H

NANYANG JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME	Solution
CLASS	
TUTOR'S NAME	
CENTRE NUMBER	S
INDEX NUMBER	

PHYSICS 9749/03

Paper 3 Longer Structured Questions

20 September 2023

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, Centre number and index number in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams, graphs.
Do not use staples, paper clips, glue or correction fluid.

The use of an approved scientific calculator is expected, where appropriate.

Section A
Answer **all** questions.

Section B
Answer **one** question only.

You are advised to spend one and a half hours on Section A and half an hour on Section B.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use	
Section A	
1	**/ 8**
2	**/ 8**
3	**/ 8**
4	**/ 8**
5	**/ 8**
6	**/ 9**
7	**/ 11**
Section B	
8	**/ 20**
9	**/ 20**
Total	**/ 80**

This document consists of **24** printed pages.

Data

speed of light in free space	$c = 3.00 \times 10^{8}$ m s^{-1}
permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
	$(1 / (36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	$e = 1.60 \times 10^{-19}$ C
the Planck constant	$h = 6.63 \times 10^{-34}$ J s
unified atomic mass constant	$u = 1.66 \times 10^{-27}$ kg
rest mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
rest mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
molar gas constant	$R = 8.31$ J K^{-1} mol^{-1}
the Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
the Boltzmann constant	$k = 1.38 \times 10^{-23}$ J K^{-1}
gravitational constant	$G = 6.67 \times 10^{-11}$ N m^{2} kg^{-2}
acceleration of free fall	$g = 9.81$ m s^{-2}

Formulae

uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$
work done on / by a gas	$W = p\Delta V$
hydrostatic pressure	$p = \rho gh$
gravitational potential	$\phi = -Gm / r$
temperature	$T / \text{K} = T / {}^\circ\text{C} + 273.15$
pressure of an ideal gas	$p = \frac{1}{3}\frac{Nm}{V} < c^2 >$
mean translational kinetic energy of an ideal molecule	$E = \frac{3}{2}kT$
displacement of particle in s.h.m.	$x = x_0 \sin \omega t$
velocity of particle in s.h.m.	$v = v_0 \cos \omega t$
	$= \pm\omega\sqrt{x_0^2 - x^2}$
electric current	$I = Anvq$
resistors in series	$R = R_1 + R_2 + \ ...$
resistors in parallel	$1/R = 1/R_1 + 1/R_2 + \ ...$
electric potential	$V = \frac{Q}{4\pi\varepsilon_0 r}$
alternating current/voltage	$x = x_0 \sin \omega t$
magnetic flux density due to a long straight wire	$B = \frac{\mu_0 I}{2\pi d}$
magnetic flux density due to a flat circular coil	$B = \frac{\mu_0 NI}{2r}$
magnetic flux density due to a long solenoid	$B = \mu_0 nI$
radioactive decay	$x = x_0 \exp(-\lambda t)$
decay constant	$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$

Section A

Answer **all** the questions in this section in the spaces provided.

1 **(a)** A student sets up the circuit shown in Fig. 1.1 to determine the resistivity of the metal of a resistance wire. He reads the current I from the ammeter and the potential difference V from the voltmeter.

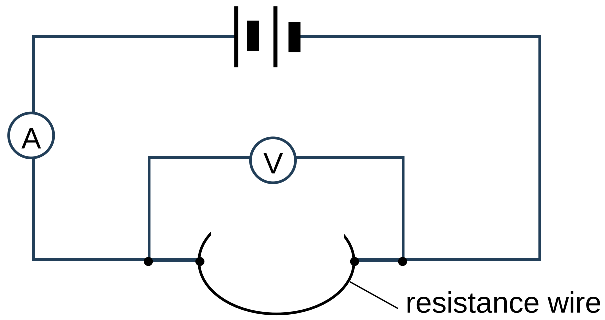


Fig. 1.1

(i) Determine the base units for resistance.

$$P = I^2R$$

$$[R] = \left[\frac{P}{I^2}\right] = \left[\frac{Fd}{I^2t}\right]$$

$$= \frac{\text{kg m s}^{-2}\text{ m}}{\text{sA}^2}$$

$$= \text{kg m}^2\text{ s}^{-3}\text{ A}^{-2}$$

base units for resistance: .... [2]

(ii) The following readings were obtained for the experiment:

Reading of voltmeter = 1.30 ± 0.01 V
Reading of ammeter = 0.76 ± 0.01 A
Length of wire = 75.4 ± 0.2 cm
Diameter of wire = 0.54 ± 0.02 mm

Calculate, with its associated uncertainty, the value of the resistivity of the metal of the resistance wire, expressing your results to an appropriate number of significant figures.

$$R = \frac{\rho L}{A}$$

$$\frac{V}{I} = \frac{\rho L}{\pi d^2/4}$$

$$\rho = \frac{V}{I}\frac{\pi d^2}{4L} = \frac{\pi(1.30)(0.54\times10^{-3})^2}{4(0.76)(75.4\times10^{-2})}$$

$$= 5.19\times10^{-7}\ \text{[C1]}\ \Omega\text{m}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta V}{V} + \frac{\Delta I}{I} + 2\frac{\Delta d}{d} + \frac{\Delta L}{L}$$

$$= \frac{0.01}{1.30} + \frac{0.01}{0.76} + 2\frac{0.02}{0.54} + \frac{0.2}{75.4}\ \text{[M1]}$$

$$\Delta\rho = 0.098\times(5.19\times10^{-7})$$

$$= 0.5\times10^{-7}\ \text{[A1]}$$

$$\rho = 5.2\ \text{[A1]} \pm 0.5\times10^{-7}\ \Omega\text{m}$$

ρ = ... ± .... Ω m [4]

(b) A second student repeated the experiment in **(a)** with the same length of the wire. In this new experiment, the supply voltage was varied and several pairs of corresponding readings on the voltmeter and ammeter were tabulated. A graph showing the variation of the current in the wire with potential difference across the wire was then plotted.

Discuss how this procedure reduces the random error and systematic error that could have occurred.

The graph of best fit will reduce the scatter caused by random errors in the experiment. [B1]

If the line of best fit does not pass through the origin, there is a systematic error present in the experiment. [B1] [2]

[Total: 8]

2 An object of mass 1.5 kg is released from a stationary hot air balloon. Fig. 2.1 shows how the vertical displacement of the object varies with time.

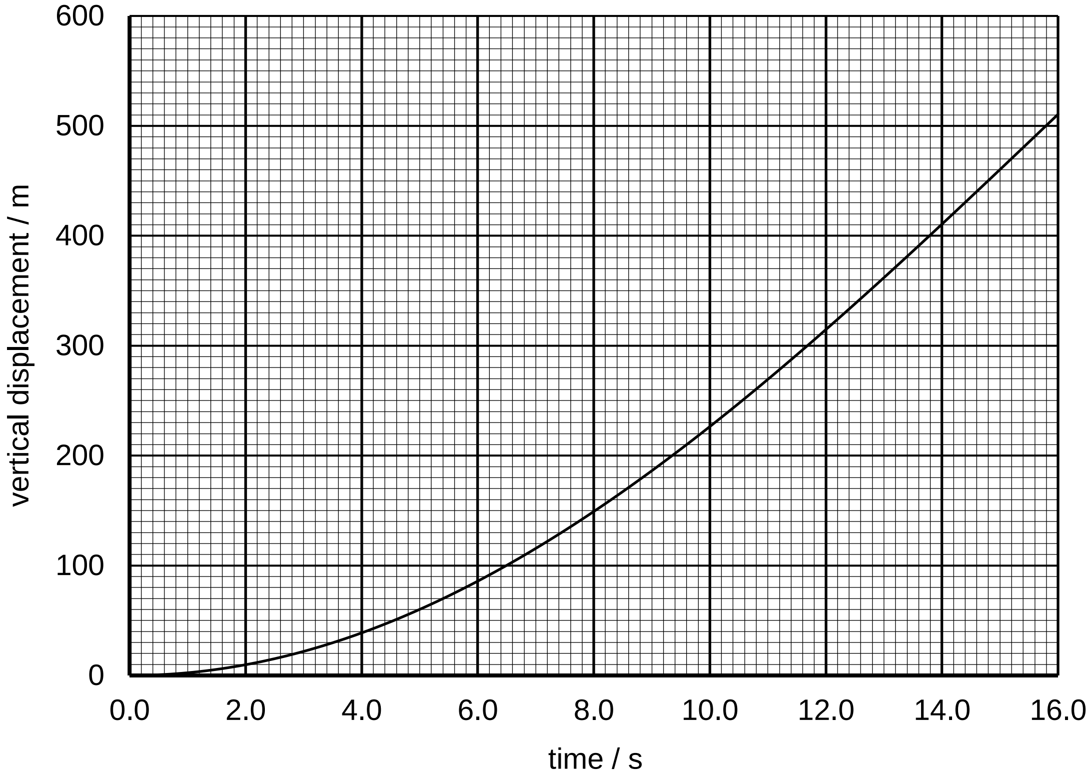


Fig. 2.1

(a) Calculate the change in gravitational potential energy ΔE_p of the object that occurred during the 16 s after it was released.

ΔE_p = mgh
= 1.5 (9.81)(-510)
= (-) 7500 J [A1]

ΔE_p = ……………….. J [1]

(b) Using Fig. 2.1, determine the speed of the object at t = 16 s.

speed = gradient of tangent to the graph at t = 16 s [M1]
= 50 m s^{-1} (acceptable range: 49 m s^{-1} to 52 m s^{-1}) [A1]

speed = ……………….. m s^{-1} [2]

(c) Calculate the change in kinetic energy ΔE_k of the object during the same period.

$$\Delta E_K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$
$$= \tfrac{1}{2}(1.5)(50)^2 - 0$$
$$= 1880 \text{ J}$$ [A1]

ΔE_k = ………………….. J [1]

(d) Explain why ΔE_p and ΔE_k are not equal to one another.

Work is done (by object) against/to overcome air resistance. [B1]
…………………………………………………………………………………………………

……………………………………………………………………………………………… [1]

(e) The object strikes the ground 16 s after it was released and penetrates 0.85 m into the ground. Determine the average resistive force acting on the object as it penetrates the ground.

Work done against resistive force = Loss of E_K (and E_P)

$F(0.85) = 1880$ [M1] (+ $1.5 \times 9.81 \times 0.85$ [B1])

$F = 2210$ N (or 2230 N) [A1]

$$f - mg = ma$$

$$\left(-1.5(9.81)^{[B1]}\right) + f = 1.5\left(\frac{50.0^2}{2 \times 0.85}\right)^{[M1]}$$

$$= 2210 \text{ N (or 2230 N)}$$ [A1]

average resistive force = ………………….. N [3]

[Total: 8]

3 Fig. 3.1 shows a 0.45 kg mass held on a horizontal air track by two identical springs that are initially unstretched. The mass is pulled 5.0 cm to the left and released. The mass oscillates horizontally on a cushion of air.

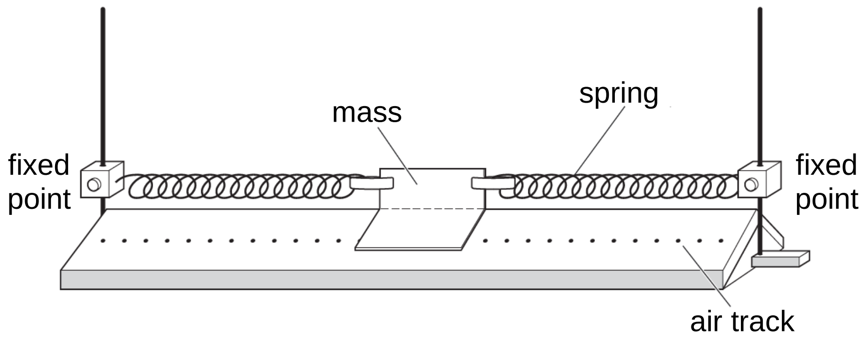


Fig. 3.1

(a) Explain why the mass oscillates with simple harmonic motion when displaced horizontally.

Resultant force on mass is proportional to displacement from equilibrium position / fixed point (as springs obey Hooke's law) so acceleration of mass is proportional to displacement from equilibrium position / fixed point [B1] and acceleration is directed towards equilibrium position/ opposite in direction to displacement from equilibrium position. [B1] [2]

The variation of kinetic energy with displacement x of the mass is shown in Fig. 3.2.

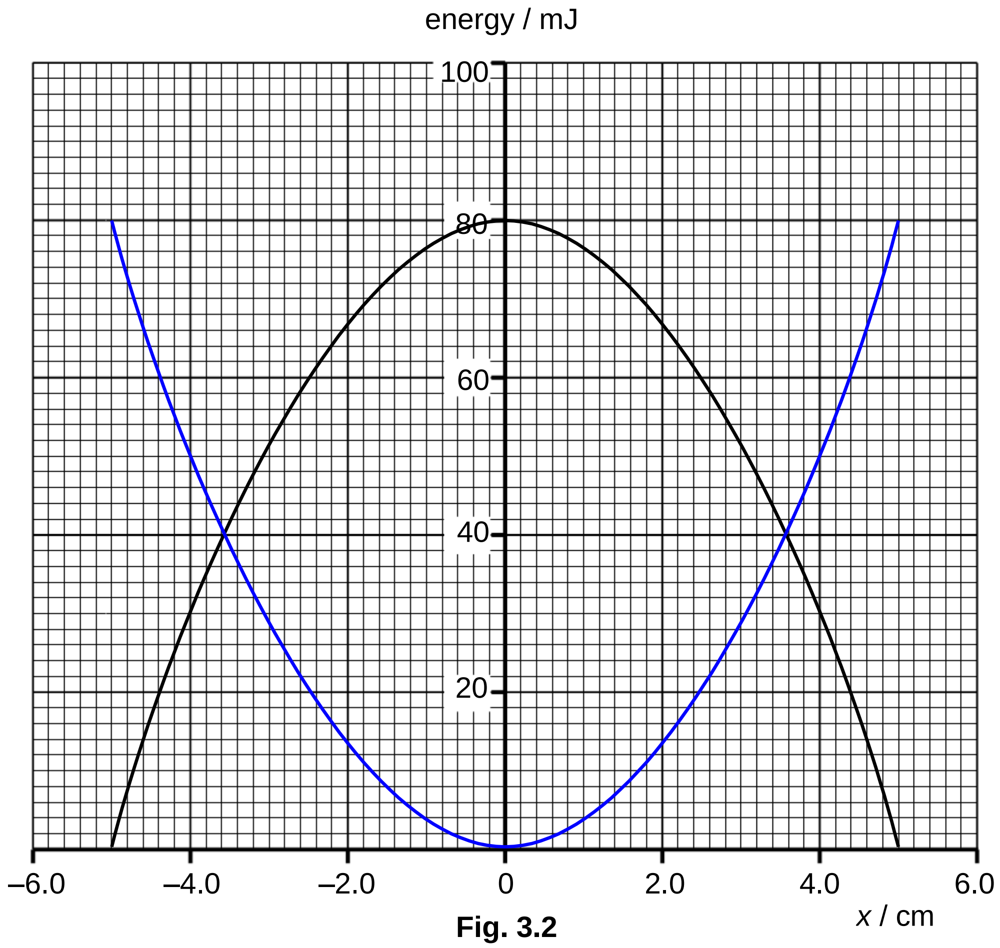


Fig. 3.2

(b) **(i)** Use Fig. 3.2 to determine the frequency of the oscillations.

$$E_{k,\max} = \frac{1}{2} m\omega^2 x_0^{\,2}$$

$$80\times10^{-3} = \frac{1}{2}(0.45)(0.050)^2(2\pi f)^2$$ [M1 - ignore POT error in substitution]

$$f = 1.90 \text{ Hz}$$ [A1]

frequency = ... Hz [2]

(ii) Draw, on Fig. 3.2, a graph to show the variation of potential energy stored in the springs with displacement *x* of the mass. [1]

correct shape, end points (5.0 cm) and intersection points (40 mJ) [1]

(c) The blower of the air track is switched off so that the mass is in direct contact with the track. One end of the spring is attached to a driver (oscillator) shown in Fig. 3.3. The amplitude of oscillation of the driver is kept constant.

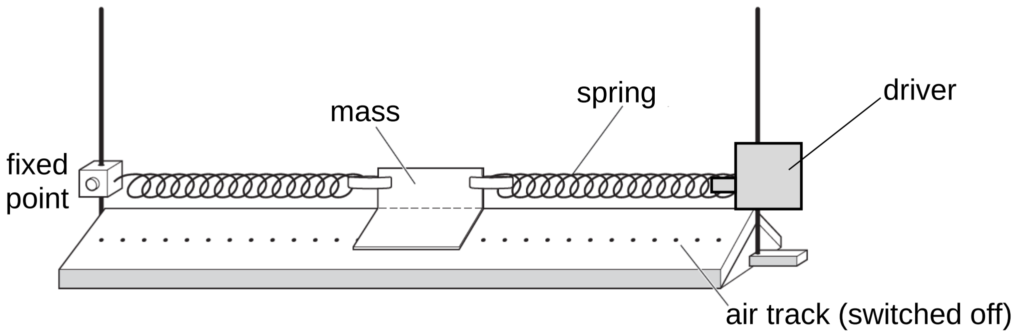


Fig. 3.3

The frequency of oscillations is gradually increased from zero. The variation of the amplitude of oscillation of the mass with driving frequency is shown in Fig. 3.4.

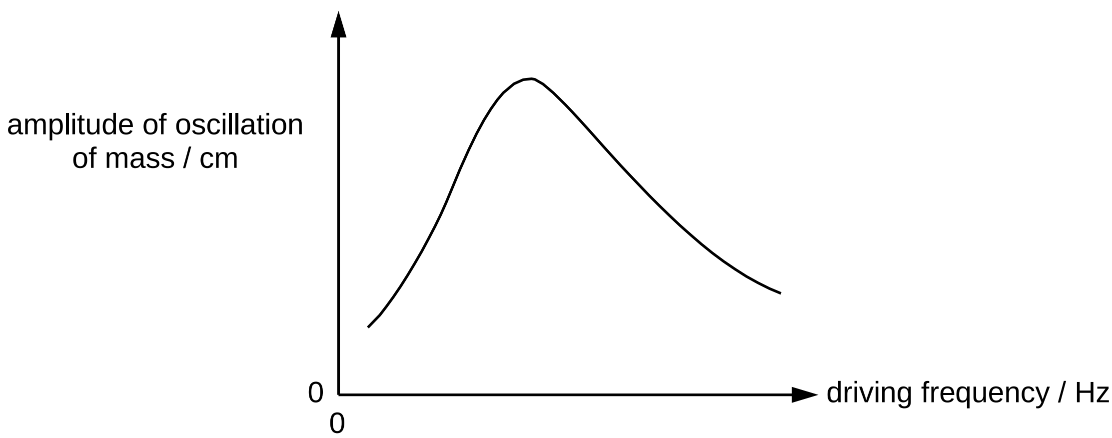


Fig. 3.4

(i) State how Fig. 3.4 shows that the mass is undergoing damped oscillations.

Peak is not infinitely high / not very sharp [B1]

.... [1]

(ii) The model in Fig. 3.3 and result in Fig. 3.4 can be applied to a practical situation.

Tall buildings, such as the Taipei 101 tower as shown in Fig. 3.5, are equipped with a damping system to reduce movement of the tower in strong winds. A mass in the form of a huge sphere on support cables oscillates with the tower to reduce the effect of the wind. The oscillations of the sphere are damped with oil-filled hydraulic pistons connected to the sphere.

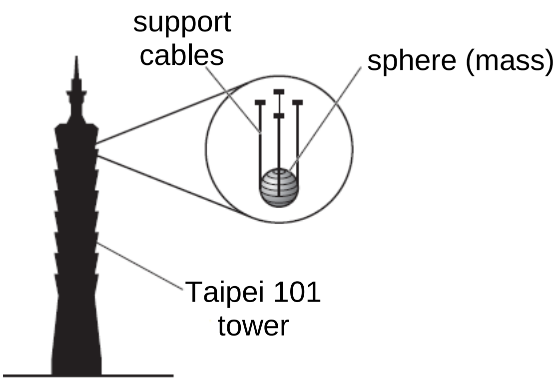


Fig. 3.5

Strong winds set the tower in oscillation. The natural frequency of oscillations of the tower is 0.15 Hz.

Suggest, by reference to energy transfer, why the frequency of the sphere-damping system must be close to 0.15 Hz to help reduce the motion of the tower.

Key points: 1) Recognise that when wind drives the tower, having similar natural frequencies allow maximum transfer of energy from tower to sphere, 2) so that this energy can be dissipated in the oil to reduce motion

- When frequency of sphere-damping system is equal/close to natural frequency of tower, sphere will resonate and there is <u>maximum</u> transfer of energy <u>from tower to sphere</u> [B1]
- where <u>oil/piston dissipates the energy</u> from the sphere to reduce the motion [B1]

[2]

[Total: 8]

4 Coherent light of wavelength 590 nm is incident normally on a double slit, as shown in Fig. 4.1.

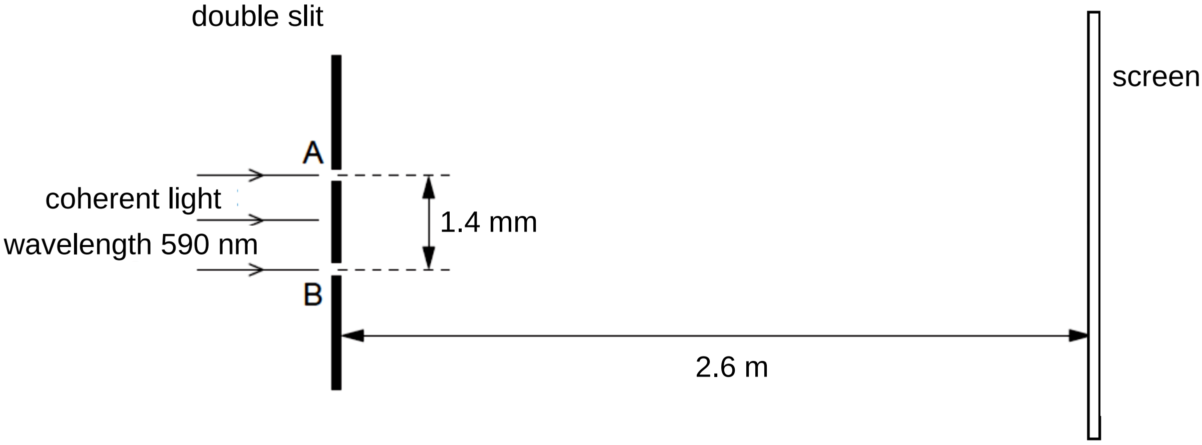


Fig. 4.1

The separation of the slits A and B is 1.4 mm and the width of each slit is 0.15mm. A screen is placed parallel to the slits at a distance 2.6 m away.

(a) One of the slits is covered.

Calculate the width of the central maximum formed on the screen by diffraction through the uncovered slit.

$$\sin\theta_{min} = \frac{\lambda}{b}$$
$$= \frac{5.90\times10^{-7}}{0.15\times10^{-3}}$$
$$= 0.00393 \text{ [M1]}$$
$$\theta_{min} = 0.225°$$

$$\tan 0.225° = \frac{\frac{1}{2}w}{2.6} \text{ [M1]}$$
$$w = 2(2.6\times\tan 0.225°)$$
$$= 0.0204 \text{ m [A1]}$$

width = ... cm [3]

(b) Now, both slits are uncovered.

(i) State Rayleigh's criterion.

Rayleigh's criterion says that the images of two-point sources can just <u>be distinguished</u>/ resolved [B1] from each other if the <u>central maximum of one</u> diffraction pattern <u>falls on the first minimum of the other</u> one. [B1] [2]

(ii) Use Rayleigh's criterion to explain whether the diffraction pattern produced by the two slits are seen as on the screen as separate.

Since the separation of the double slits is 1.4 mm, the central maxima of the two diffraction patterns will be 1.4 mm apart. [B1]

From (a) the first minimum of each pattern is at a distance of 10.2 mm away from the central maximum of the pattern. [B1]

Hence, the maximum of the first diffraction will fall within the 1st minimum of other diffraction pattern. The two patterns are not distinguished/ resolved and will not be seen as separate. [B1] [3]

[Total: 8]

5 Fig. 5.1 shows the arrangement of the apparatus used in the measurement of the magnetic flux density between the poles of a U-shaped magnet. The magnet rests on top of an electronic balance. A stiff rectangular frame measuring 8.0 cm by 5.0 cm carries a current I between the magnetic poles. The bottom edge of the frame is lowered into the region between the poles of the U-shaped magnet and is entirely within the uniform magnetic field.

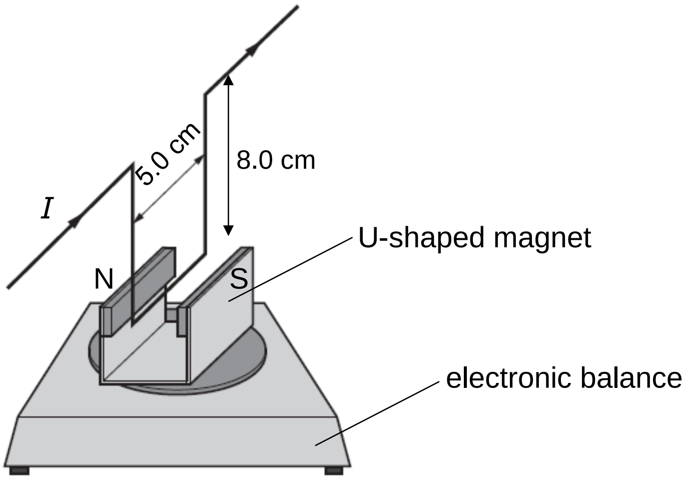


Fig. 5.1

Fig. 5.2 shows the apparatus when viewed from the front. The poles of the U-shaped magnet have been indicated.

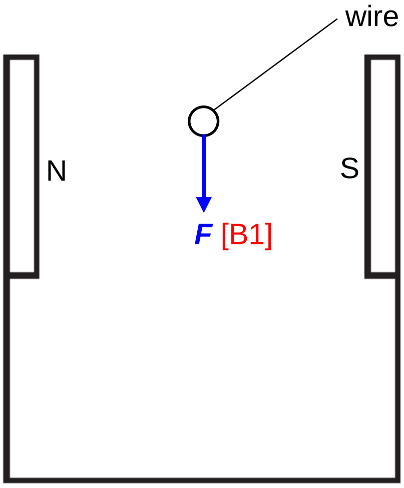


Fig. 5.2

(a) **(i)** Draw, on Fig. 5.2, a vertical force acting on the wire when current passes through it. Label the force *F*. [1]

(ii) When current passes through the wire, the balance reading changes.

Explain why there is a change in reading, and whether the change is an increase or decrease.

By Newton's 3rd Law, the interaction between wire and magnet causes an upward force on magnet [M1] which causes balance reading to decrease. [A1 – allow e.c.f. from (a)(i)] [2]

(b) The current I is varied and the change in the balance reading is recorded as shown in Fig. 5.3.

	Change in balance reading / g				
***I* / A**	**Trial 1**	**Trial 2**	**Trial 3**	**Mean change / g**	***F* / × 10^{-3} N**
0.5	0.08	0.05	0.06	0.06	0.6
1.0	0.14	0.16	0.16	0.15	1.5
1.5	0.22	0.20	0.23	0.22	2.2
2.0	0.31	0.29	0.31	0.30	2.9
2.5	0.38	0.39	0.34	0.37	3.6
3.0	0.44	0.48	0.48	0.47	4.6

Fig. 5.3

(i) Complete the table in Fig. 5.3. [1]

(ii) On Fig. 5.4, plot the missing data point from the table in Fig. 5.3 and draw a line of best fit. [1]

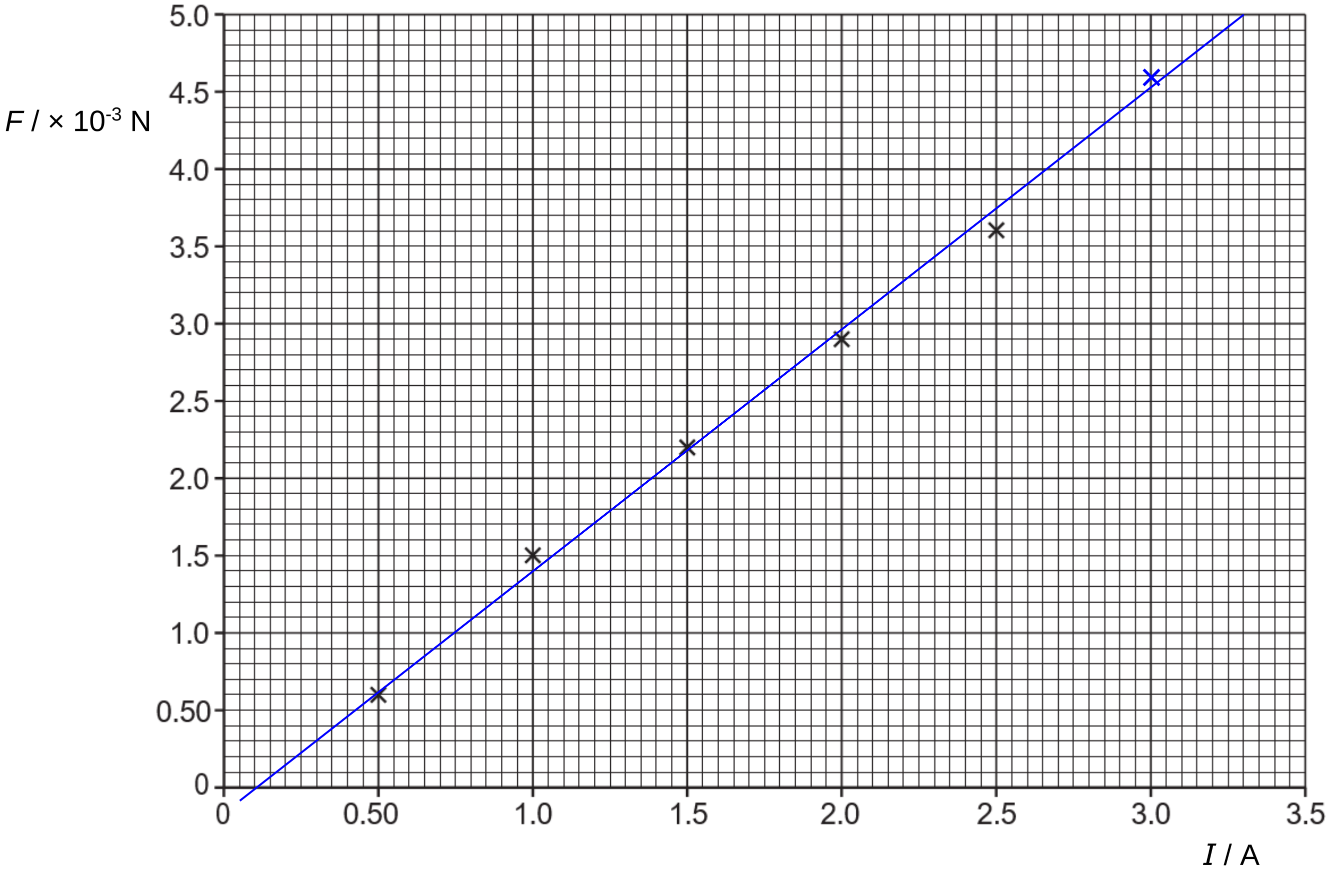


Fig. 5.4

(iii) Use your graph to determine the value of the magnetic flux density *B*, in mT, between the U-shaped magnet.

$$\text{gradient} = \frac{5.0\times10^{-3}-0}{3.3-0.10}$$
$$= 0.00156$$ [M1 - ignore POT error]

$$BL = 0.00156$$
$$B = \frac{0.00156}{0.050}$$
$$= 0.0312 \text{ T}$$
$$= 31 \text{ mT (acceptable range: 28 mT to 34 mT)}$$ [A1]

B = ... mT [2]

(iv) The value of the magnetic flux density in **(b)(iii)** is slightly inaccurate for a reason which has nothing to do with human error or meter inaccuracies.

Suggest one reason that might be the cause of this inaccuracy.

Effect of the Earth's magnetic field / Presence of external magnetic field [B1]

.... [1]

[Total: 8]

6 An a.c. power supply is connected to a resistor *R*, as shown in Fig. 6.1.

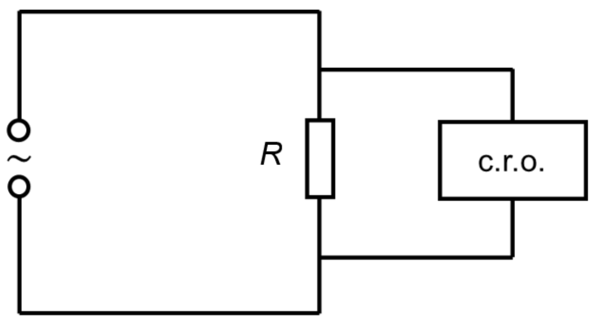


Fig. 6.1

A cathode ray oscilloscope (c.r.o.) is used to show the potential difference (p.d.) across *R*. The screen of the c.r.o. displays the variation with time of the p.d. across *R*, as shown in Fig. 6.2.

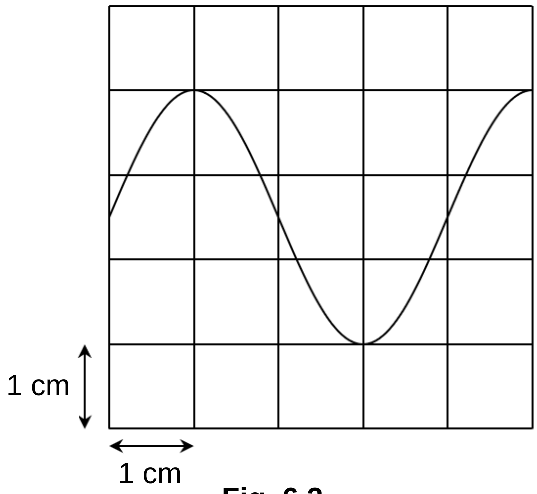


Fig. 6.2

(a) Explain, by reference to direct current, what is meant by the *root-mean-square* (r.m.s.) value of an alternating current.

The r.m.s. value of an alternating current is the value of the steady current that will dissipate heat at the same average rate as the alternating current in a resistive load [B1].

.... [2]

(b) The voltage V of the power supply is given by the expression

$$V = 6.0\sin(100\pi t)$$

V is measured in volts and the time t is measured in seconds.

Determine the Y-gain and time-base of the c.r.o.

12.0 V corresponds to 3 cm.

$12 \div 3 = 4.0$ V/cm [A1]

$\frac{2\pi}{T} = 100\pi$

$T = 20$ ms

20 ms corresponds to 4 cm.

$20 \div 4 = 5$ ms [A1]

Y-gain = ... V / cm

time-base = ... ms / cm

[2]

(c) A diode is then connected in series with the resistor *R*, as shown in Fig. 6.3.

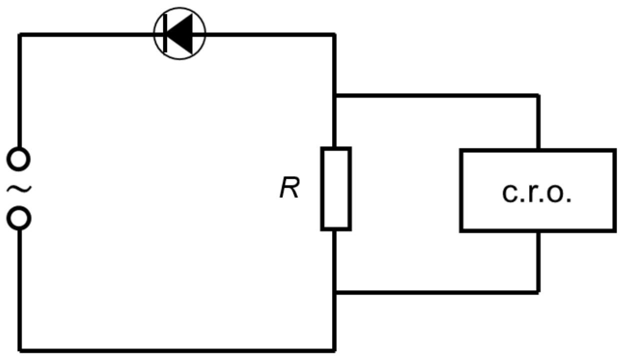
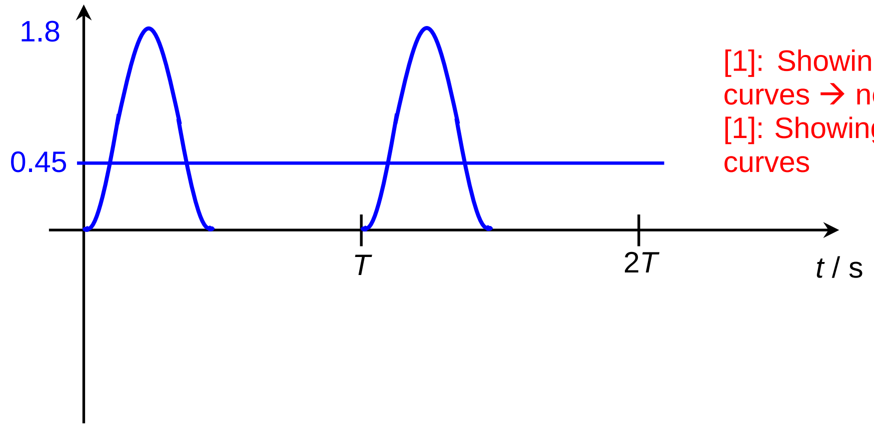


Fig. 6.3

(i) Sketch in Fig. 6.4, for 2 periods of the alternating p.d., the variation with time of the power *P* dissipated in *R* when *R* is 20 Ω, where *T* is the period. Indicate the peak power value in your sketch. [2]

$$P_{max} = \frac{V_0^2}{R} = \frac{6.0^2}{20} = 1.8 \text{ W}$$



[1]: Showing sinusoidal square curves → not McDonald curves
[1]: Showing half wave rectified curves

Fig. 6.4

(ii) Draw a line in Fig. 6.4 to represent the average power dissipated in *R*. Indicate its value on the y-axes. [1]

(iii) Calculate the root-mean-square current in *R*.

$$\langle P \rangle = I_{rms}{}^2 R$$
$$0.45 = I_{rms}{}^2 (20)$$
$$I_{rms} = 0.15 \text{ A}$$

root-mean-square current = ………………….. A [2]

[Total: 9]

7 Fig. 7.1 shows what is observed when a parallel beam of electrons, accelerated by a potential difference V, is incident on a fluorescent screen.

Fig. 7.1

A carbon film is then placed perpendicularly to the path of the electron beam as shown in Fig. 7.2.

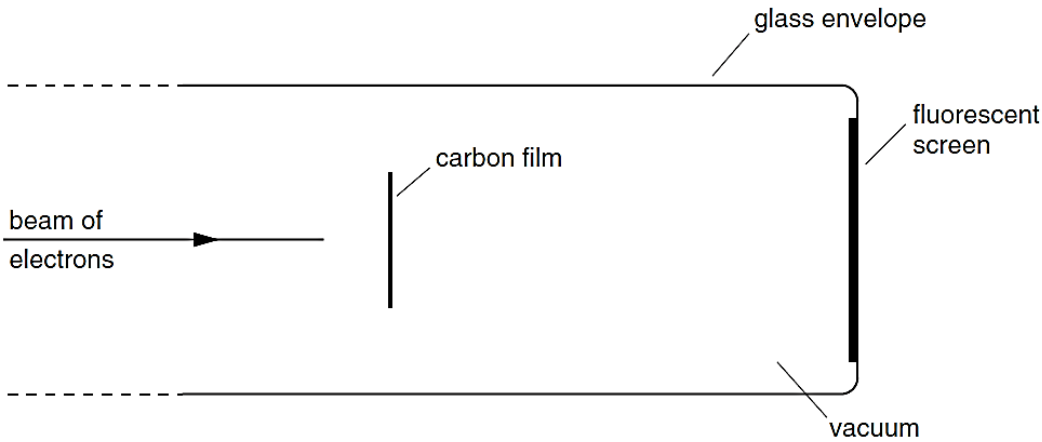


Fig. 7.2

The pattern observed on the screen is shown in Fig. 7.3.

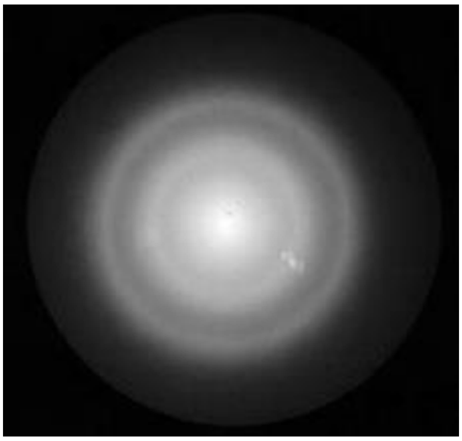


Fig. 7.3

(a) Identify and explain two key features in Fig. 7.3 that provide evidence for the wave nature of the electrons.

Pattern spreading/occupy over a large area demonstrates diffraction of the beam. [B1] (don't accept diffract w/o explanation)

Bright rings (with alternating with dark regions between them) indicates regions of constructive and destructive interference. [B1] (don't accept interference w/o explanation)

Since both interference & diffraction are behaviours exhibited by waves, hence the electrons which are usually treated as particles, have wave-like behaviours. [B1] .... [3]

(b) The electrons were accelerated through a potential difference of 2000 V.

(i) Calculate the momentum of an electron after being accelerated through a potential difference of 2000 V.

E_k of electron = $\dfrac{p^2}{2m}$

By conservation of energy,

Loss in electric potential energy = Gain in kinetic energy

$$2000 \times 1.60 \times 10^{-19} = \frac{p^2}{2(9.11\times10^{-31})} \text{ [M1]}$$

$$p = 2.41 \times 10^{-23} \text{ N s [A1]}$$

momentum = **......................** N s [2]

(ii) Hence, calculate the de Broglie wavelength of the electrons.

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{2.41\times10^{-23}} \text{ [C1]}$$

$$= 2.75\times10^{-11} \text{ m [A1]}$$

wavelength = **......................** m [2]

(c) State and explain the changes, if any, that is observed in the pattern on the screen when the following changes are made separately.

(i) The potential difference used to accelerate the electrons is increased.

With a larger potential difference used, the kinetic energy/speed of the electrons will become larger and hence momentum becomes larger. With a larger momentum, the de Broglie wavelengths of the electrons become smaller. [M1] The rings will now have a smaller diameter/ closer together. [A1] [2]

(ii) The current of the electron beam is increased.

A greater rate of electrons is incident on the fluorescent screen. [M1]

This will cause the bright rings to appear brighter/greater contrast. [A1]

[2]

[Total: 11]

Section B

Answer **one** question from this section in the spaces provided.

8 **(a)** Fig. 8.1 shows the variation of the gravitational potential ϕ with distance d from the surface of Pluto along a line joining the centers of Pluto and Charon.

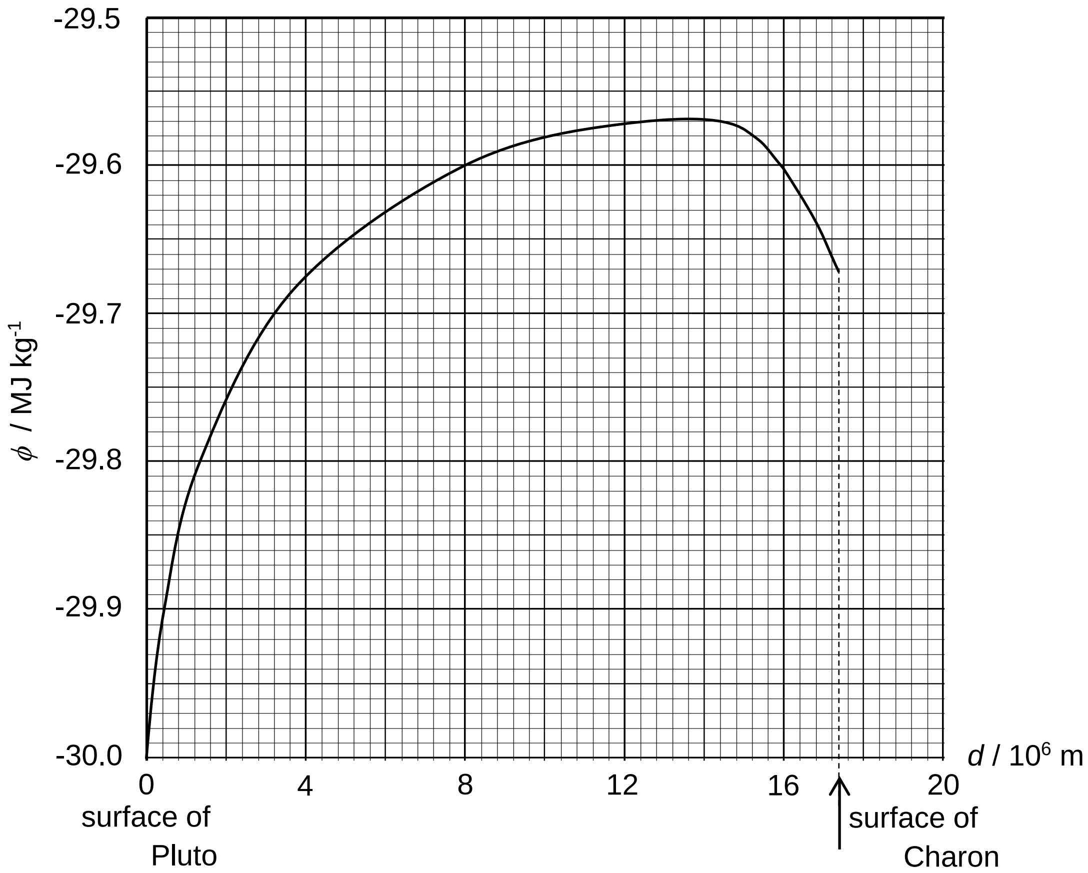


Fig. 8.1

The gravitational potential is taken as being zero at infinity.

(i) By considering the definition of gravitational potential, suggest why all values of ϕ in Fig. 8.1 are negative and explain how they may be obtained.

The direction of the external force to bring a (test) mass from infinity to a point in the field without a change in kinetic energy is opposite in direction to displacement. [B1]

Hence the work done per unit mass (which is the gravitational potential) is negative.

The resultant gravitational potential at that point is the (numerical) sum of the individual negative values of gravitational potential due to each planet. [B1] [2]

(ii) Explain how Fig. 8.1 can be used to determine the resultant gravitational force acting on an object with mass m placed at a point between Pluto and Charon.

The resultant gravitational force acting on an object at a point is the product of m [B1] and the (negative of the) gradient of the tangent at that point [B1] on the graph.

.... [2]

(iii) A lump of rock of mass 5.0 kg is ejected from the surface of Charon such that it travels towards Pluto.

1. Using data from Fig. 8.1, determine the minimum speed of the rock as it hits the surface of Pluto.

Loss in gravitational potential energy = gain in kinetic energy

$$m(\phi_i - \phi_f) = \frac{1}{2} m v_{minimum}^2 - 0 \quad \text{[M1]}$$

$$5.0\,(-29.57 - (-30.00)) \times 10^6 = \frac{1}{2}(5.0) \times v_{minimum}^2 \quad \text{[C1]}$$

$$v_{minimum} = 927 \text{ m s}^{-1} \quad \text{[A1]}$$

minimum speed = ... m s^{-1} [3]

2. The rock is now projected from Pluto to Charon.

State and explain how its minimum speed when it hits the surface of Charon will be different from the answer in **(a)(iii)1**.

The loss in gravitational potential energy from the maximum gravitational potential energy point to the surface of Charon is less than that to the surface of Pluto. Thus, the gain in kinetic energy of the rock on impact on Charon will be lesser [M1] and the minimum speed of impact on Charon will be smaller. [A1]

.... [2]

(b) In another isolated planetary system, X and Y are two stars of equal mass M and separated by a distance $2L$. The two stars rotate about their common centre of mass.

(i) State Newton's Law of Gravitation.

The gravitational force of attraction between any two point masses is directly proportional to the product of their masses and inversely proportional to the square of their separation. [B1] [1]

(ii) Explain why the two stars will not collide with each other even though the gravitation force acting on each other are attractive.

The gravitational force acts perpendicular to the motion of the star. [B1] Hence the gravitation force provides the centripetal force for circular motion. [B1]

.... [2]

(iii) Derive an expression, in terms of M, L and the gravitational constant G, for the period T of their rotation.

Gravitational force provides the centripetal force for circular motion.

$$\frac{GMM}{(2L)^2} = ML\left(\frac{2\pi}{T}\right)^2 \quad \text{[B1]}$$

$$T = \sqrt{\frac{16\pi^2 L^3}{GM}} \quad \text{[B1]}$$

[2]

(c) Fig. 8.2 shows three points A, B and C relative to the stars X and Y. All three points are equidistant from both X and Y and point B lies on the straight line joining the centres of mass of X and Y.

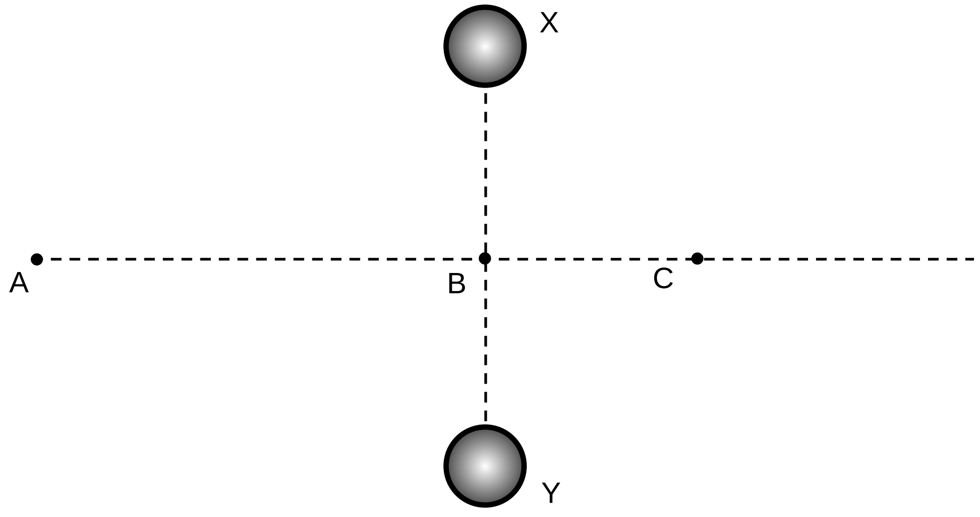


Fig. 8.2

(i) Draw arrows on Fig. 8.2 to represent the gravitational forces due to each star acting on a small mass placed separately at points A, B and C. The length of the arrow should be relative to their respective magnitudes. [2]

arrows directed towards centres of X and Y [B1]
shortest arrows for A and largest for B [B1]

(ii) Fig. 8.3 shows another schematic of X and Y. P is a point equidistant from X and Y and is at a distance *s* from the straight line joining the centers of mass of X and Y.

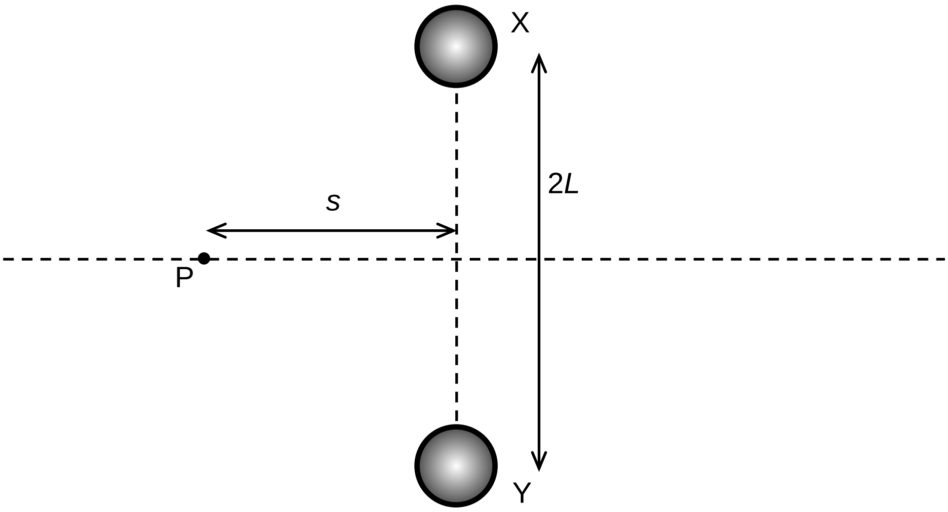


Fig. 8.3

1. Show that the magnitude of the resultant gravitational force acting on a mass *m* placed at P is

$$F_G = \frac{2GMms}{\left(s^2+L^2\right)^{\frac{3}{2}}}$$

Components of forces along XY cancels each other [B1]

$$F_G = 2\frac{GMm}{\left(\sqrt{L^2+s^2}\right)^2}\left(\frac{s}{\sqrt{L^2+s^2}}\right) \text{ [B1]}$$

$$= \frac{2GMms}{\left(s^2+L^2\right)^{\frac{3}{2}}} \text{ (shown)}$$

[2]

2. Hence, state and explain if the subsequent motion of a mass placed at rest at point P is simple harmonic.

From the above equation, it may be deduced that the acceleration of the mass is not directly proportional to its displacement from equilibrium *s* [M1], the subsequent motion of the mass is not simple harmonic [B1] [2]

9 **(a)** One assumption of the kinetic theory of gases is that the particles of the gas make perfectly elastic collisions among themselves and with the wall of the container. State two other assumptions of the kinetic theory of gases.

1. Any gas is made up of a large number of particles in constant and random motion.
2. The volume of each particle is negligible compared to the volume of the gas. This also means that any 2 particles are considered far apart (relative to their size).
3. The forces between the particles are negligible except during the time of collision. Having negligible forces between particles implies that the microscopic potential energy of the particles is constant and we can set it as 0 J.
4. The duration of collisions is negligible compared with the time interval between collisions.

(b) Consider a single molecule of mass *m* in a cuboidal container of internal side length L. The molecule is travelling with velocity *v* directly towards one of the walls W as shown in Fig 9.1.

Collisions between the molecule and the walls are elastic. This molecule makes multiple collisions in unit time. The pressure the molecule exerts on the wall is dependent on the frequency of the collisions with the wall.

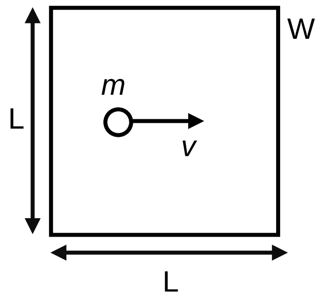


Fig. 9.1

Write expressions for

(i) the change in momentum of the molecule when it collides with wall W.

change in momentum $= p_f - p_i = -(mv) - (+mv) = -2mv$ [1]

(ii) time between collisions of the molecule and the wall W.

time between collisions $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{2L}{v}$ [1]

(iii) the frequency of the collisions of the molecule with the wall W.

frequency of collisions $= \dfrac{1}{t} = \dfrac{v}{2L}$ [1]

(iv) the average momentum change per unit time for this molecule.

total momentum change $= f(\Delta p) = \dfrac{v}{2L}(-2mv) = -\dfrac{mv^2}{L}$ [1]

(v) the average force on wall W as a result of impacts by the molecule.

from (iv) and according to Newton's second law, the average force on the molecule $= \dfrac{mv^2}{L}$ [B1]

according to Newton's Third Law, average force on wall $= \dfrac{mv^2}{L}$ [B1]

[2]

(vi) the average pressure of wall W.

average pressure on wall $= \dfrac{\text{average force on wall}}{L^2} = \dfrac{mv^2}{L^3}$ [1]

(c) A cuboidal container contains *N* molecules of an ideal gas has volume V and pressure *p* given by the relation

$$p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$$

where *m* is the mass of each molecule.

Explain briefly how the expression in **(b)(vi)** leads to this relation.

The volume of the container V is equal to L^3. [B1]

The equation was assuming every molecule was moving horizontally.

In a sample of gas of *N* molecules, each molecule could be moving randomly in any three directions within the container so that its actual velocity c is given by

$c^2 = c_x{}^2 + c_y{}^2 + c_z{}^2$

The molecules are moving with a range of velocities, $\langle c^2 \rangle$ is the mean square speed of the molecules, so $\langle c^2 \rangle = \langle c_x{}^2 \rangle + \langle c_y{}^2 \rangle + \langle c_z{}^2 \rangle$ [B1]

Since the number of particles is large and they are in random motion, the particles have an equal chance of being in any direction $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle = \frac{1}{3}\langle c^2 \rangle$ [B1]

The expression in **(b)(vi)** for average pressure due to one molecule moving in one direction is modified to be an expression for the average pressure due to N molecules in all directions.

average pressure on wall $= N\dfrac{m}{V}\left(\dfrac{1}{3}\langle c^2 \rangle\right) = \dfrac{1}{3}\dfrac{Nm\langle c^2 \rangle}{V}$ [B1] [4]

(d) (i) Use the equation of state for an ideal gas, together with the equation given in **(c)** to show that K, the average translational kinetic energy of a molecule is proportional to its absolute temperature [3]

$$pV = nRT$$

from (c) $$pV = \frac{1}{3}Nm\langle c^2\rangle$$

$$nRT = \frac{1}{3}Nm\langle c^2\rangle$$ [M1]

$$= \frac{1}{3}nN_A m\langle c^2\rangle$$

$$3RT = N_A m\langle c^2\rangle$$

$$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}\frac{R}{N_A}T$$ [M1]

$$pV = NkT$$

from (c) $$pV = \frac{1}{3}Nm\langle c^2\rangle$$

$$NkT = \frac{1}{3}Nm\langle c^2\rangle$$ [M1]

$$kT = \frac{1}{3}m\langle c^2\rangle$$

$$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$ [M1]

Since R and N_A are constants / Since k is constant [B1], the kinetic energy of a molecule is proportional to its absolute temperature. [A0]

(ii) Calculate , for oxygen and hydrogen at the same temperature, the ratio

$$\frac{\text{root mean square speed of a hydrogen molecule}}{\text{root mean square speed of an oxygen molecule}}.$$

The masses of the molecules are given the following table:

Mass of a hydrogen molecule = 3.34×10^{-27} kg

Mass of an oxygen molecule = 5.34×10^{-26} kg

Kinetic energy of a molecule is proportional to its absolute temperature.

So when temperature is the same, mean square speed is inversely proportional to mass

$$\frac{v_H^2}{v_O^2} = \frac{m_O}{m_H}$$

$$\frac{v_H}{v_O} = \sqrt{\frac{m_O}{m_H}} = \sqrt{\frac{5.34\times10^{-26}}{3.34\times10^{-27}}}$$ [M1]

$$= 3.98$$ [A1]

ratio = = ... [2]

(iii) Suggest why the atmosphere of the Earth contains very little hydrogen.

Hydrogen molecules have higher speed [B1]

So a greater proportion/fraction of hydrogen molecules have a speed greater than escape speed [B1]

.... [2]

[Total: 20]

End of Paper