Class | Index Number

Name : ______________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2024
Secondary 4

Tuesday **ADDITIONAL MATHEMATICS** **4049/02**

13 August 2024 **PAPER 2** **2 hours 15 mins**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Solve the equation $\sqrt{2x+1}-\sqrt{x}=1$. [3]

$\sqrt{2x+1}-\sqrt{x}=1$

$\sqrt{2x+1}=1+\sqrt{x}$

$2x+1=1+2\sqrt{x}+x$ M1

$x=2\sqrt{x}$

$x^2=4x$ M1

$x(x-4)=0$

$x=0$ or $x=4$ A1

(b) Express $\frac{\left(2\sqrt{3}-5\right)^2}{\sqrt{3}-2}$ in the form $p\sqrt{3}+q$ where p and q are integers. [4]

$\frac{\left(2\sqrt{3}-5\right)^2}{\sqrt{3}-2}$

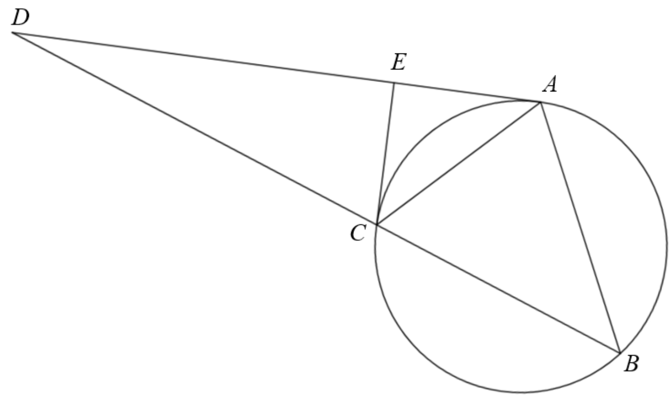
$=\frac{12-20\sqrt{3}+25}{\sqrt{3}-2}\times\frac{\sqrt{3}+2}{\sqrt{3}+2}$ M1

$=\frac{\left(37-20\sqrt{3}\right)\left(\sqrt{3}+2\right)}{3-4}$ M1

$=-\left(37\sqrt{3}+74-60-40\sqrt{3}\right)$ M1

$=3\sqrt{3}-14$ A1

2 The diagram shows a circle passing through the vertices of a triangle ABC. The tangent to the circle at A meets BC extended at point D. The tangent at C meets AD at E.



(i) Prove that angle CED is twice of angle ABC. [3]

Let angle $EAC = x$ and angle $ECD = y$
angle $ECA = x$ (base angles, isosceles triangle) M1
angle $ABC = x$ (alternate segment theorem) M1
angle $CED = 2x$ (exterior angle = sum of interior opposite angles) A1
Therefore, angle CED is twice of angle ABC.

(ii) By proving a pair of similar triangles, show that $BD \times CD = AD^2$. [3]

angle ABD = angle CAD (alternate segment theorem) M1
angle ADB = angle CDA (common)
Therefore, triangle ABD is similar to triangle CAD. M1

$$\frac{AD}{CD} = \frac{BD}{AD}$$

$BD \times CD = AD^2$ A1

3 The mass, m grams, of a radioactive substance detected in a piece of stone is given by the formula $m = \alpha e^{-kt}$, where $\alpha \neq 0$, k is a constant and t is the time interval in months.

(i) The mass of the substance is reduced to half its original value four months after it was first being detected, find the value of k. [2]

$m = \alpha e^{-kt}$

$\frac{1}{2}\alpha = \alpha e^{-k(4)}$

$\ln\left(\frac{1}{2}\right) = -4k$ M1

$\frac{1}{2}\alpha = \alpha e^{-k(4)}$

$k = 0.1732867...$

$k = 0.173$ A1

(ii) Find the initial mass of the substance given its mass after 1 month is 0.25 g. [2]

$0.25 = \alpha e^{-0.1732867(1)}$

$\frac{0.25}{e^{-0.1732867}} = \alpha$ M1

$\alpha = 0.29730177...$

$\alpha = 0.297$ A1

(iii) Calculate the time taken for the mass to reduce to 0.01 g. [2]

$0.01 = 0.29730177e^{-0.1732867t}$

$\frac{0.01}{0.29730177} = e^{-0.1732867t}$ M1

$\ln\left(\frac{0.01}{0.29730177}\right) = -0.1732867t$

$t = 19.575...$

$t = 19.6$ A1

4 **(a)** Solve the equation $3\cos 2x = \sin x + 2$, for $0° \le x \le 360°$. [4]

$3(1 - 2\sin^2 x) = \sin x + 2$ M1

$6\sin^2 x + \sin x - 1 = 0$

$(3\sin x - 1)(2\sin x + 1) = 0$ M1

$\sin x = \frac{1}{3}$ $\sin x = -\frac{1}{2}$ A2

$x = 19.5°, 160.5°$ $x = 210°, 330°$

(b) Find all the angles between 0 and 5 which satisfy the equation $\sin 2x + 3\cos^2 x = 0$. [4]

$\sin 2x + 3\cos^2 x = 0$

$2\sin x \cos x + 3\cos^2 x = 0$ M1

$\cos x(2\sin x + 3\cos x) = 0$ M1

$\cos x = 0$ $2\sin x = -3\cos x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\tan x = -1.5$ A2

$x = 2.16$

5 **(a)** **(i)** Given that m is a constant, expand $(3+mx)^4$, in ascending powers of x, simplifying each term in your expansion. [2]

$(3+mx)^4$

$= 3^4 + \binom{4}{1}(3^3)mx + \binom{4}{2}(3^2)(mx)^2 + \binom{4}{3}(3)(mx)^3 + (mx)^4$ M1

$= 81 + 108mx + 54m^2x^2 + 12m^3x^3 + m^4x^4$ A1

(ii) Given also that the coefficient of x is equal to the coefficient of x^2, find the value of m. [1]

$108m = 54m^2$ A1

$m = 2$

(b) **(i)** By considering the general term in the binomial expansion of $\left(3x - \frac{1}{2x}\right)^9$, explain why there are no even powers of x in this expansion. [3]

$\binom{9}{r}(3x)^{9-r}\left(-\frac{1}{2x}\right)^r$ M1

Powers of $x = 9 - r - r = 9 - 2r$ M1

$2r$ is always even for all integer values of r

The difference between an odd and even number is always odd. A1

Thus, there are no even powers of x in the expansion

(ii) Using the value of m found in **part (a)(ii)**, find the term independent of x in the expansion of $(3+mx)^4\left(3x-\frac{1}{2x}\right)^9$. [4]

$(81+108mx+54m^2x^2+12m^3x^3+m^4x^4)\left(3x-\frac{1}{2x}\right)^9$ M1

$=(81+216x+216x^2+96x^3+16x^4)(.....\ x^{-1}\ .....\ x^{-3}\ .....)$

$x^{9-2r}=x^{-1}$	$x^{9-2r}=x^{-3}$
$9-2r=-1$	$9-2r=-3$
$2r=10$	$2r=12$
$r=5$	$r=6$

M1

cofficient $=216\times\binom{9}{5}(3)^4(-\frac{1}{2})^5+96\times\binom{9}{6}(3)^3(-\frac{1}{2})^6$ M1

$=-\frac{130977}{2}$ A1

6 A circle C has equation $x^2 + y^2 + 6x - 4y = 12$.

(i) Find the centre and the radius of C. [3]

$x^2 + y^2 + 6x - 4y = 12$

$(x+3)^2 + (y-2)^2 = 25$ M1

centre is $(-3, 2)$

radius = 5 units A2

The points P (–8, 2) and Q (1, –1) lie on the circumference of C.

(ii) Determine whether PQ is a diameter of C. [2]

$\sqrt{(-8-1)^2 + (2+1)^2} = 9.4868 \neq 10$ M1 A1

PQ is not a diameter of C

(iii) Find the equation of tangent to the circle at the point R (0, 6). [3]

$m=\frac{6-2}{0+3}=\frac{4}{3}$ M1

$m\perp=-\frac{3}{4}$ M1

eqn of tangent,

$y=-\frac{3}{4}x+6$ A1

(iv) The circle C is reflected in the tangent line at R obtained in **part (iii)**. Write down the equation of the new circle. [2]

new centre = (3, 10) M1

new equation : $(x-3)^2+(y-10)^2=25$ A1

7 **(a)** Express $\dfrac{2x^3+x^2+x}{(x-1)(x^2+1)}$ in partial fractions. [5]

$(x-1)(x^2+1)=x^3-x^2+x-1$ M1

$$\begin{array}{r} 2 \\ x^3-x^2+x-1 \overline{\big)\, 2x^3+x^2+x+0} \\ -)\ \underline{2x^3-2x^2+2x-2} \\ 3x^2-x+2 \end{array}$$

$\dfrac{2x^3+x^2+x}{(x-1)(x^2+1)} = 2+\dfrac{3x^2-x+2}{(x-1)(x^2+1)}$ M1

$\dfrac{3x^2-x+2}{(x-1)(x^2+1)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$

$3x^2-x+2=A(x^2+1)+(x-1)(Bx+C)$

at $x=1$,	at $x=0$,	$3=A+B$	M2
$4=2A$	$2=2-C$	$1=B$	
$2=A$	$C=0$		

$\dfrac{2x^3+x^2+x}{(x-1)(x^2+1)} = 2+\dfrac{2}{x-1}+\dfrac{x}{x^2+1}$ A1

(b) Differentiate $\ln(x^2+1)$ with respect to x. [1]

$\dfrac{\mathrm{d}\left[\ln(x^2+1)\right]}{\mathrm{d}x}=\dfrac{2x}{x^2+1}$

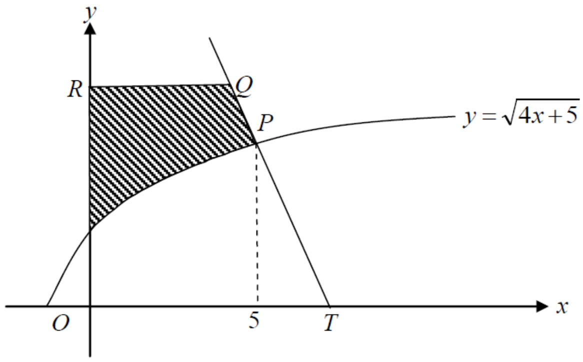
(c) Hence, find $\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x$. [4]

$$\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x = \int 2+\frac{2}{x-1}+\frac{x}{x^2+1}\,\mathrm{d}x$$ M1

$$\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x = \left[2x+2\ln(x-1)\right]+c_1+\frac{1}{2}\int \frac{2x}{x^2+1}\,\mathrm{d}x$$ M1

$$\int \frac{2x^3+x^2+x}{(x-1)(x^2+1)}\,\mathrm{d}x = 2x+2\ln(x-1)+\frac{1}{2}\ln(x^2+1)+c_2$$ M1 A1

8 The diagram shows part of the curve $y=\sqrt{4x+5}$. The normal of the curve at P meets the x-axis at T. The x-coordinate of P is 5. Given that PT is twice of PQ and RQ is parallel to the x-axis,



(i) find the equation of the normal at P, [3]

$y=\sqrt{4x+5}$

$\frac{dy}{dx}=\frac{1}{2}(4x+5)^{-\frac{1}{2}}(4)=\frac{2}{\sqrt{4x+5}}$ M1

at $x=5$, $\frac{dy}{dx}=\frac{2}{5}$ M1

$y=5$

eqn of normal,

$y-5=-\frac{5}{2}(x-5)$

$5x+2y=35$ A1

(ii) explain why the curve has no stationary points, [2]

$\frac{dy}{dx}=\frac{2}{\sqrt{4x+5}}$

since $\sqrt{4x+5}>0$,

$2>0$, M1

$\frac{dy}{dx}>0$ A1

so, the curve has no stationary points.

(iii) find the coordinates of Q, [2]

at $y = 0, x = 7$
$T\,(7, 0)$ M1

Since $PT = 2\,PQ$,
$x_Q = 4$
$Q(4,\ 7.5)$ A1

(iv) find the area of the shaded region. [3]

$$\int_0^5 \sqrt{4x+5}\ \mathrm{d}x$$

$$= \left[\frac{2(4x+5)^{\frac{3}{2}}}{3(4)}\right]_0^5 = 18.9699...$$ M1

Area of shaded region $= 5(5) + \frac{1}{2}(4+5)(2.5) - 18.9699... = 17.3$ M1 A1

9 **(a)** It is given that $y = a\sin bx + c$, for $0° \le x \le 360°$, where a, b, c are integers and $a > 0$.
The period is $120°$ and the maximum and minimum value of y is 3 and – 5.

(i) State the value of a, of b and of c. [3]

$3 = a(1) + c$ $\quad\quad$ $-5 = a(-1) + c$

$3 = a + c$ $\quad\quad$ $-5 = -a + c$

$-2 = 2c$

$-1 = c$ $\quad\quad$ A1

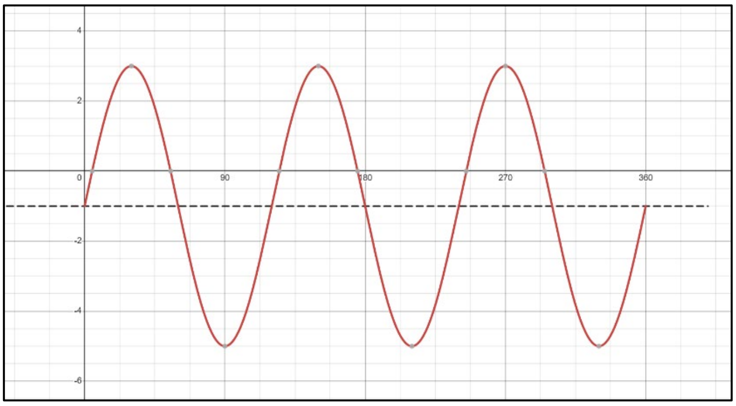
$a = 4$ $\quad\quad$ A1

$$\text{period} = \frac{360°}{b}$$

$$120° = \frac{360°}{b}$$

$b = 3$ $\quad\quad$ A1

(ii) Sketch the graph of $y = a\sin bx + c$. [2]



(b) Prove that $\frac{1+\cos x}{1-\cos x} = (\operatorname{cosec} x + \cot x)^2$. [3]

$(\operatorname{cosec} x + \cot x)^2$

$= = \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right)^2$

$= \frac{(1+\cos x)^2}{\sin^2 x}$ M1

$= \frac{(1+\cos x)^2}{1-\cos^2 x}$

$= \frac{(1+\cos x)^2}{(1-\cos x)(1+\cos x)}$ M1

$= \frac{1+\cos x}{1-\cos x}$ A1

(c) Without the use of a calculator, find x such that $\cos 2x = \sin 230°$ where $0° < x < 180°$. [3]

$\cos 2x = \sin 230° = -\sin 50°$ $0° < 2x < 360°$

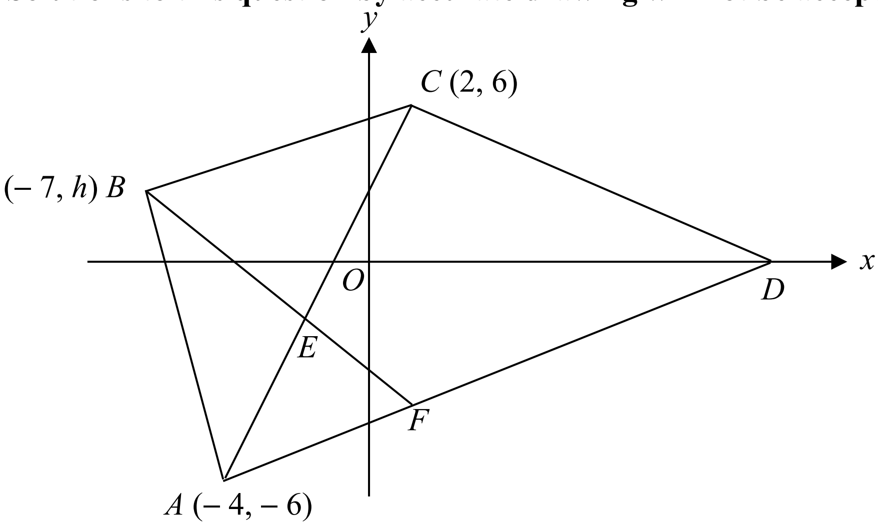
$\cos 2x = -\cos 40°$ M1

basic angle = $40°$

$2x = 140°,\ 220°$ M1

$x = 70°,\ 110°$ A1

10 Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a quadrilateral $ABCD$ with vertices $A(-4,-6)$, $B(-7,\ h)$ and $C(2, 6)$ and D, which lies on the x-axis. AD is parallel to BC. The point E lies on AC such that $AE : AC = 1 : 3$. The line BE produced meets the line AD at F.

(i) Given that $AB = BC$, show that $h = 3$. [2]

$AB = BC$

$(h+6)^2 + (-7+4)^2 = (6-h)^2 + (2+7)^2$ M1

$h^2 + 12h + 36 + 9 = 36 - 12h + h^2 + 81$

$24h = 72$

$h = 3\,(\text{shown})$ A1

(ii) Show that $\angle ABC = 90°$. [2]

$m_{AB} = \dfrac{3+6}{-7+4} = -3$ M1

$m_{BC} = \dfrac{6-3}{2+7} = \dfrac{1}{3}$

Since gradient of AB × gradient of $BC = -1$, A1

$\angle ABC = 90°$.

(iii) Find the coordinates of D. [2]

$m_{AD} = \frac{1}{3}$ and D lies on the x-axis

$\frac{0+6}{x+4} = \frac{1}{3}$ M1

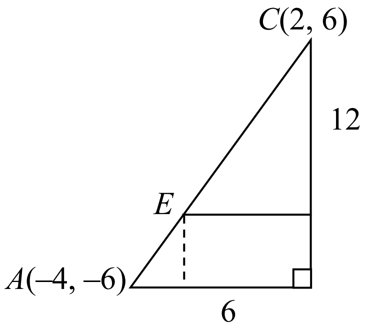
$x = 14$

$D(14, 0)$ A1

(iv) Find the coordinates of E. [2]

$E(-4+2, -6+4)$ M1

$E(-2, -2)$ A1



(v) Find the coordinates of G such that $ABCG$ is a square. [2]

Let the coordinates of G be (x, y).

Midpoint of $BG = \left(\frac{-7+x}{2}, \frac{3+y}{2}\right) = (-1, 0)$ M1

$\frac{-7+x}{2} = -1$ and $\frac{3+y}{2} = 0$

$x = 5$ $y = -3$

Therefore, $G(5, -3)$ A1

(vi) Find the area of triangle ABC. [2]

$$A = \frac{1}{2}\begin{vmatrix} -7 & -4 & 2 & -7 \\ 3 & -6 & 6 & 3 \end{vmatrix}$$

$A = \frac{1}{2} \times [(42 - 24 + 6) - (-12 - 12 - 42)]$ M1

$A = 45$ A1

END OF PAPER

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