right]_2^6$ $\int_2^6 \frac{x}{\sqrt{4x+1}}\,\mathrm{d}x = \frac{1}{6}\left[(x-1)\sqrt{4x+1} + \frac{\sqrt{4x+1}}{2}\right]_2^6$ $\int_2^6 \frac{x}{\sqrt{4x+1}}\,\mathrm{d}x = \frac{1}{6}\left[\left(((6)-1)\sqrt{4(6)+1} + \frac{\sqrt{4(6)+1}}{2}\right) - \left(((2)-1)\sqrt{4(2)+1} + \frac{\sqrt{4(2)+1}}{2}\right)\right]$ $= 3\frac{5}{6}$	M1 (using (i)) M1 (integrate *their* $\frac{q}{\sqrt{4x+1}}$ correctly) A1 (correct $\int_2^6 \frac{x}{\sqrt{4x+1}}\,\mathrm{d}x$ including $\frac{1}{6}$) M1 (substitute correct values in correct order) A1
4(a)	$x < -2$ or $x > 3$ $(x+2)(-x+3) < 0$ $-x^2 + x + 6 < 0$ $-3x^2 + 3x + 18 < 0$ $p = 3$ $q = 18$	M1 (accept $(x+2)(x-3) > 0$; $y = -3(x+2)(x-3)$) A1 A1
4(b) (i)	$qx + 1 = -3x^2 + x + q$ $3x^2 + (q-1)x + 1 - q = 0$ $(q-1)^2 - 4(3)(1-q) = 0$ $q^2 - 2q + 1 - 12 + 12q = 0$ $q^2 + 10q - 11 = 0$ $(q+11)(q-1) = 0$ $q = -11$ or $q = 1$	M1 (eliminate *x* or *y*) M1 (correct D and = 0) M1 (factorise or quadratic formula) A1
4(b) (ii)	$3x^2 - 12x + 12 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $x = 2$ $y = -21$ $R(2,\ -21)$	M1 (factorise or quadratic formula A1 (correct *x*) A1 (correct *y*)

<table>
<tr><th>Qn</th><th>Solution</th><th>Marks</th></tr>
<tr><td>5(a)
(i)</td><td>
<table>
<tr><td>T</td><td>2.2</td><td>5.0</td><td>7.8</td><td>10</td><td>12.8</td></tr>
<tr><td>lg t</td><td>0.146</td><td>0.283</td><td>0.420</td><td>0.526</td><td>0.662</td></tr>
</table>


</td><td>B2 (minus 1 for each incorrect point; minus 1 if axes not labelled)</td></tr>
<tr><td>5(a)
(ii)
(a)</td><td>When $T=0$
$\lg t = 0.04$
$t = 1.10$ (accept 1.07 to 1.13)</td><td>M1 (equate vertical intercept to lg t)
A1
(both marks only given if graph is extended to find $\lg t$-intercept)</td></tr>
<tr><td>5(a)
(ii)
(b)</td><td>$\lg t = \lg a + \lg\left(1.064\right)^{kT}$
$\lg t = \lg a + kT\lg\left(1.064\right)$

Gradient = 0.0487
$k\lg\left(1.064\right) = 0.0487$
$k = \dfrac{0.0487}{\lg\left(1.064\right)}$
$= 1.807611629$
$= 1.81$ $\left(\text{accept } 1.7 \text{ to } 1.9\right)$</td><td>M1 (correct linear equation or equating gradient to $k\lg\left(1.064\right)$)
M1 (find gradient using line drawn)

A1</td></tr>
<tr><td>5(a)
(iii)</td><td>Physical attributes such body fat of the diver different, pre-existing health conditions, materials of diving suit.</td><td>B1 (accept other logical reason based in context of question. Reject answers like values not accurate; different bodies take different time etc)</td></tr>
</table>

Qn	Solution	Marks
5(b)	Gradient $= \dfrac{3-(-1)}{1-(-1)} = 2$ $Y-3=2(X-1)$ $Y=2X+1$ $y^2=2x^2y+1$	B1 (correct gradient) M1 (subs point into equation using *their gradient*) A1
6(a)	 $\sin\theta = \dfrac{DG}{15}$ $DG = 15\sin\theta$ $\cos\theta = \dfrac{HB}{4}$ $HB = 4\cos\theta$ Height of D from ground = $(4\cos\theta + 15\sin\theta)$ cm	M1 (to find *DG*, only given with correct θ and correct right-angle) M1 (to find *HB*, only given with correct θ and correct right-angle)
6(b)	$4\cos\theta + 15\sin\theta = R\cos(\theta-\alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $R\cos\alpha = 4$ $R\sin\alpha = 15$ $R=\sqrt{4^2+15^2}$ $=\sqrt{241}$ $\tan\alpha = \dfrac{15}{4}$ $\alpha = 75.06858282°$ $4\cos\theta + 15\sin\theta = \sqrt{241}\cos(\theta - 75.1°)$	M1 (all correct, minus 1 if never write) M1 (find *R*) M1 (form trigo equation, accept $\cos\alpha = \dfrac{4}{\sqrt{241}}; \sin\alpha = \dfrac{15}{\sqrt{24}}$) A1 (accept $15.5\cos(\theta - 75.1°)$)

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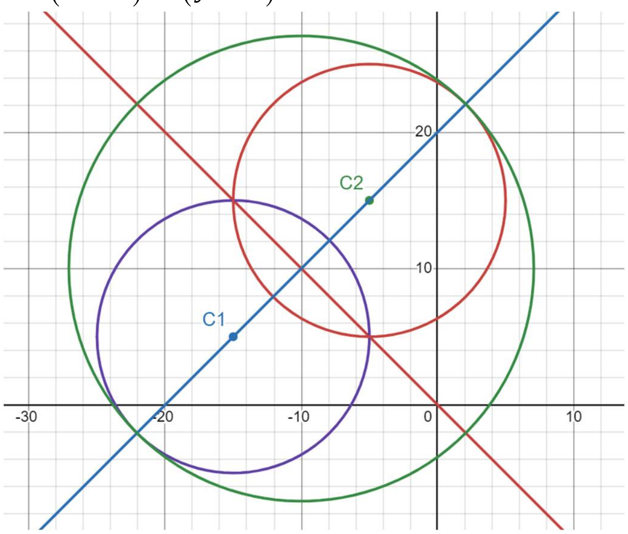
6(c)	$\sqrt{241}\cos(\theta-75.06°)=14$ $\cos(\theta-75.06°)=\frac{14}{\sqrt{241}}$ $\theta-75.06°=-25.60175686$ $\theta=49.46682596°$ $=49.5°$ (1 d.p.) Since $40°<49.5°<50°$, the chock can secure the aircraft wheel.	M1 for $\cos(\theta-their\alpha)=\frac{14}{their\ R}$ A1 A1 (only given for correct θ)
7(a)	$12=ke^{0.24(0)}+8$ $12=k+8$ $k=4$	B1 (substitute $t=0$ and equate to 12)
7(b)	$21=4e^{0.24t}+8$ $e^{0.24t}=\frac{13}{4}$ $\ln e^{0.24t}=\ln\frac{13}{4}$ $0.24t=\ln\frac{13}{4}$ $t=4.911062485$ $=4.91$ s (3sf) Since $t>3$, he did not manage to pass point B before the traffic light turned red.	B1 M1 (take ln on both sides) M1 A1 (only given for correct t and comparison with 3 s)
7(c)	$s=\int 4e^{0.24t}+8\ \mathrm{d}t$ $s=\frac{50}{3}e^{0.24t}+8t+c$ Substitute s = 0 and t = 0 $\frac{50}{3}e^{0.24(0)}+8(0)+c=0$ $c=-\frac{50}{3}$ $s=\frac{50}{3}e^{0.24t}+8t-\frac{50}{3}$ $s=\frac{50}{3}e^{0.24(4.911)}+8(4.911)-\frac{50}{3}$ $=76.78849988$ m Average speed $=\frac{76.78849988}{4.911062485}$ $=15.63582221$ $=15.6$ m/s (3sf)	M1 (without c) A1 (with correct c) M1 (sub their t from (ii)) M1 (divide their t by their s) A1

Qn	Solution	Marks
8(a)	${}^{n}C_r x^{n-r}\left(\frac{k}{2x}\right)^r$ Power of x $= n-r-r$ $= n-2r$ If ***n* is an odd integer**, since **2*r* is even**, $n-2r$ is an odd integer. Hence x only has odd powers in every term.	B1 (correct general term, don't need to expand) B1 (explain using power)
8(b)	$11-2r=7$ $2r=4$ $r=2$ Term in x^7 $= {}^{11}C_2(x)^9\left(\frac{k}{2x}\right)^2$ $=\frac{55}{4}k^2x^7$ Coefficient of $x^7=\frac{55}{4}k^2$	M1 (using *their* general term to find x^7) A1 (or B2)
8(c)	$\left(x^2-\frac{k}{2}+\frac{k^2}{4x^2}\right)\left(x+\frac{k}{2x}\right)^{12}$ $=\left(x^2-x\left(\frac{k}{2x}\right)+\left(\frac{k}{2x}\right)^2\right)\left(x+\frac{k}{2x}\right)\left(x+\frac{k}{2x}\right)^{11}$ $=\left(x^3+\left(\frac{k}{2x}\right)^3\right)\left(x+\frac{k}{2x}\right)^{11}$ $=\left(x^3+\frac{k^3}{8x^3}\right)\left(x+\frac{k}{2x}\right)^{11}$ Alternative method: $\left(x^2-\frac{k}{2}+\frac{k^2}{4x^2}\right)\left(x+\frac{k}{2x}\right)^{12}$ $=\left(x^2-\frac{k}{2}+\frac{k^2}{4x^2}\right)\left(x+\frac{k}{2x}\right)\left(x+\frac{k}{2x}\right)^{11}$ M1 $=\left(x^3+\frac{kx}{2}-\frac{kx}{2}-\frac{k^2}{4x}+\frac{k^2}{4x}+\frac{k^3}{8x^3}\right)\left(x+\frac{k}{2x}\right)^{11}$ M1 (show expansion $=\left(x^3+\frac{k^3}{8x^3}\right)\left(x+\frac{k}{2x}\right)^{11}$	M1 (separate into $\left(x+\frac{ky}{2}\right)\left(x+\frac{ky}{2}\right)^{11}$) M1 (using sum of cubes) No mark if they did not show $\left(x^3+\left(\frac{ky}{2}\right)^3\right)$

Qn	Solution	Marks
8(d)	In the expansion of $\left(x+\dfrac{k}{2x}\right)^{11}$, Term in $x = {}^{11}C_5\left(x\right)^6\left(\dfrac{k}{2x}\right)^5 = \dfrac{231}{16}k^5x$ $\dfrac{55}{4}k^2\left(\dfrac{k^3}{8}\right)+\dfrac{231}{16}k^5\left(1\right)=-577$ $\dfrac{517}{32}k^5=-577$ $k^5=-\dfrac{18464}{517}$ $k=-2.044405542$ $=-2.04$ (3sf)	 M1 (using *their* general term to find x) M1 (correct products to form equation) A1
9(a)	The points of intersection are $\left(-15,\ 15\right)$ and $\left(-5,\ 5\right)$. Midpoint of these points of intersection $=\left(\dfrac{-15+\left(-5\right)}{2},\ \dfrac{15+5}{2}\right)=\left(-10,\ 10\right)$ Equation of line passing through the centres of C_1 and C_2: $y-10=1\left(x+10\right)$ $y=x+20$	 B1 (correct midpoint) M1 (find equation of line passing pass through centre)
9(b)	Let the centre of C_1 be $\left(a,\ a+20\right)$ $\sqrt{\left(a-\left(-5\right)\right)^2+\left(a+20-5\right)^2}=10$ $\left(a+5\right)^2+\left(a+15\right)^2=100$ $a^2+10a+25+a^2+30a+225=100$ $2a^2+40a+150=0$ $a^2+20a+75=0$ $\left(a+15\right)\left(a+5\right)=0$ $a=-5$ or -15 Centres of C_1 and C_2 are $\left(-5,\ 15\right)$ and $\left(-15,\ 5\right)$.	 M1 (sub *their equation* into length formula) M1 (factorise or quadratic formula) A1, A1

Qn	**Solution**	**Marks**
9(c)	Centre of C_3 is $(-10,\ 10)$ Distance from $(-10,\ 10)$ to $(-15,\ 5)$ $=\sqrt{(-10-(-15))^2+(10-5)^2}$ $=\sqrt{50}$ units Radius of C_3 $=\sqrt{50}+10$ units Equation of circle C_3 is $(x-(-10))^2+(y-10)^2=\left(\sqrt{50}+10\right)^2$ $(x+10)^2+(y-10)^2=50+20\sqrt{50}+100$ $(x+10)^2+(y-10)^2=150+100\sqrt{2}$ 	M1 (find distance from *their centre* to midpoint) (or find distance between centres of C_1 and C_2) M1 (*their distance* + 10 units) (or (distance between centres of C_1 and C_2)/2 +10) M1 (form equation using *their radius* and $(-10,\ 10)$) A1

Qn	Solution	Marks
10 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{\sqrt{3}x}\left(\frac{d}{dx}\cos x\right) + \cos x\left(\frac{d}{dx}e^{\sqrt{3}x}\right)$ $= -e^{\sqrt{3}x}\sin x + \sqrt{3}e^{\sqrt{3}x}\cos x$ At stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $-e^{\sqrt{3}x}\sin x + \sqrt{3}e^{\sqrt{3}x}\cos x = 0$ $e^{\sqrt{3}x}\left(-\sin x + \sqrt{3}\cos x\right) = 0$ $e^{\sqrt{3}x} = 0$ (rejected) or $-\sin x + \sqrt{3}\cos x = 0$ $\tan x = \sqrt{3}$ $x = \frac{\pi}{3}$ $y = e^{\sqrt{3}\left(\frac{\pi}{3}\right)}\cos\left(\frac{\pi}{3}\right)$ $= \frac{1}{2}e^{\frac{\pi}{\sqrt{3}}}$ $C\left(\frac{\pi}{3}, \frac{1}{2}e^{\frac{\pi}{\sqrt{3}}}\right)$	M1 for product rule M1 for $-\sin x$ M1 for $\sqrt{3}e^{\sqrt{3}x}$ M1 (equate *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0) M1 (factorise) (minus 1 from Q10 if never reject $e^{\sqrt{3}x} = 0$) M1 (change to tan) A1 (correct *x*) A1 (correct y)
10 (b)	$e^{\sqrt{3}x}\cos x = 0$ $e^{\sqrt{3}x} = 0$ (rejected) or $\cos x = 0$ $x = -\frac{\pi}{2}, \frac{\pi}{2}$ Coordinates of *A* and *B* are $\left(-\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 0\right)$. Area of triangle ABC $= \frac{1}{2}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)\frac{1}{2}e^{\frac{\pi}{\sqrt{3}}}$ $= = \frac{1}{2}(\pi)\frac{1}{2}e^{\frac{\pi}{\sqrt{3}}}$ $= \frac{\pi}{4}e^{\frac{\pi}{\sqrt{3}}}$	M1 (equate *y* to 0 and solve) (minus 1 from Q10 if never reject $e^{\sqrt{3}x} = 0$) A1, A1 (or only *x*-coordinates) M1 (using *their* *x*-coordinates of *A* and *B* and their *y*-coordinate of *C*) M1 (all correct)