SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2017
SECONDARY FOUR
O-LEVEL PROGRAMME

ADDITIONAL MATHEMATICS **4047/01**
Paper 1

Wednesday **2 August 2017** **2 hours**

Additional Materials: Answer Paper
Cover Page

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\displaystyle\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

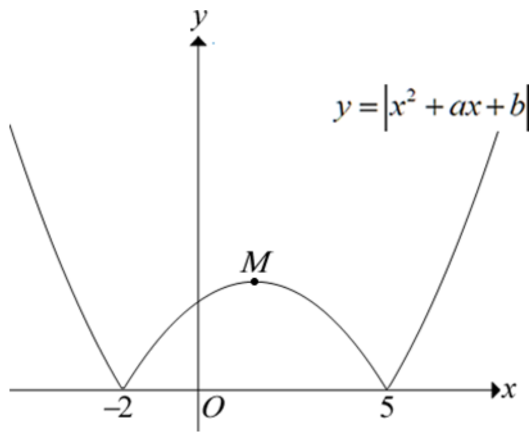
$$\Delta = \frac{1}{2} bc \sin A$$

1. Find the value of k for which the line $y + 2x = k$ and the curve $y^2 = x - 2$ do not intersect. [4]

2. **(i)** On the same axes sketch the curves of $y^3 = 27x$ and $y = 3x^3$. [3]

(ii) Find the length of the line segment which joins all the points of intersection of the two curves. [3]

3. The diagram shows part of the graph of $y = \left|x^2 + ax + b\right|$. The curve touches the x-axis at (–2, 0) and at (5, 0) and has a maximum point at $M(p, q)$.



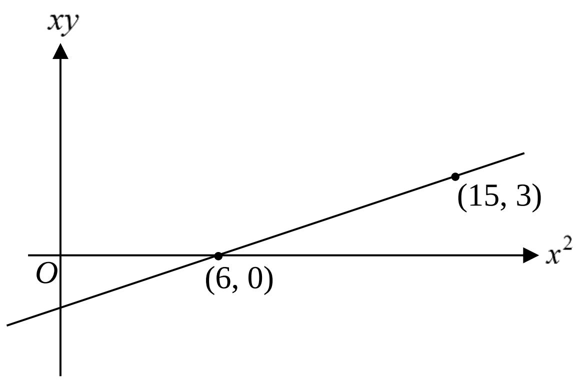
(i) Find the value of a and of b. [2]

(ii) Find the coordinates of M. [2]

(iii) Solve the equation $\left|x^2 + ax + b\right| = 2x + 4$.

Hence, solve the inequality $\left|x^2 + ax + b\right| < 2x + 4$. [3]

4. The diagram shows part of a straight line drawn to represent the equation $x + \dfrac{p}{x} = qy$.



Calculate the value of p and of q. [4]

5. **(a)** **Without using a calculator**, show that $\cos^4 15^\circ - \sin^4 15^\circ = \frac{\sqrt{3}}{2}$. [2]

(b) Given that $0 \le 2x \le 2\pi$ and $\cos 2x = -\frac{23}{49}$, calculate the exact value of $\sin x$. [2]

6. Without the use of a calculator, find the values of the integers p and q for which the solution of the equation $x\sqrt{24} + \sqrt{96} = \sqrt{108} + x\sqrt{12}$ is $\sqrt{p} + q$. [4]

7. **(a)** Find the term independent of x in the expansion of $\left(x - \frac{1}{5x^2}\right)^9$. [3]

(b) Obtain the first four terms in the expansion, in ascending order of x, of $\left(2 - \frac{x}{3}\right)^6$. [2]

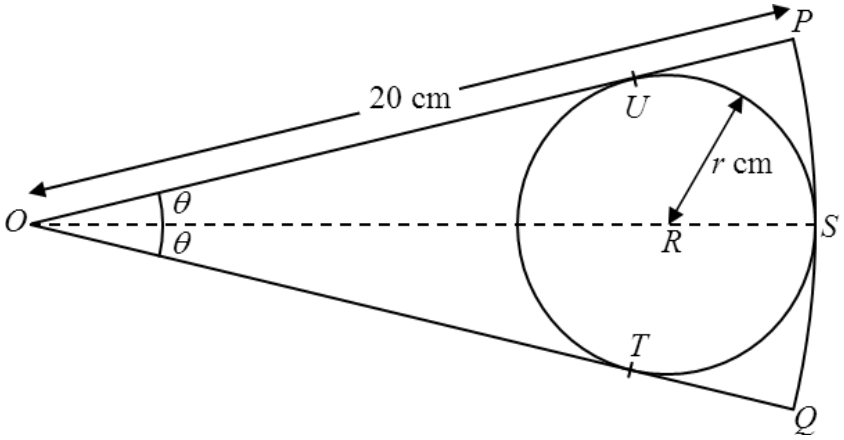
Hence, find the coefficient of x^3 in the expansion of $\left(2 - \frac{x}{3}\right)^6 (3 + x)^2$. [3]

8. **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\sqrt{2x-1}\right]$ can be expressed in the form $\frac{ax+b}{\sqrt{2x-1}}$ where a and b are integers. [4]

(ii) Integrate $\frac{3x}{\sqrt{2x-1}}$ with respect to x. [3]

(iii) Given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{5}{2}, 8\right)$ and is such that $\mathrm{f}'(x) = \frac{3x}{\sqrt{2x-1}}$, find $\mathrm{f}(x)$. [2]

9. The figure shows a sector OPQ of a circle, centre O, radius 20 cm. Angle $POQ = 2\theta$ radians where $0 < \theta < \frac{\pi}{2}$. A circle centre R, radius r cm, touches the arc PQ at the point S. The lines OP and OQ are tangents to the circle at the points U and T respectively.



(i) Write down, in terms of r, the length of OR. [1]

(ii) Hence show that $r = \dfrac{20\sin\theta}{1+\sin\theta}$. [2]

(iii) Given that r is increasing at 2 cm s^{-1}, find the rate at which θ is increasing when $\theta = \dfrac{\pi}{6}$. [4]

10. The points A and B lie on a circle with centre C. The coordinates of A and B are (1, 7) and (– 3, 9) respectively. The line $y = 8x + 4$ passes through the centre of the circle.

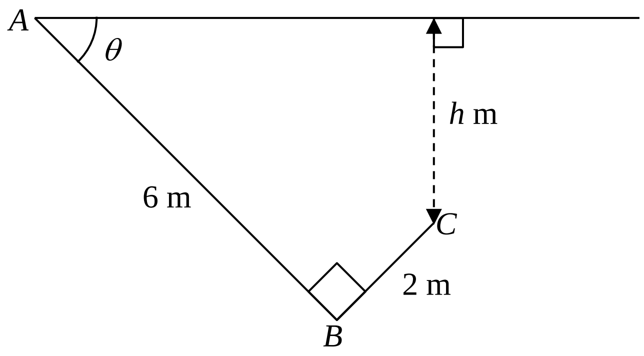
(i) Find the coordinates of C and the radius of the circle. [5]

(ii) Hence find the equation of the circle. [1]

Another circle, with centre $D(-3, 6)$, has a radius of 6 units.

(iii) Do the two circles intersect? Support your answer with working. [2]

11. The diagram shows two rods, AB and BC, of length 6 m and 2 m respectively. The rods are fixed at B such that angle $ABC = 90^{\circ}$ and hinged at the ceiling, at A so that they can rotate in a vertical plane. The rod AB makes an acute angle θ with the ceiling.



(i) Obtain an expression, in terms of θ, for h, where h m is the vertical distance from the ceiling to C. [2]

(ii) Express h in terms of $R\sin(\theta-\alpha)$ where $R>0$ and $0^{\circ}<\alpha<90^{\circ}$. [4]

(iii) Find the value of θ for which C is 1.6 m below the ceiling. [2]

12. The velocity, v ms^{-1}, of a particle P, travelling in a straight line, at time t seconds after leaving a fixed point O, is given by

$$v = 2e^{2t} - 19e^{t} + 35\text{, where } t \geq 0.$$

Find

(i) the value of t when the particle is instantaneously at rest, [3]

(ii) the acceleration of the particle when $t = \ln 7$. [2]

The displacement of the particle P, at time t seconds after leaving a fixed point O, is denoted by s metres.

(iii) Find an expression, in terms of t, for s. [3]

Hence,

(iv) find the distance travelled by the particle in the first 1.5 seconds. [3]

End of Paper 1