Marking Scheme
2024 Sec 4 Express PRELIM AM Paper 1

			Marker's Comments
1(a)	$y=-2x^2+3x+5$ $=-2\left[x^2-\frac{3}{2}x\right]+5$ --- M1 $=-2\left[\left(x-\frac{3}{4}\right)^2-\frac{9}{16}\right]+5$ --- M1 $=-2\left(x-\frac{3}{4}\right)^2+\frac{18}{16}+5$ $=-2\left(x-\frac{3}{4}\right)^2+\frac{49}{8}$ --- A1	**3 marks**	
(b)	For all real values of x, $\left(x-\frac{3}{4}\right)^2\geq 0$ --- M1 $-2\left(x-\frac{3}{4}\right)^2\leq 0$ $-2\left(x-\frac{3}{4}\right)^2+\frac{49}{8}\leq\frac{49}{8}$ --- A1 $\therefore x=\frac{3}{4}, y_{maxi}=\frac{49}{8}$	**2 marks**	
	Alternative method $y=-2\left(x-\frac{3}{4}\right)^2+k$ When $x=\frac{3}{4}, y=-2(0)+k$ $=k$ $\frac{dy}{dx}=-4(x-\frac{3}{4})$ When $\frac{dy}{dx}=0$ --- M1 $x=\frac{3}{4}$ $\frac{d^2y}{dx^2}=-4(<0)$ --- A1 $\therefore y$ is a maximum value, k	**2 marks**	

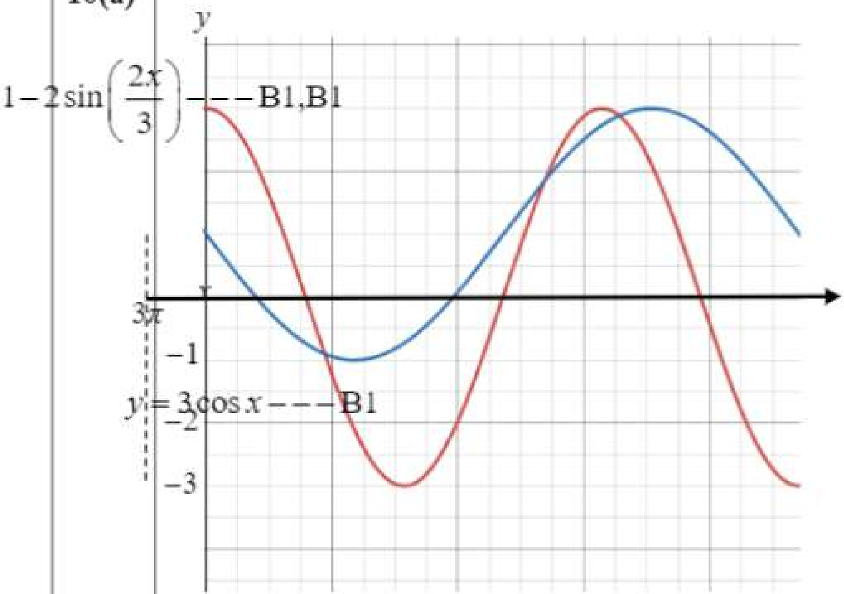
2(a)	$64-(x+1)^3$ $=4^3-(x+1)^3$ $=[4-(x+1)][4^2+4(x+1)+(x+1)^2]$---M1 $=(3-x)(16+4x+4+x^2+2x+1)$ $=(3-x)(x^2+6x+21)$---A1	**2 marks**	
(b)	$(3-x)(x^2+6x+21)=15(3-x)$ $(3-x)(x^2+6x+21)-15(3-x)=0$ $(3-x)(x^2+6x+21-15)=0$ $(3-x)(x^2+6x+6)=0$---M1 $(3-x)=0$ or $(x^2+6x+6)=0$ $x=3$ or $x=\frac{-6\pm\sqrt{36-4(6)}}{2}$ $x=3$ or $x=\frac{-6\pm\sqrt{12}}{2}$ $x=3$ or $x=\frac{-6\pm2\sqrt{3}}{2}$ $x=3$ or $x=-3\pm\sqrt{3}$---A1, 0	**2 marks**	
3.	$\frac{x+7}{(x^2-9)(x-3)}$ $=\frac{x+7}{(x+3)(x-3)(x-3)}$ $\frac{x+7}{(x+3)(x-3)^2}=\frac{A}{(x+3)}+\frac{B}{(x-3)}+\frac{C}{(x-3)^2}$----M1 $A(x-3)^2+B(x+3)(x-3)+C(x+3)=x+7$----M1 Sub $x=3$ $6C=10$ $C=\frac{5}{3}$----A1 Sub $x=-3$ $36A=4$ $A=\frac{1}{9}$---A1 Sub $x=0$ $7=9A-9B+3C$ $9\left(\frac{1}{9}\right)-9B+3\left(\frac{5}{3}\right)=7$ $B=-\frac{1}{9}$---A1 $\frac{x+7}{(x-3)(x-3)(x+3)}=\frac{1}{9(x+3)}-\frac{1}{9(x-3)}+\frac{5}{3(x-3)^2}$----A1	**6 marks**	

4(a)	$y = x^3 + kx^2 + kx + 8$ $\frac{dy}{dx} = 3x^2 + 2kx + k$ --- M1 Let $\frac{dy}{dx} > 0$ $3x^2 + 2kx + k > 0$ Let discriminant < 0 $(2k)^2 - 4(3)(k) < 0$ --- M1 $4k^2 - 12k < 0$ $k^2 - 3k < 0$ $k(k-3) < 0$ $0 < k < 3$ --- A1	**3 marks**	
(b)	$\frac{dy}{dx} = 3x^2 + 2kx + k$ $\frac{dy}{dx} = 0$ $3x^2 + 2kx + k = 0$ Let discriminant $= 0$ $(2k)^2 - 4(3)(k) = 0$ $4k^2 - 12k = 0$ $k^2 - 3k = 0$ } M1 $k(k-3) = 0$ $k = 0$ or $k = 3$ --- A1	**2 marks**	
5(a)	$\left(2x^2 + \frac{3}{x}\right)^9$ $T_{r+1} = \binom{9}{r}(2x^2)^{9-r}\left(\frac{3}{x}\right)^r$ $= \binom{9}{r}(2)^{9-r}(x^2)^{9-r}(3)^r x^{-r}$ $= \binom{9}{r}(2)^{9-r}(3)^r x^{18-3r}$ --- M1 Let $18 - 3r = 0$ $r = 6$ --- A1 $T_7 = \binom{9}{6}(2)^3(3)^6$ $= 489888$ --- A1	**3 marks**	

5(b)	$(2-3x)^5$ $=2^5+\binom{5}{1}(2)^4(-3x)+\binom{5}{2}(2)^3(-3x)^2+....$ $=32-240x+720x^2+...----B2,1,0$	2 marks	
(c)	$(2-3x)^5(1-3x)^n$ $=(32-240x+720x^2+...)(1-3x)^n$ $=(32-240x+720x^2+...)\left(1+\binom{n}{1}(-3x)+...\right)---M1$ Coefficient of $x=32\binom{n}{1}(-3x)-240x---M1$ $32\times n\times(-3x)-240x=-720x$ $-96nx-240x=-720x$ $-96n=-720+240$ $n=5---A1$	3 marks	
6(a)	$\frac{dy}{dx}=36(2x+1)^{-2}$ $y=\int 36(2x+1)^{-2}\,dx$ $=36\times\frac{(2x+1)^{-1}}{-1(2)}+c$ (c : constant) $=\frac{-18}{(2x+1)}+c---M1$ Sub $\left(\frac{1}{2},2\right)$ $2=\frac{-18}{(1+1)}+c---M1$ $2=-9+c$ $c=11$ $y=\frac{-18}{(2x+1)}+11---A1$	3 marks	
(b)	$\frac{dy}{dx}=36(2x+1)^{-2}$ At $x=1$ $\frac{dy}{dx}=\frac{36}{3^2}$ $=4---M1$ $\tan\theta=4$ $\theta=76.0^\circ---A1$	2 marks	

7	$y=\ln\left(\frac{x+1}{x-1}\right)$ $y=\ln(x+1)-\ln(x-1)$ $\frac{dy}{dx}=\frac{1}{x+1}-\frac{1}{x-1}$ ---M1 $2y=3x+2$ $y=\frac{3}{2}x+1$ Gradient of normal at $A=\frac{3}{2}$ $\therefore$ Gradient of tangent $=-\frac{2}{3}$ ---M1 $\frac{1}{x+1}-\frac{1}{x-1}=-\frac{2}{3}$ $\frac{x-1-(x+1)}{(x+1)(x-1)}=\frac{-2}{3}$ $\frac{-2}{(x+1)(x-1)}=\frac{-2}{3}$ ---M1 $x^2-1=3$ $x^2=4$ $x=2$ or $x=-2$ $\therefore x=2$ ---A1 At $x=2$, $y=\ln\left(\frac{2+1}{2-1}\right)$ $y=\ln 3$ ---A1 $P(2,\ln 3)$ Equation of tangent $y=-\frac{2}{3}x+c$ Sub $(2,\ln 3)$ $\ln 3=-\frac{2}{3}(2)+c$ ---M1 $c=\ln 3+\frac{4}{3}$ $y=-\frac{2}{3}x+\ln 3+\frac{4}{3}$ ---A1		

	At $x = 0$ $y = \ln 3 + \frac{4}{3}$ $Q(0, \ln 3 + \frac{4}{3})$ Area of triangle POQ $= \frac{1}{2} \times 2 \times \left(\ln 3 + \frac{4}{3} \right)$ --- M1 $= \ln 3 + \frac{4}{3}$ $= 2.43$ --- A1	**9 marks**	
8(a)	$OQ = OP = OR = OT$ (radii of a circle) --- M1 $\angle QOP = \angle ROT$ (vertically opp angles) --- M1 ΔQOP is congruent to ΔROT (SAS congruency) --- A1 Hence, $TR = PQ$ (Shown)	**3 marks**	
(b)	$\angle TUR = \angle PUT$ (Common angle) $\angle PTU = \angle TRU$ (Tangent-chord theorem) --- M1 ΔUTR is similar to ΔUPT (AA similarity) --- M1 $\frac{TP}{RT} = \frac{UT}{UR} = \frac{PU}{TU}$ $RT = QP$ (From (i)) $\frac{TP}{QP} = \frac{UT}{UR}$ --- M1 $QP \times UT = TP \times UR$ (Shown)	**3 marks**	
9(a)	$D\left(6.5 + \frac{7.5}{3} \times 2, 8 + \frac{6}{3} \times 2 \right)$ --- M1 $= D(11.5, 12)$ --- A1 $M(6.5, 0)$ --- B1 $\left(\frac{x + 8.5}{2}, \frac{y + 4}{2} \right) = (6.5, 0)$ --- M1 $x = 4.5, y = -4$ $B(4.5, -4)$ --- A1	**5 marks**	

(b)	$A(-1,2), B(4.5,-4), C(8.5,4), D(11.5,12)$ Area = $\frac{1}{2}\begin{bmatrix} -1 & 4.5 & 8.5 & 11.5 & -1 \\ 2 & -4 & 4 & 12 & 2 \end{bmatrix}$ $=\frac{1}{2}\left[4+4.5(4)+8.5(12)+2(11.5)-(2(4.5)-4(8.5)+4(11.5)+12(-1)\right]$ ----M1 $=\frac{1}{2}(147-9)$ $=\frac{138}{2}$ $= 69$ units^2 ----A1	**2 marks**	
10(a)	 B1 - $y=3\cos x$, shape, 1.5 oscillations over 3π 3 and -3 to be marked on y axis π, 2π and 3π to be marked on x axis	**B1 Shape** **B1 Shifting curve vertically up by 1 unit**	
(b)(i)	3	**1 mark**	
(b)(ii)	9	**1 mark**	

11(a)	Area of cross-sectional area of prism $=6\times\frac{1}{2}\times\left(5+\sqrt{3}\right)^2\sin 60^\circ$ ---M1 $=3\times\left(25+10\sqrt{3}+3\right)\times\frac{\sqrt{3}}{2}$ $=3\left(28+10\sqrt{3}\right)\times\frac{\sqrt{3}}{2}$ ---M1 $=\frac{3}{2}\left(28\sqrt{3}+30\right)$ $=42\sqrt{3}+45$ ---A1	**3 marks**	
(b)	Volume of the prism $=3\left(32\sqrt{3}+138\right)\text{ cm}^3$ $3\left(14\sqrt{3}+15\right)\times h=3\left(32\sqrt{3}+138\right)$ $\left(14\sqrt{3}+15\right)\times h=\left(32\sqrt{3}+138\right)$ $h=\frac{32\sqrt{3}+138}{14\sqrt{3}+15}\times\frac{14\sqrt{3}-15}{14\sqrt{3}-15}$ ---M1 $=\frac{32\sqrt{3}\left(14\sqrt{3}-15\right)+138\left(14\sqrt{3}-15\right)}{\left(14\sqrt{3}\right)^2-15^2}$ $=\frac{448(3)-480\sqrt{3}+1932\sqrt{3}-2070}{\left(14\sqrt{3}\right)^2-15^2}$ ---M1 $=\frac{1452\sqrt{3}-726}{363}$ $=4\sqrt{3}-2$ ---A1	**3 marks**	
12(a)	$v=\frac{3}{t+2}-\frac{t+2}{3}$ Let $v=0$ $\frac{3}{t+2}-\frac{t+2}{3}=0$ ---M1 $(t+2)^2=9$ $t+2=3$ or $t+2=-3$ $t=1$ or $t=-5$(rej) ---A1	**2 marks**	

(b)	$s=\int \frac{3}{t+2}-\frac{t+2}{3}\,dt$ $=3\ln(t+2)-\frac{t^2}{6}-\frac{2}{3}t+c---\text{M1}$ At $t=0, s=0$ $3\ln(0+2)+c=0$ $c=-3\ln 2$ $s=3\ln(t+2)-\frac{t^2}{6}-\frac{2}{3}t-3\ln 2--\text{A1}$	**2 marks**	
(b)	At $t=1$ $s=3\ln 3-\frac{1}{6}-\frac{2}{3}-3\ln 2$ $=0.383\text{ m}----\text{A1}$ At $t=2$ $s=3\ln 4-\frac{4}{6}-\frac{4}{3}-3\ln 2$ $=0.0794---\text{A1}$ Distance travelled $=0.383+(0.383-0.0794)$ $=0.687\text{ m}\ ---\text{A1}$	**5 marks**	
(c)	$a=\frac{dv}{dt}$ $=\frac{-3}{(t+2)^2}-\frac{1}{3}---\text{M1}$ At $t=1$ $a=-\frac{3}{9}-\frac{1}{3}$ $=-\frac{2}{3}<0---\text{A1}$ Since $a<0$, the velocity of the particle is decreasing.	**2 marks**	
13(a)	$\log_a 125-3\log_a\sqrt{b}+\log_a c=3$ $\log_a 125-\log_a b^{\frac{3}{2}}+\log_a c=3$ $\log_a\frac{125c}{b^{\frac{3}{2}}}=3---\text{M1}$ $a^3=\frac{125c}{b^{\frac{3}{2}}}---\text{M1}$ $a=\frac{5\sqrt[3]{c}}{\sqrt{b}}---\text{A1}$	**3 marks**	

(b)	$5^{n+1}-4\left(5^{n}\right)+5^{n-1}$ $=5^{n-1}\left(5^{2}\right)-4\left(5^{n-1}\right)(5)+5^{n-1}$ ---M1 $=5^{n-1}\left(5^{2}-20+1\right)$ ---M1 $=6\left(5^{n-1}\right)$ ---A1 Alternative Method $5\times5^{n}-4\left(5^{n}\right)+\frac{5^{n}}{5}$ ---M1 $=\frac{1}{5}\left(25\left(5^{n}\right)-20\left(5^{n}\right)+5^{n}\right)$ $=\frac{6}{5}\left(5^{n}\right)$ ---M1 $=6\left(5^{n-1}\right)$ ---A1 Since the expression has a factor of 6 which is divisible by 2, hence $5^{n+1}-4\left(5^{n}\right)+5^{n-1}$ is divisible by 2 for all positive integers of n	**3 marks**	
14	Refer to solution on the graph paper.	**8 marks**	