Paya Lebar Methodist Girls' School (Secondary)
Preliminary Examination 2017
Secondary 4 Express / 5 Normal Academic

Name: ____________________________ (   ) Class: ________

Centre Number	S				

Index Number				

ADDITIONAL MATHEMATICS 4047/02

Paper 2 24 August 2017

Additional Materials: Answer Paper (10 sheets) 2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number, name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

My Target is:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

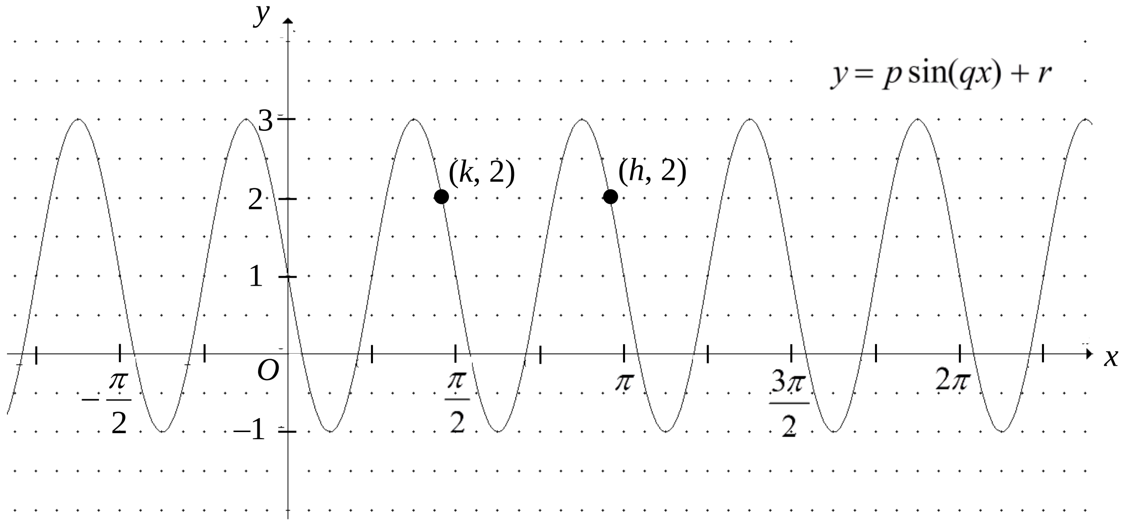
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The diagram shows part of the graph of $y = p\sin(qx) + r$.



(i) Find the value of each of the constants p, q and r. [3]

(ii) Given that $(k, 2)$ and $(h, 2)$ lie on the curve as shown, find an equation connecting k, h and π. [1]

2 50 milligrams of a volatile substance is placed in a laboratory.

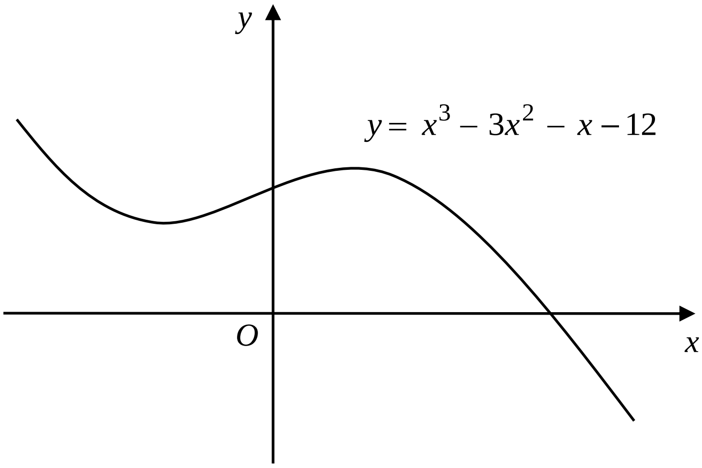
Its mass, M milligrams, t minutes later, is given by $M = 120 - ke^{\frac{1}{2}t}$, where k is a constant.

(i) Explain why $k = 70$. [1]

(ii) Determine the value of t the moment the substance has completely evaporated. [2]

(iii) Sketch the graph of M against t. [2]

3



The diagram shows the graph of $y = x^3 - 3x^2 - x - 12$.

(i) Explain why $x^3 - 3x^2 - x - 12$ cannot be written as a product of 3 linear factors. [1]

(ii) Show that $x - 4$ is a factor of $x^3 - 3x^2 - x - 12$. [1]

(iii) Factorise $x^3 - 3x^2 - x - 12$ completely and hence solve the equation $x^3 - 3x^2 - x - 12 = 5(x - 4)$. [3]

4 The area of a triangle *ABC*, right-angled at *B*, is $\left(7 + \sqrt{192}\right)$ cm^2.
AB has a length of $\left(1 + \sqrt{3}\right)$ cm.

(i) Find the exact length of *BC* in the form $\left(a - \sqrt{b}\right)$ cm, where a and b are integers. [4]

(ii) Find an expression for $(AC)^2$ in the form $\left(c + d\sqrt{3}\right)$ cm^2, where c and d are integers. [2]

5 The roots of the quadratic equation $x^2 + px + 4 = 0$ are α^2 and β^2, where $\alpha > 0$, $\beta > 0$ and $\alpha > \beta$.

(a) Find an expression, in terms of p, for $\alpha - \beta$. [3]

(b) In the case where $p = -5$, find a quadratic equation with roots $\alpha - 1$ and $1 - \beta$. [5]

6 **(a)** Given that lg 2 = m, express in terms of m,

(i) $\log_5 32$, [3]

(ii) 2^{10}. [2]

(b) Express $2\log_3 x - \log_3 2 = \log_3(x-4)$ as a quadratic equation in x and explain why the quadratic equation has no real solutions. [4]

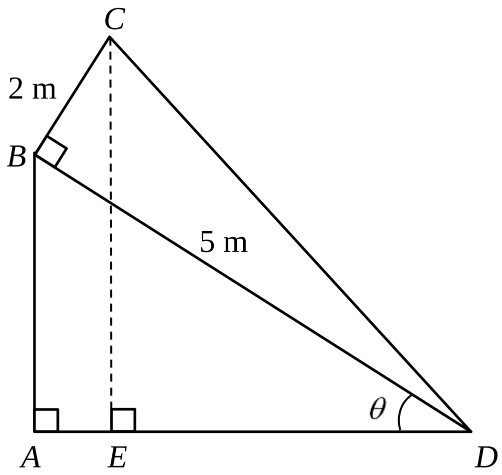
7 A curve has the equation $y = 3(4-x)^3 + 5$.

(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Determine the nature of the stationary point. [3]

(iii) Hence sketch the graph of $y = 3(4-x)^3 + 5$, showing all the intercepts. [3]

8



The diagram shows two triangles DAB and DBC.
It is given that BD = 5 m and BC = 2 m.
Angle DAB = angle DBC = angle DEC = 90°. Angle $ADB = \theta$ and θ can vary.

(i) Show that DE can be expressed in the form $(a\cos\theta - b\sin\theta)$ metres, where a and b are constants to be found. [2]

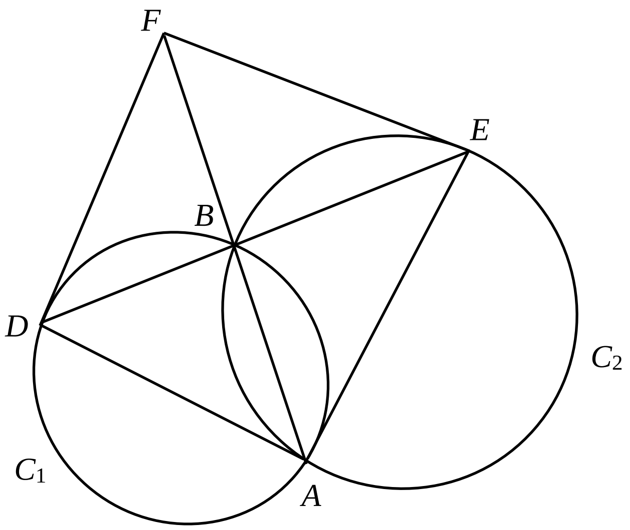
(ii) Express DE in the form $R\cos(\theta + \alpha)$ metres, where $R > 0$ and α is an acute angle. [2]

(iii) Express CE in the form $R\sin(\theta + \alpha)$ metres. [2]

(iv) Hence show that the area of triangle CDE can be expressed in the form $p\sin(2\theta + 2\alpha)$ m^2, where p is a constant to be found. [3]

[Turn over

9



The diagram shows two circles, C_1 and C_2, intersecting at A and at B.
F is a point on AB produced such that FD and FE are tangents to C_1 at D and C_2 at E respectively.
DBE is a straight line.

(i) Prove that triangles FDB and FAD are similar. [2]

(ii) Name another pair of similar triangles and hence show that $FD = FE$. [4]

(iii) If the line ABF is perpendicular to DE, explain why a circle with AF as a diameter passes through D and E. [4]

10 A circle C_1, whose centre lies on the line $2x + y = 0$, passes through the points $P(3, -1)$ and $Q(-4, -2)$.

(i) Find the equation of the perpendicular bisector of PQ. [3]

(ii) Hence find the equation of C_1. [4]

(iii) Show that the point $R(2, 5)$ lies inside C_1. [2]

(iv) Find the equation of another circle C_2, which is a reflection of C_1 in the y-axis. [2]

11 A scooter travelling up and down a straight horizontal road passes a fixed point O with a speed of p ms^{-1}. The velocity, v ms^{-1}, of the scooter, t seconds after passing O, is given by

$$v = 7 - 8\sin\frac{1}{3}t.$$

(i) State the value of p. [1]

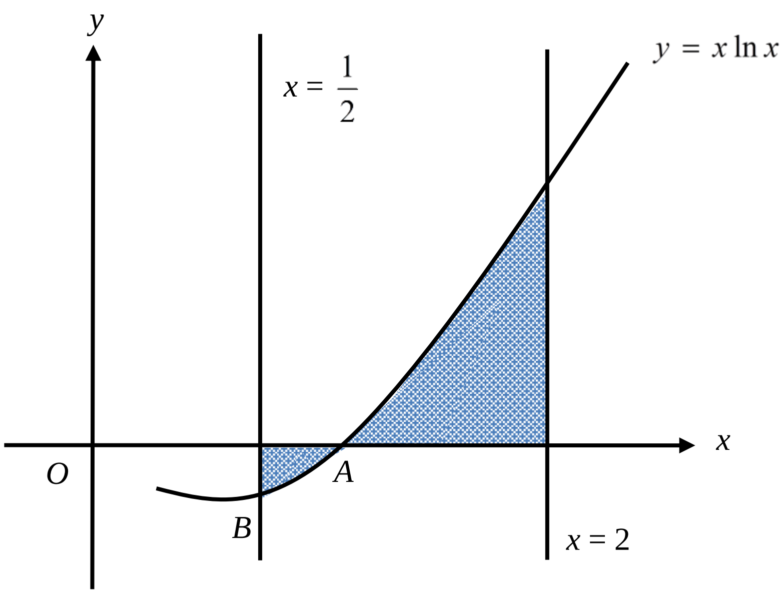
(ii) Find the values of t for the first two instances when the scooter changes its direction of motion. [4]

(iii) Find the distance travelled by the scooter in the third second. [7]

12 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 \ln x\right)$. [2]

(b) Hence find $\int x \ln x \, \mathrm{d}x$. [3]

(c)



The diagram shows the line $x = 2$ and part of the curve $y = x\ln x$.

The curve meets the x-axis at the point A and the line $x = \frac{1}{2}$ at the point B.

(i) Find the x-coordinate of A. [2]

(ii) Find the total area of the shaded region. [5]

End of Paper