#### Secondary 4 Mathematics: Sets

#### 1. Definition of a Set

- A set is a collection of distinct objects such as numbers, letters, symbols, etc.
- The objects in a set are called the members or elements of the set.
- Common examples of sets
  - (a) Set of whole numbers,  $W = \{0, 1, 2, 3, ...\}$
  - (b) Set of integers,  $Z = \{..., -2, -1, 0, 1, 2, ...\}$
  - (c) Set of prime numbers,  $P = \{2, 3, 5, 7, 11, ...\}$

#### 2. Ways of Defining a Set

- (a)  $A = \{a, b, c\}$   $\rightarrow$  A is the set containing elements *a*, *b* and *c*.
- (b)  $B = \{x : x \text{ is a prime number}\} \rightarrow B$  is the set containing set of prime numbers
  - B can also be expressed as  $B = \{2, 3, 5, 7, 11, ...\}$
- (c) C = {x : x is a integer number such that  $5 \le x \le 9$ }
  - C can also be expressed as  $C = \{5, 6, 7, 8\}$

#### 3. Null or Empty Sets

• A null or empty set, denoted by  $\{ \}$  or  $\emptyset$ , is a set which contains **no elements**.

#### 4. Equal Sets

• Two sets are equal if they have the same elements.

**Example:** Set  $A = \{1, 2, 3\}$  and set  $B = \{3, 1, 2\}$ 

Then A = B (Note that the order in which the elements appear in the set is not important).

#### 5. <u>Subset and Subset Symbols</u>

Symbol	Meaning	Example
$A \subset B$	Set <i>A</i> is a proper subset of set <i>B</i> .	Set $A = \{1, 2, 3\}$ and
	i.e., every element of A is also an element	set $B = \{3, 1, 2, 4, 5\}$ , then
	of $B$ , and $A \neq B$ .	$A \subset B$
$A \subseteq B$	<i>A</i> is subset of <i>B</i>	If $A \subseteq B$ and
	i.e., every element of A is also an element	Set B = $\{3, 1, 2, 4, 5\},\$
	of <i>B</i> , and set <i>A</i> can be equal set <i>B</i> .	then Set A can be = $\{1, 2, 3\}$
		or Set A can be = $\{1, 2, 3, 4, 5\}$
$A \not\subset B$	A is not subset of B.	If Set $A = \{1, 2, 5\}$ and
		Set $B = \{1, 3, 6, 8\}$ , then
		$A \not\subset B$
$A \cap B$	The <b>intersection</b> of sets <i>A</i> and <i>B</i> .	If Set $A = \{1, 2, 3\}$ and
	It refers to the <b>common elements found in</b>	Set $B = \{1, 3, 6, 8\}$ , then
	both sets.	$A \cap B = \{1, 3\}$
$A \cup B$	The <b>union</b> of sets <i>A</i> and <i>B</i> .	If Set $A = \{1, 2, 3\}$ and
	This refers to the set of elements which	Set $B = \{1, 3, 6, 8\}$ , then
	belongs to either set A or set B or both/	$A \cup B = \{1, 2, 3, 6, 8\}$
n( <i>A</i> )	Represents the number of elements in set $A$ .	If Set $A = \{1, 2, 3\}$ , then
		n(A) = 3

### 6. Element of a Set

- If a is an element of set A, we write  $a \in A$ .
- If b is not an element of set A, we write  $b \notin A$ .

#### 7. <u>Universal Set</u>

- A universal set is a set which contains all the available elements in a particular discussion.
- It is the largest set in the discussion and is denoted by  $\xi$ .

#### 8. Complement of Set

The complement of a set A, denoted by A', relative to the universal set ξ contains elements that are not elements of A but are elements of ξ. i.e. it is the set that contains all elements in ξ except those in A.
Example: If ξ = {1, 2, 3, 4, 5}, A = {1, 2}, then A' = {3, 4, 5}

#### 9. Disjoint Sets

• Two sets are said to be disjoint if they do not have any common element. **Example**: If  $A = \{1, 2, 3, 4, 5\}, B = \{6, 7\}$ , then A and B are disjoint.

#### 10. Venn Diagram

• A Venn diagram is pictorial representation of relationships involving sets.

**Example**: If  $\xi = \{1, 2, 3, 4, 5, 6, 7, 8\}, A = \{1, 2, 3, 4\}, B = \{2, 3, 6, 7\}.$ 

Venn diagram of example:



Venn diagram of (1) proper subset, (2) disjoint sets and (3) complement of a set:

 $(3) \ B \subset A$ 

(2) A and B are disjoint

 Complement of A (Shaded region)







Venn diagram representing (4) Intersection of sets and (5) Union of sets.



### **Other Examples:**

Write down the set notation that represents the shaded region in the Venn diagram below:



### **Answer:** $P' \cup Q$

Hint: Dissect the diagram into its individual parts:



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Venn diagram of *Q* 





# Sets (Worksheet 1)

On each of the Venn diagram, shade the region(s) represented by the set notation
 (a) B' ∩ A



(b)  $B' \cap A'$ 





(e)  $(P' \cup Q)'$ 



(d)  $P \cup Q'$ 



- 2. It is given that  $\xi = \{x : x \text{ is an integer such that } 1 \le x \le 15\}, A = \{x : x \text{ integers divisible by } 2\}$  and  $B = \{x : x \text{ integers divisible by } 3\}.$ 
  - (a) List all the elements in  $A \cup B$  in set notation.

(b) List all the elements in  $A' \cup B$  in set notation.

(c) List all the elements in  $A \cap B$  in set notation.

(d) List all the elements in  $A' \cap B'$  in set notation.

- 3. It is given that ξ = {x: x is an integer such that 1 ≤ x ≤ 10}, A = {x: x is an odd number} and B = {x: x integers multiple of 3}.
  (a) List all the elements in A ⊂ B in set notation
  - (a) List all the elements in  $A \cap B$  in set notation.

(b) List all the elements in  $A' \cap B'$  in set notation.

- (c) Draw a venn diagram showing  $\xi$ , *A* and *B* and place each of the members/elements in appropriate parts of the diagram.
  - ξ

4. There are 27 children in a class. Out of these children, 19 own a smartphone, 15 own a computer and 3 neither own a smartphone no a computer. *x* number of student own both a smartphone and computer. Using a Venn diagram, find the number of children who own a smartphone but not a computer.

- 5. It is given that
  - $\xi = \{x: x \text{ is an integer such that } 10 \le x \le 23\},\$
  - $E = \{x: x \text{ is an even number}\},\$
  - $P = \{x: x \text{ is a prime number}\}, \text{ and }$
  - $M = \{x: x \text{ is multiple of 5}\},\$

The Venn diagram shows the universal set  $\xi$  and the subsets *E* and *M*. Two elements of  $\xi$  are shown in their appropriate subsets.



(a) Complete the Venn diagram shown above and draw subset *P*.

Write the remaining elements of  $\xi$  in the appropriate subsets of your Venn diagram.

(b) Hence, simplify  $(E' \cup M) \cap P'$ 

6.  $\xi = \{x : x \text{ is an integer and } 3 < x \le 15\},\$   $A = \{x : x \text{ is a multiple of 5}\}$  $B = \{x : x \text{ is a multiple of 3}\}$ 

(a) List the elements in *A*.

Answer: (a) \_\_\_\_\_

(b) Fill the members of  $\xi$ , *A* and *B* in the spaces in the Venn diagram below.



(c) List all possible subsets of *A*.

- 7.  $A = \{\text{points lying on the line } 2x + y = 8\}$ 
  - $B = \{\text{points lying on the line } 3x 4y = 12\}$
  - $C = \{\text{points lying on the line } mx 4y = c\}$
  - (a) Is  $(-1, 6) \in A$ ? Explain clearly.

(b) Find the element *p* such that  $p \in (A \cap B)$ .

Answer: (b) *p* = \_\_\_\_\_

(c) Write down a possible value of *m* and of *c* such that  $B \cap C = \emptyset$ .

Answer: (c) *m* = \_\_\_\_\_

*c* = \_\_\_\_\_

8.  $\xi = \{ \text{integers } x : 1 \le x \le 7 \}$ 

The Venn diagram shows the elements of  $\xi$  and three sets, *A*, *B* and *C*.



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Use one of the symbols below to complete each statement.

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- (a)  $\{4, 5\}$  \_\_\_\_\_\_B
- (b) 2 \_\_\_\_\_  $(A \cup B)$
- (c)  $B \cap C =$  \_\_\_\_\_

# Sets (Worksheet 2)

1. (a) On the Venn diagram, shade the region which represents  $A' \cap B$ .



(b) Given that *P* is a subset of *Q*, simplify  $(P \cup Q)'$ .

You may use the space below to help in your investigation.



Answer: (b)  $(P \cup Q)' =$  \_\_\_\_\_

- 2.  $\xi = \{ \text{integers } x : 1 \le x \le 15 \},$ 
  - $A = \{$ integers divisible by 2 $\}$
  - $B = \{$ integers divisible by 3 $\}$
  - (a) Draw a Venn diagram to illustrate this information.

(b) List the elements contained in the set  $(A \cup B)'$ .

Answer: (b)  $(A \cup B)' =$ \_\_\_\_\_

(c) Describe, as simply as possible, in words, the elements contained in the set  $A \cap B$ .

3.  $\xi = \{ \text{integers } x : 1 \le x \le 8 \},\$ 

The Venn diagram shows the elements of  $\xi$  and three sets, *A*, *B* and *C*.



Use one of the symbols below to complete each statement.

- $\emptyset \subset \not\subset \in \notin \xi$
- (a) 3 \_\_\_\_\_ *C*
- (b)  $\{1\}$  \_\_\_\_\_  $(A \cap B)$
- (c)  $(A \cup B \cup C)' =$
- 4.  $\varepsilon = \{x: x \text{ is a positive integer and } x \le 12\},\$ 
  - $A = \{x: x \text{ is an odd integer and } 3x + 5 > 11\}$
  - $B = \{x: x \text{ is a factor of } 28\}$
  - $C = \{x: x \text{ is a composite number}\}$
  - (a) List the elements of
    - (i)  $A \cup B$ ,

(ii)  $B \cap C$ .

Answer: (a)(ii)  $B \cap C =$ 

(b) Represent the elements of  $\varepsilon$  and sets *A*, *B* and *C* in the Venn Diagram below:



- 5.  $\varepsilon = \{$ students in Class 4B $\},$ 
  - *H* = {students who study History}
  - *G* = {students who study Geography}

There are 32 students in Class 4B.

13 students study History, 23 students study Geography and only 11 students study both subjects.

(a) (i) Draw a Venn Diagram to illustrate this information

(ii) Find the total number of students who study only one of these two subjects

(b) *D* = {students who play drums}

 $P = \{$ students who play the piano $\}$ 

(i) Express, in set notation, 'All students who play the drums also play the piano'.

Answer: (b)(i)

(ii) Write the set notation  $D \cap P \neq \emptyset$ .

6. Place each of the five numbers 39, 42, 45, 49 and 51 in the correct position in the Venn diagram below.



 (a) Write down the set notation for the set represented by the shaded region in the Venn diagram below.



Answer: (a)

- (b)  $\varepsilon = \{x: x \text{ is an integer, } 1 \le x \le 9\},\$ 
  - $A = \{x: x \text{ is an odd number}\}\$
  - $B = \{x: x \text{ is a factor of } 6\}$
  - (i) Draw a Venn diagram in the box below to illustrate the above information.



(ii) List the elements in the set  $(A \cup B)'$ 

Answer: (b)(ii)

8. A, B and C are three non-empty sets satisfying the following conditions.

 $A \subset B$ ,  $A \cap C \neq \emptyset$ ,  $A \not\subset C$  and  $C \not\subset B$ 

Draw a clearly labelled Venn diagram to illustrate the above information.

9. (a) On the Venn Diagram, shade the set  $A \cap B'$ .



(b) 
$$\varepsilon = \{x : x \text{ where } 0 < x \le 30\},\$$
  
 $M = \{x : x \text{ is a perfect cube}\}$   
 $N = \{x : x = 2k + 1, k \text{ integer}\}$   
Find  $(M \cap N)$ 

# Sets (Worksheet 3)

- 1.  $\varepsilon = \{x: x \text{ is a positive integer and } x < 25\},\$ 
  - $A = \{x: x \text{ is a perfect square}\}$
  - $B = \{x: x \text{ is an odd number}\}\$
  - (a) Find  $n (A \cap B)$

Answer: (a)  $n (A \cap B) =$ 

(b) On the Venn diagram below, shade the region that represents  $(A \cap B')'$ .



2. (a) Describe the shaded region in set notation.



Answer: (a)

(b) Given that

 $\varepsilon = \{x : x \text{ is a positive integer and } 0 < x < 10\},\$   $A = \{x : x \text{ is a prime number}\}\$   $B = \{x : x \text{ is a factor of } 12\}\$   $C = \{x : \frac{50}{x} < 8\}\$ 

(i) Find  $B \cap C$ .

Answer: (b)(i)  $B \cap C =$ 

(ii) List the elements of  $(A \cup B)'$ .

Answer: (b)(ii)  $(A \cup B)' =$ 

(iii) Draw a Venn diagram to illustrate the given information.



3. On the Venn diagram, shade the region which represents  $A' \cap B$ .



4. There are 30 students in a class. 14 students are in the NCC and 20 students are in the Soccer team. 4 students are neither members of the NCC nor the Soccer team.

Let

 $\varepsilon = \{ \text{Students in the class} \}$ 

*A* = {Students in NCC}

*B* = {Students in Soccer Team}

(a) Draw a Venn Diagram to illustrate the above information. Show on the Venn Diagram the number of elements in each distinct region.



(b) It is also given that

 $C = \{$ Chinese students in the class $\}$ 

 $M = \{$ Malay students in the class $\}$ 

 $I = \{$ Indian students in the class $\}$ 

(i) Describe in words the meaning of the set notation  $M \cap S \neq \{\}$ .

(ii) Describe what you can deduce from the set notation  $I \subset N$ .

(iii) Express in set notation:

{Chinese students who are neither in NCC nor the Soccer team}.

Answer: (b)(iii)

5.  $\varepsilon = \{x: x \text{ is an integer and } 1 \le x < 11\},\$ 

 $A = \{x: x \text{ is a multiple of } 2\}$ 

 $B = \{x: x \text{ is a multiple of } 3\}$ 

 $C = \{x: x \text{ is a perfect square}\}$ 

Draw a Venn diagram to illustrate this information. Hence, or otherwise, find  $n(A \cup B \cup C)'$ .



Answer:  $n(A \cup B \cup C)' =$ 

6.  $\varepsilon = \{x : x \text{ is an integer and } 1 \le x < 10\},\$   $A = \{x : x \text{ is a prime number}\}\$  $B = \{x : x \text{ is a factor of } 6\}$ 

(a) List the elements in  $A \cap B'$ .

Answer: (a)  $A \cap B' =$ 

(b) List the elements in  $(A \cup B)'$ .

Answer: (b)  $(A \cup B)' =$ \_\_\_\_\_

(c) Is  $\{2, 3\} \in A$ . Justify your answer.

(d) Draw a Venn diagram to represent the sets  $\varepsilon$ , A and B.

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7. The Venn diagram shows the sets E and F with their members.



(a) Write down, as simply as possible, the set notation which represents the set that contains irrational numbers only.

Answer: (a)

(b) Find the value of  $n(E \cap F')$ .

Answer: (b)  $n(E \cap F') =$ \_\_\_\_\_

8.  $\varepsilon = \{x: x \text{ is a positive integer smaller than 10}\},\$ 

 $A = \{x: x \text{ is a prime number}\}$ 

 $B = \{x: x \text{ is an even number}\}$ 

Write down all the numbers in the universal set in the Venn diagram below.



- 9.  $\varepsilon = \{x: x \text{ is an integer } 10 < x \le 23\},\$ 
  - $A = \{x: x \text{ is a prime number}\}$
  - $B = \{x: x \text{ is a multiple of } 3\}$
  - (a) Complete the Venn diagram below to illustrate this information.



(b) List the elements of  $(A \cup B)'$ .

Answer: (b)  $(A \cup B)' =$ \_\_\_\_\_